Effect of Sampling and Testing Methods on Determination of Fibre Length in Jute Slivers and Yarns

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The expected frequency distributions of fibre length in jute slivers and yarns for different methods of sample preparation have been deduced analytically. For each such distribution, the expected average length of fibres obtained on the basis of number and weight of fibres has been estimated. Some test results are presented to show their extent of agreement with the values estimated analytically. The causes for discrepancies between experimental values and those expected from analytical relations are discussed. For routine testing of fibre length, the need for standardizing the methods of sampling and testing has been stressed.

The determination of fibre length in slivers at different stages of preparation is necessary for adjusting the 'reach', i.e. distance between the front and back rollers of the drawing and spinning frames. Since the fibres in jute slivers are produced by the carding action applied to raw jute reeds in the carding machine, the average length of fibres obtained after carding is expected to vary with the carding action as well as with the softening treatment given to the raw jute prior to carding. The measurement of fibre length in slivers is, therefore, important for optimizing the carding action by adjustment of machine settings and the softening treatment on raw jute by modifying the formulation and application of softening emulsion.

There may, however, be differences in the methods of sampling and testing followed for determining the distribution and average length of fibres in slivers. As a result, the average length obtained by different methods may vary appreciably. It, therefore, seems important to examine in detail the effect of sampling and testing methods on the distribution and average value of fibre length in slivers and yarns and the relationship between the results obtained by different methods, because when such a relationship is known, the results of one method can be converted into those of another, depending on the requirement in practice.

The present paper deals with the expected variation in the distribution and average value of fibre length in jute slivers and yarns arising from the difference in the methods of sampling and testing adopted.

Methods of Sampling and Testing
The methods employed for sampling fibres from a sliver or yarn may be either end-biased or length-biased. In end-biased sampling, the fibres whose leading or trailing ends lie over a short length of the sliver are taken, while in length-biased sampling, the fibres which cross a given section of the sliver are selected for the test.

After the sampling is done, the fibres are divided into different groups according to their lengths. The average length and distribution of length of fibres of all the groups may then be obtained on the basis of either number or weight of the fibres in each group. Accordingly, the results obtained may be termed number-based or weight-based values.

Thus, there may be basically four different results for average length and distribution depending on the two methods of sampling and the two methods of testing.

Basic Length Distribution
When the meshy structure of raw jute reeds is broken down randomly into fibrous elements in a carding machine, the frequency of such elements by number can be expected to decrease exponentially with increase in their length. The relationship between the frequency (by number) and length has been derived for flax by Spencer-Smith and Todd, which should be applicable for jute also, since both flax and jute are bast fibres consisting of a meshy structure of fibrous elements separated from one another by mechanical processes of splitting and rupturing. The derivation of the relationship is given below.

Assuming that the weak points where the filaments in a raw jute reed are likely to rupture in the carding process are distributed at random along the length of the reed and that the average number of such points per unit length of reed is \( m \), the expected average length \( l \) of the filaments obtained after carding is \( \frac{1}{m} \), which is small compared to the length of reed. In segments of length \( l \) of the reed, there are on an average \( ml \) number of such
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points, but their distribution in different segments can be expected to follow the Poisson law, according to which the probability of occurrence of \( n \) such points in a segment is given by

\[
P(n) = \frac{1}{n!} (ml)^n \exp(-ml) \quad \ldots (1)
\]

From Eq.(1), the probability of a segment having zero, i.e. no weak points at all, which is also the probability of the filaments obtained after carding being longer than \( l \) is obtained by putting \( n = 0 \), as

\[
P(0) = \exp(-ml) \quad \ldots (2)
\]

Eq.(2), therefore, gives the cumulative relative frequency by number of the fibres which are longer than \( l \). The relative frequency \( f(l) \) of fibres of length \( l \) by number is, therefore, obtained by differentiating Eq.(2)

\[
f(l) = -\frac{d}{dl} \left[ \exp(-ml) \right] \quad \ldots (3)
\]

\[
f(l) = ml \exp(-ml) \quad \ldots (4)
\]

Since \( I = -\frac{1}{m} \) or \( m = \frac{1}{I} \)

\[
f(l) = \frac{1}{I} \exp(-l/I) \quad \ldots (5)
\]

Eq.(5), therefore, gives the basic length distribution of fibres by number expected to be found in a sliver at the delivery of the carding machines. Some further rupturing of the filaments takes place in the subsequent drawing and drafting processes, but the basic relationship for length distribution is expected to hold good\(^1\) for slivers at all the processing stages as well as for the yarns, although there is likely to be a gradual diminution of the average filament length with the progress of processing operations because of further rupture of the filaments. The basic length distribution by number given by Eq.(5) is represented graphically in Fig. 1(A) assuming the average length \( I \) to be 5 cm.

