Multi-product FPR model with rework and multi-shipment policy resolved by algebraic approach

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This study resolves a multi-product finite production rate (FPR) model with rework and a multi-shipment delivery policy using algebraic approach. Conventional method for solving multi-product FPR model is to use differential calculus on system cost function to prove its convexity first, and then derive optimal common production cycle time that minimizes the long-run average system cost. Whereas the proposed approach obtains the optimal common cycle time in simplified algebraic way without using differential calculus. Simplification may enable managing practitioners to resolve real-life multi-item FPR systems more effectively.

Keywords: Multi-product FPR system, Optimization, Common cycle time, Algebraic approach, Finite production rate model, Rework, Multi-delivery.

Introduction

A multi-item finite production rate model with rework and a multi-shipment delivery policy is reexamined and simplified in this study using an algebraic approach. In real world manufacturing environments, production manager commonly has multi-product production plan on a single production machine with common cycle policy in order to maximize machine utilization and production efficiency. Economic lot-sizes for a group of products are produced at a single work center. Product requirements were assumed to be known in advance. There was a separate linear production system having fixed setup cost and negligible setup time for every product. All production rates and costs may vary from product to product. Determination of lot-sizes without backlogging of products for minimizing cost was a major criterion for required capacity of production system. A simple heuristic was developed with a relatively small amount of computational effort for arriving at feasible solution. Zahorik et al. solved a multi-item multi-level production scheduling problem with linear cost function with production and inventory constraints. Two multi-item problems were considered, one in which the constraint was on shipping capability and the other in which there was a final stage bottleneck in production machine. A multi-item facilities-in-series problem was formulated as a linear programming model. Byrne proposed an approach for multi-item production lot sizing problem based on use of simulation search model for cost minimization in the production process system. Aggarwal simplified multi-item inventory control by creating subgroups with a common order cycle for all product items in each group to find optimal values in computationally efficient way. Additional various related aspects of multi-item production planning and optimization issues may be found elsewhere. Due to various unpredictable factors, production of defective items in any production cycle run is almost inevitable. Gopalan and Kannan viewed manufacturing process, inspection activities and rework activities as a two-stage transfer-line production system. Transient state characteristics of this production system subjected to an initial buffer of infinite capacity, inspection at both inter- and end-stages, and rework were analyzed. A stochastic model was developed to investigate the production system. Explicit analytical expressions for some of the system characteristics were obtained using the state-space method and regeneration point technique. Many studies were conducted to address different aspects of imperfect production systems as well as quality assurance...
issues in production. Multiple or periodic delivery policy for transporting finished goods to customers is commonly adopted in real-life vendor-buyer integrated systems. Hahm and Yano studied frequency of production and delivery of a single component with the objective of minimizing the long-run average cost per unit time. Production setup costs, inventory-holding costs for both the supplier and customer and transportation costs were considered. Hill considered a problem of a vendor supplying a product to a buyer. The vendor manufactures the product in batches at a finite rate and ships the output to the buyer, who then consumes the product at a fixed rate. The objective is to determine a purchasing and production schedule which minimizes the overall costs of purchasing, manufacturing, and stockholding. Chiu et al. derived the optimal common production cycle time for a multi-item finite production rate model with rework and a multi-shipment delivery policy. Mathematical model involving differential calculus was formulated to prove firstly the convexity of system cost function and secondly deriving a closed-form optimal production cycle time for the studied model. Additional studies addressing various aspects of periodic or multiple deliveries issues in vendor-buyer integrated systems may be found elsewhere. Grubbström and Erdem presented algebraic approach to the economic order quantity (EOQ) model with backlogging without using differential calculus. A few studies used similar algebraic approach to deal with various specific production lot-size and vendor-buyer integrated problems. This paper extends such an algebraic method to reexamine the problem in Chiu et al., and demonstrates that the optimal common production cycle time may be obtained without using models involving complexity of differential calculus.

Experimental Section

Problem formulation and modelling

Reconsider the multi-item finite production rate model with rework and multi-shipment policy as follows: a production manager has planned for L products to be made on a single production machine with common production cycle policy. During production process for each product i (where i = 1, 2, . . . , L), there is a xi portion of defective items being randomly produced at a rate di. All produced items are screened and the inspection cost is included in unit production cost Cj. It is assumed that all defective items are repairable at a reworking rate of Pi when regular production process ends in each cycle with unit reworking cost CRi. In order to avoid shortage occurrences it is assumed that constant production rate for product i, Pi must satisfies (Pi - di - λi) > 0; where λi is the annual demand rate for product i, and di can be expressed as d = x; Pi. A multi-shipment delivery policy is adopted here. Under such a policy, finished product items for each product i can only be delivered to customers if the whole production lot is quality assured at the end of rework process. Fixed quantity installments n of the finished batch are delivered at a fixed interval of time during delivery time t3i (Fig.1). Other notation used in the modeling and analysis of this paper is listed at ends. Total production-inventory-delivery cost per cycle TC(Qi) for L products, consists of setup cost, variable production cost, rework cost, fixed and variable delivery cost, holding cost during production uptime t1i, and rework time t2i, and holding cost for finished goods kept during the delivery time t3i is can be obtained. By taking the randomness of x into account and with further derivations, the expected E[TCU(T)] can be obtained as follows:

\[
E[TCU(T)] = \sum_{i=1}^{L} \{ C_Pi \cdot \lambda_i \cdot E(x_i) + C_WRi + K/T + nK_i/T + (h_1T\lambda_i/2)\} \{1/\lambda_i - 1/P_i - E(x_i)/P_i\} \}
\]

