Static analysis of thick skew laminated composite plate with elliptical cutout

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The behaviour of a laminated composite skew plate with elliptical hole subjected to transverse pressure loading has been investigated in the present analysis. A finite element method which works on the basis of three-dimensional theory of elasticity is employed to evaluate the transverse deflection, in-plane stresses and interlaminar stresses. The results obtained by varying the skew angle, size of the cutout and the effect of elliptical cutout orientation are discussed. The magnitudes of the transverse deflection and in-plane stresses for pressure loading are observed to be less at higher skew angles and for larger size of the cutout. It is also observed that the configuration 3 of the ellipse (major axis of the ellipse parallel to inclined sides of the skew plate) is found to be better with respect to the major stresses point of view. The solutions of skew structures considered in the present analysis may be useful for the construction of safe and efficient structures like skew bridges and swept wings of aircraft structures.

Keywords: FEM, Skew laminate, Cutout, Interlaminar stresses

The increasing use of fibre reinforced laminates in space vehicles, aircrafts, automobiles, ships and chemical vessels have necessitated the rational analysis of structures for their mechanical response. In addition, the anisotropy and non-homogeneity and larger ratio of longitudinal to transverse moduli of these new materials demand improvement in the existing analytical tools. As a result, the analysis of laminated composite structures has attracted many research workers, and has been considerably improved to achieve realistic results. In the design of modern high-speed aircraft and missile structures, swept wing and tail surfaces are extensively employed. Moreover, some of the structural elements are provided with cutouts of different shapes to meet the functional requirements like (i) for the passage of various cables, (ii) for undertaking maintenance work and (iii) for fitting auxiliary equipment. Depending upon the nature of application, these structural elements are acted upon by mechanical and thermal loads of varied nature. Usually, the anisotropy in laminated composite structures causes complicated responses under different loading conditions by creating complex couplings between extension, bending, and shear deformation modes. To capture the full mechanical behaviour, it must be described by three-dimensional elasticity theories.

In solving the three-dimensional elasticity equations of rectangular plates, quite a number of solution approaches have been proposed. Wu and Kuo\textsuperscript{1} developed an inter-laminar stress mixed finite element method, based on local high order lamination theory wherein the local displacement fields are expanded in terms of high order polynomial series through thickness within each ply and the displacement continuity requirements at the interface between layers are regarded as the constraints introduced into the formulation.

Lee et al.\textsuperscript{2} studied the behaviour of simply supported rectangular symmetric cross-ply laminated composite plates subjected to bidirectional bending using an improved zig-zag displacement model. The zig-zag model is based upon a layer wise cubic variation of the in-plane displacements and a parabolic variation of the transverse shear stresses with zero values at the free surfaces. Zhang and Zhang\textsuperscript{3} presented a new concise procedure for obtaining the static exact solution of composite laminates with piezo-thermo-elastic layers under cylindrical bending using the basic coupled thermo-electro-elastic differential equations. Setoodeh and Karami\textsuperscript{4} employed a three-dimensional elasticity based layer-wise finite element method (FEM) to study the static, free vibration and buckling responses of general laminated thick composite plates. Rastgaar et al.\textsuperscript{5} evaluated deformations of a laminated
composite plate due to mechanical loads. Third order shear deformation theory of plates, which is categorized in equivalent single layer theories, is used to derive linear dynamic equations of a rectangular multi-layered composite plate. Kong and Cheung⁶ proposed a displacement-based, three-dimensional finite element scheme for analyzing thick laminated plates by treating the plate as a three-dimensional inhomogeneous anisotropic elastic body. Prasad and Shuart⁷ presented a closed form solution for the moment distributions around holes in symmetric laminates subjected to bending moments. Ukadaonker et al.⁸ gave a general solution for bending of symmetric laminates with holes. Pradhan and Panda⁹ investigated the influence of ply lay up and the interaction of residual thermal stresses and mechanical loading on the interlaminar asymmetric embedded delamination crack growth behaviour. Karami et al.¹⁰ has applied differential quadrature method (DQM) for static, free vibration, and stability analysis of skewed and trapezoidal composite thin plates without hole. From the review of available literature it is observed that the static analysis of skew plates with cutouts using elasticity theory has not been studied. The behaviour of a laminate with skew edges and having various types of cutouts is different from the one without skew edges and/or cutouts. So, it is necessary to analyse this kind of problem using elasticity theory based finite element method to evaluate for the most accurate behaviour of thick laminated skew plates with cutouts.