Effect of Sampling and Testing Methods on Distribution and Average Length of Fibres

End-biased number-based method—The fibres selected in end-biased sampling are expected to be of length distribution (by number) similar to the basic type given by Eq.(5). So, the frequency of fibres of length \( l \), denoted as \( f_{EN}(l) \) and the average length, denoted as \( I_{EN} \), in this method may be written as

\[
f_{EN}(l) = \frac{1}{I} \exp(-l/I) \quad \ldots (6)
\]

\[
I_{EN} = I \quad \ldots (7)
\]

Length-biased number-based method—In length-biased sampling, the probability of a fibre being included in the sample increases as the length of the fibre increases. So, the frequency of fibres of length \( l \), denoted as \( f_{LN}(l) \), is related to the basic distribution pattern in Eq.(5) as

\[
f_{LN}(l) = k.l.f(l) = k.l \exp(-l/I) \quad \ldots (8)
\]

where \( k \) is a constant.

Integrating Eq.(8) between \( l = 0 \) and \( l = l_m \), the total number of fibres is found to be

\[
N = k.I \quad \ldots (9)
\]

assuming \( l_m \) to be large.

Taking \( N = 1, k = 1/I \)

Eq.(8) reduces to

\[
f_{LN}(l) = \frac{l}{I} \exp(-l/I) \quad \ldots (11)
\]

represented in Fig. 1(B) taking \( I = 5 \) cm.

![Fig. 1](image-url) — Theoretical length distribution of fibres in slivers and yarns [(A) end-biased number-based method; (B) length-biased number-based method; (C) end-biased weight-based method; and (D) length-biased weight-based method)
The average length, denoted as $\bar{L}_{LN}$, in this method is obtained as

$$\bar{L}_{LN} = \int_0^{l_m} l \cdot f_{LN}(l) \, dl$$

from Eq.(11)

$$= \frac{1}{l_0} \int_0^{l_m} l^2 \exp(-l/l_0) \, dl$$

$$\approx 2T$$  \hspace{1cm} \text{(12)}

Thus, the average length obtained in this method is about double that obtained in the end-biased number-based method or the population number-based average value.

**End-biased weight-based method**—The end-biased weight-based distribution can be obtained from the end-biased number-based one from the following considerations.

The weight frequency $f_{EW}(l)$ of fibres of length $l$ is related to their number frequency $f_{EN}(l)$ as

$$f_{EW}(l) = f_{EN}(l) \cdot l \cdot c$$  \hspace{1cm} \text{(13)}

since the weight is the product of number $f_{EN}(l)$, length $l$ and weight per unit length $c$ of the fibres. But in the case of jute fibres in slivers and yarns, the linear density $c$ of the fibres is found to increase almost linearly with increase in the length of fibres$^{2,3}$. So, $c$ may be written as

$$c = c' \cdot l$$  \hspace{1cm} \text{(14)}

where $c'$ is a constant.

From Eqs.(13) and (14)

$$f_{EW}(l) = f_{EN}(l) \cdot l^2 \cdot c'$$  \hspace{1cm} \text{(15)}

Using Eq.(6)

$$f_{EW}(l) = \frac{1}{l^2} \exp\left(-l/l_0 \cdot l^2 \cdot c'\right)$$  \hspace{1cm} \text{(16)}

Integrating Eq.(16) between $l = 0$ and $l = l_m$, the total weight $W$ of the fibres sampled is given by

$$W = 2T \cdot c'$$  \hspace{1cm} \text{(17)}

Taking $W = 1$, $c' = \frac{1}{6T^2}$ as

$$c = \frac{1}{2T^2}$$  \hspace{1cm} \text{(18)}

From Eqs.(16) and (18), the end-biased weight-based distribution of length is obtained as

$$f_{EW}(l) = \frac{l^2}{2T^3} \cdot \exp\left(-l/l_0\right)$$  \hspace{1cm} \text{(19)}

The distribution given by Eq.(19) is shown in Fig. 1(C) taking $T = 5$ cm. The average length $\bar{L}_{EW}$ of fibres obtained in the end-biased weight-based method is

$$\bar{L}_{EW} = \int_0^{l_m} l \cdot f_{EW}(l) \, dl$$

$$= \frac{1}{2T^3} \int_0^{l_m} l^3 \cdot \exp\left(-l/l_0\right) \, dl$$

$$\approx 3T$$  \hspace{1cm} \text{(20)}

Thus, the average length obtained in this method is about three times greater than that obtained in the end-biased number-based method, i.e. the population number-based average value.

**Length-biased weight-based method**—Since the weight of fibres of length $l$ is given by the product of their number, length and weight per unit length, following the steps as in Eqs.(13), (14) and (15), the frequency $f_{LW}(l)$ can be obtained

$$f_{LW}(l) = f_{LN}(l) \cdot l^2 \cdot c'$$  \hspace{1cm} \text{(23)}

$$= \frac{l}{7T^2} \exp\left(-l/l_0\right) \cdot l^2 \cdot c'$$  from Eqs.(11) and (23)

$$= c' \cdot \frac{l^3}{7T^2} \exp\left(-l/l_0\right)$$  \hspace{1cm} \text{(24)}

Integrating Eq.(24) between $l = 0$ and $l = l_m$, the total weight $W$ of the fibres sampled is given by

$$W = c' \cdot 6T^2$$  \hspace{1cm} \text{(25)}

Taking $W = 1$, $c' = \frac{1}{6T^2}$ as

$$c = \frac{1}{2T^2}$$  \hspace{1cm} \text{(26)}

From Eqs.(24) and (26)

$$f_{LW}(l) = \frac{1}{6T^2} \cdot \frac{l^3}{7T^2} \exp\left(-l/l_0\right)$$  \hspace{1cm} \text{(27)}

The variation of $f_{LW}(l)$ with $l$ is shown in Fig. 1(D). The average fibre length, $\bar{L}_{LW}$, obtained in this method is

$$\bar{L}_{LW} = \int_0^{l_m} l \cdot \frac{l^3}{6T^2} \exp\left(-l/l_0\right) \, dl$$

$$= \frac{1}{6T^2} \int_0^{l_m} l^4 \exp\left(-l/l_0\right) \, dl$$  \hspace{1cm} \text{(28)}

$$\approx 4T$$  \hspace{1cm} \text{(29)}

Thus, the average length obtained in this method is expected to be four times that obtained in end-biased
Results

The results derived are summarized in Table 1.

Using the results given in Table 1, the frequency distribution and average value of length obtained in one method can be converted into those expected in another method. For example, if the average length in method 3 is found to be 10 cm, then that in method 4 will be \( \frac{3}{2} \times 10 = 15 \) cm.

The distribution of fibre length in slivers from different stages of processing, namely from finisher card to finisher drawing, as well as in the corresponding yarns of hessian warp quality has been studied following both the length-biased number-based and length-biased weight-based methods. In carrying out the tests, pieces of slivers and yarns of about 1.2 m length were cut from different parts of each sample. The fibres at the central region of these cut pieces were coloured red over a band of width about 0.5 cm. In the case of yarn samples, the central regions of the cut pieces were untwisted before colouring was done.

The fibres having coloured marks were carefully separated from the cut pieces (in the case of yarns, this was done after untwisting the pieces). Certain fibres were found to have more than one coloured marks because of their lying in a folded form in the specimen. Such fibres and also those obtained from under-redded root parts of raw jute (and so having much higher weight per unit length) were excluded from the tests.

After the fibres were grouped in accordance with their lengths, the number of fibres and their weight in each group were determined. From these values, the frequency distributions of length on number and weight basis were obtained. The results are presented in Figs. 2(A-J).

The average values of length on number and weight basis, obtained from the length distribution for different samples, are given in Table 2, which includes also the ratio of average length obtained on weight basis to that obtained on number basis.