The present work aims at filling of the knowledge gaps in the existing literature. The research problem deals with the static analysis of thick skew laminated plate with elliptical cutout by elasticity theory based on finite element method.

**Problem Modeling**

**Geometric modeling**

Figure 1 shows the in-plane dimensions of the laminate considered for the present analysis. The dimensions for ‘l’ and ‘b’ are taken as 20 mm. d is the length of the minor axis of the ellipse. Major axis of the ellipse is taken as twice the length of the minor axis.

The value of d is determined from the ratio of d/l which is varied from 0.1 to 0.4, and the skew angle α is varied from 0° to 50°, the thickness of the plate is fixed from the length to thickness ratio l/h (s = 10). The individual layers are arranged so that the total thickness of the layers oriented in x-direction (θ = 0°) is equal to the total thickness of the layers oriented in y-direction (θ = 90°). Effect of orientation of the elliptical cutout in the skew plate is analyzed for five different positions.

**Configuration 1:** Major axis of the ellipse parallel to the horizontal side of the skew plate (Fig. 2).

**Configuration 2:** Major axis of the ellipse collinear with the longer diagonal of the skew plate (Fig. 3).
Configuration 3: Major axis of the ellipse parallel to the inclined side of the skew plate (Fig. 4).

Configuration 4: Major axis of ellipse perpendicular to the horizontal side of the skew plate (Fig. 5).

Configuration 5: Major axis of the ellipse collinear with the shorter diagonal of the skew plate (Fig. 6).

Finite element modeling

The finite element mesh is generated using a three-dimensional brick element ‘SOLID 95’ of ANSYS. This element (Fig. 7) is a structural solid element designed based on three-dimensional elasticity theory and is used to model thick orthotropic solids. The element is defined by 20 nodes having three degrees of freedom per node: translations in the nodal x-, y-, and z-directions. The element may have any spatial orientation.

Boundary conditions

All the edges of the skew plate are clamped, i.e., all the three degrees of freedom (displacements in global x-, y- and z-directions) of the nodes attached to the side faces of the plate are constrained.

Loading

A transverse pressure of 1MPa is applied on the top surface of the plate.

Material properties (graphite-epoxy)

\[ E_1 = 172.72 \, \text{GPa}, \quad E_2 = E_3 = 6.909 \, \text{GPa} \]
\[ G_{12} = G_{13} = 3.45 \, \text{GPa}, \quad G_{23} = 1.38 \, \text{GPa}, \quad \nu_{12} = \nu_{13} = \nu_{23} = 0.25 \]

Validity of the Present Analysis

To validate the finite element results, a square plate with simply supported edges and subjected to a sinusoidal load of \( p = p_0 \sin \left( \frac{\pi x}{a} \right) \sin \left( \frac{\pi y}{b} \right) \), where \( a \) and \( b \) are the length and width of the plate, is modeled with SOLID95 element. The results obtained from this model are compared with the exact elasticity solution for various lengths to thickness ratios of the plate (Table 1). It is observed that the finite element results are in close agreement with the exact elasticity solution.

In the present work the transverse deflection and stresses (including the inter-laminar stresses at the free edge of the elliptical cutout for five different orientations) of a clamped skew laminated plate with elliptical cutout at the centre of the plate and...
subjected to a transverse pressure loading is evaluated by varying the size of the elliptical cutout, skew angle and the effect of orientation of elliptical cutout in the skew plate.

Results and Discussion

Numerical results are obtained for pressure loading case. Variation of the stresses and deflection with respect to the skew angle ($\alpha$), the ratio of length of the minor axis of the elliptical cutout to the side length of the plate ($d/l$) is shown in Fig. 8-16. The following observations are made.

Effect of skew angle

The in-plane normal stresses $\sigma_x$ and $\sigma_y$ decrease with the increase in skew angle. The increase in skew angle increases the length of the longer diagonal and decreases the length of the shorter diagonal of the skew plate. The first factor (increase in the length of the longer diagonal) increases the flexibility of the plate whereas the second factor (decrease in the length of the shorter diagonal) increases the stiffness of the plate. The reduction in the stresses $\sigma_x$ and $\sigma_y$ is due to the domination of stiffness effect. There is no significant variation of the in-plane shear stress $\tau_{xy}$ with respect to the variation of the skew angle (Fig. 8).