It is seen from Fig. 2 that the actual length distribution relationships based on number and weight of fibres are more or less similar to those obtained theoretically and represented by Eqs.(11) and (27), and Figs 1(B) and 1(D) respectively.

But the ratio of the weight-based average length to the number-based average length is slightly less than 2, the value expected from the theoretical relationships (3) and (5) in Table 1. The actual value is found to be around 1.8. This discrepancy, though not large, may arise from the fact that in the theoretical derivations

<table>
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<tr>
<th>Method followed</th>
<th>Relationship for frequency distribution</th>
<th>Average length</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic population, number-based</td>
<td>( f(l) = \frac{1}{l} \exp (-l/l) )</td>
<td>( T )</td>
</tr>
<tr>
<td>End-biased sampling, number-based</td>
<td>( f_{\text{EN}}(l) = \frac{1}{l} \exp (-l/l) )</td>
<td>( T )</td>
</tr>
<tr>
<td>Length-biased sampling, number-based</td>
<td>( f_{\text{LN}}(l) = \frac{l}{l^2} \exp (-l/l) )</td>
<td>( 2T )</td>
</tr>
<tr>
<td>End-biased sampling, weight-based</td>
<td>( f_{\text{EW}}(l) = \frac{l^3}{2l^2} \exp (-l/l) )</td>
<td>( 3T )</td>
</tr>
<tr>
<td>Length-biased sampling, weight-based</td>
<td>( f_{\text{W}}(l) = \frac{l^3}{6l^2} \exp (-l/l) )</td>
<td>( 4T )</td>
</tr>
</tbody>
</table>
the maximum fibre length $l_m$ was assumed to be large, while in practice it has a value limited to 30-40 cm only.

Regarding changes in the average length of fibres both on the number and weight basis there is a gradual diminution with the progress of the processing operation, which is in accordance with the expectation, because some additional ruptures of fibres can take place at each stage of processing. It was also found to be so in the work of Bandyopadhyay and Sil.

In regard to the distribution patterns, however, there is hardly any change with the progress of the processing operation, which should be so from theoretical consideration also.

Beard Method of Tests

Besides the methods discussed above, there is yet another method of determining the average fibre length in a sliver or yarn, the beard method. In this method, pieces of slivers or yarns of about 1.2 m length are held at the centre in a suitable clamp. The loose fibres from the two ends of the pieces not held at the clamp are removed carefully by gentle combing (in the case of yarns, the pieces are untwisted before combing). The two tufts on the two sides of the clamp are then cut along the edges of the clamp and their weights taken. The fibres held in the clamp are weighed separately. The average length is determined from the following relationship

$$T_b = \frac{(W_T + W_C)b}{W_C}$$

where $T_b$ is the average length in cm obtained by the 'beard' method; $W_T$, weight of the two tufts in g; $W_C$, weight of fibres held in the clamp in g; and $b$, the width of the clamp in cm.

It may appear that the average length obtained in the ‘beard’ method should be the same as that obtained in the length-biased weight-based method. But actually these two values are not exactly the same. In both the length-biased weight-based method and beard method respectively, where $W_i$ denotes the weight of fibre of length $l_i$.

Some values of average length obtained in the beard method on sliver samples at different stages are given in Table 3 from which it is seen the the values obtained by the beard method are slightly lower than even the number-based values.

It may also be noted that the values obtained in the beard method have an increasing trend with the progress of the processing operation, which is contrary to the trend in the other methods, namely length-biased number-based and length-biased weight-based methods. The reason for this discrepancy in the beard method may be that while combing the fibres not held at the clamp, the additional rupture of fibres may be much more in the slivers at the early stage of processing compared with the slivers at the late stage of processing, because the entanglement and disorientation of fibres are greater for the sliver at the early stage compared with that at the late stage of processing. As a result, the weight of the tufts obtained after combing will be relatively low for the sliver at the early stage and hence the length value will also be accordingly short.

Conclusion

Depending on the method of sampling and testing both the distribution patterns and the average values of fibre length in jute slivers and yarn may vary appreciably; the variation of the latter may be as high as 1:4. The standardization of the methods is, therefore, considered necessary.

References