The inter-laminar stresses at the free edge of the elliptical cutout $\sigma_z$ and $\tau_{yz}$ increase with increase in skew angle up to $\alpha = 40^\circ$ and then decreases (Fig. 9). There is no significant variation of the shear stress $\tau_{xz}$ with increase in $\alpha$. However, the variation is very small when compared to the magnitude of the in-plane stresses.

The transverse deflection ‘$w$’ decreases with increase in skew angle $\alpha$ (Fig. 10). The reduction in ‘$w$’ with respect to $\alpha$ may be due to the domination of the stiffness factor.

<table>
<thead>
<tr>
<th>$S = l/h$</th>
<th>Normalized $\sigma_x$ $(a/2,a/2, \pm 1/2)$</th>
<th>Normalized $\sigma_y$ $(a/2,a/2, \pm 1/3)$</th>
<th>Normalized $\tau_{yz}$ $(0,a/2,0)$</th>
<th>Normalized $\tau_{xz}$ $(a/2,0,0)$</th>
<th>Normalized $w$ $(a/2,a/2,0)$</th>
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<tr>
<td>10</td>
<td>EL 0.545</td>
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Fig. 8 — Variation of in-plane stresses with respect to $\alpha$

Fig. 9 — Variation of inter-laminar stresses with respect to $\alpha$
Effect of d/l ratio

When the size of the ellipse increases, the area supporting the load decreases. Due to this the net force acting on the plate decreases causing for the reduction in stresses. At the same time the resisting volume of the material decreases and as a result the induced stresses increase.

The in-plane stresses decrease with increase in d/l ratio. This is due to the reduction in stress concentration with increase in size of the cutout. There is no significant variation of in-plane shear stress \( \tau_{xy} \) with respect to d/l ratio (Fig. 11).

The inter-laminar stresses at the free edge of the elliptical cutout \( \sigma_z, \tau_{yz} \) and \( \tau_{zx} \) increase with increase in d/l ratio (Fig. 12). The forces causing the inter-laminar stresses form in couples to balance the forces for equilibrium. When the size of the cutout increases,
the moment arm of these forces decreases and this may be the reason for increase in interlaminar stresses.

The transverse deflection ‘$w$’ decreases with increase in $d/l$ ratio due to the reason that the resultant force due to the applied pressure decreases with increase in the size of the cutout (Fig. 13).

**Effect of ellipse configuration (orientation)**

Five different configurations of the elliptical cutout are analyzed to evaluate the better configuration with minimum magnitudes of major stresses ($\sigma_x$ and $\sigma_y$).

The in-plane normal stress $\sigma_x$ is minimum at configuration 1. The in-plane normal stress $\sigma_y$ is minimum at configuration 3 (Fig. 14). There is no significant variation of $\tau_{xy}$ with respect to the configuration of the elliptical cutout. Since the major stresses $\sigma_x$ is not too high at configuration 3 when compared to the magnitude of $\sigma_y$ at configuration 1, configuration 3 is considered to be the better configuration.

The inter-laminar stress $\sigma_z$ is minimum for configuration 1. $\tau_{yz}$ and $\tau_{zx}$ are minimum for configuration 2 (Fig. 15). If the design is based on in-plane strength, configuration 3 is preferred. If the design is based on interlaminar strength configuration 2 is preferred.

The transverse deflection ‘$w$’ is minimum for configuration 2 as compared to other configurations (Fig. 16). This configuration may be preferred for the design of the skew plate based on stiffness.

**Conclusions**

Static analysis of a laminated composite skew plate with an elliptical cutout at the centre of the plate for five different configurations has been carried out in the present work. The transverse deflection, maximum in-plane stresses and maximum interlaminar stresses at the free edge of the cutout have been evaluated using 3-dimensional theory of elasticity based finite element analysis. The results obtained for uniform transverse pressure loading are analyzed for the variation of skew angle of the plate,
size of the ellipse, and configuration of the cutout. The magnitudes of the in-plane normal stresses and the transverse deflection due to pressure loading are greatly affected by the skew angle variation and their magnitudes are observed to be minimum at higher value of the skew angle and $d/l$ ratio. Configuration 3 is observed to be better in view of the in-plane normal stresses.

References