Alternative approach to the problem

Unlike conventional method which uses differential calculus on the cost function E[TCU(T)] to find the optimal common cycle time, an alternative
algebraic approach is proposed here. It may be clearly seen that in the right-hand side of Eq. (1) single decision variable \( T \) is in different forms, namely \( T^0 \), \( T^1 \), and \( T^2 \). Hence, Eq. (1) can be rearranged as

\[
E[T_{CU}(T)] = \delta_1 + T^1 [(\delta_2)^{1/2} - T(\delta_3)^{1/2}]^2 + 2(\delta_2)^{1/2}(\delta_3)^{1/2}
\]

where \( \delta_1 = \sum_{i=1}^{L} \{ C_{Ri} + C_{Ei} E(x_i) + C_{Ti} \lambda_i \} ; \quad \delta_2 = \sum_{i=1}^{L} \{ K_i + nK_{i1} \} ; \quad \delta_3 = \sum_{i=1}^{L} \left\{ (h_{1,i}/2)[\lambda_i E(x_i)/P_{2i}] + [h_{1}(\lambda_i)^2/2][E(x_i)/P_{2i} - E(x_i)^2/P_{2i}] + 1/\lambda_i \right\} - [h_{1}(\lambda_i)^2/2n] \}

Applying Eq. (3) one obtains optimal common production cycle time \( T^* \) as follows:

\[
T^* = \left\{ \sum_{i=1}^{L} [K_i + nK_{i1}] / \sum_{i=1}^{L} \left\{ (h_{1,i}/2)[\lambda_i E(x_i)/P_{2i}] + [h_{1}(\lambda_i)^2/2][E(x_i)/P_{2i} - E(x_i)^2/P_{2i}] + 1/\lambda_i \right\} - [h_{1}(\lambda_i)^2/2n] \left\{ 1/\lambda_i - 1/P_{2i} - E(x_i)/P_{2i} \right\} \right\}^{1/2}
\]

It may be seen that Eq. (3) is identical to that obtained using the conventional differential calculus method\(^1\). Applying the optimal common production cycle time \( T^* \) (i.e., to substitute Eq. (3) in Eq. (2)) one obtains a simplified formula

\[
E[T_{CU}(T)] = \delta_1 + 2(\delta_2)^{1/2}(\delta_3)^{1/2}
\]

Results and Discussion

Numerical Example and Discussion

In order to verify the aforementioned results, in this section the same numerical example is solved\(^1\). A routine production plan is to manufacture five products on a single machine under the common production cycle policy. Production rate \( P_{2i} \) for each product \( i \) is 58000, 59000, 60000, 61000, and 62000 respectively, and annual demand rate \( \lambda_i \) for five different products are 3000, 3200, 3400, 3600, and 3800, respectively. Defective rates \( x_i \) in production uptime for each product \( i \), follow the Uniform distribution over the intervals of \([0, 0.05]\), \([0, 0.10]\), \([0, 0.15]\), \([0, 0.20]\) and \([0, 0.25]\), respectively. All defective items are assumed to be repairable at the reworking rates \( P_{2i} \) of 1800, 2000, 2200, 2400, and 2600, respectively. Unit reworking costs are $50, $55, $60, $65, and $70, respectively. Values for other variables used in the proposed study include:

\[
h_1 = \text{unit holding costs} ($10, $15, $20, $25, and $30) ; \quad h_{1i} = \text{unit holding costs per reworked} ($30, $35, $40, $45, and $50) ; \quad C_i = \text{unit production costs} ($80, $90, $100, $110, and $120) ; \quad K_i = \text{the production set up costs} ($3800, $3900, $4000, $4100, and $4200) ; \quad n = \text{number of shipments per cycle} 4 ; \quad K_{1i} = \text{fixed costs per delivery} ($1800, $1900, $2000, $2100, and $2200) ; \quad C_{2i} = \text{unit transportation costs} ($0.1, $0.2, $0.3, $0.4, and $0.5) .
\]

Applying Eq. (3) one obtains optimal common production cycle time \( T^* = 0.6026 \) (years). By applying \( E[T_{CU}(T)] = \delta_1 + 2(\delta_2)^{1/2}(\delta_3)^{1/2} \), total expected production-inventory-delivery costs per unit time for producing \( L \) products \( E[T_{CU}(T^*)] = $2,008,926 \) can be obtained. These results are identical to that obtained in Chiu et al.\(^1\).

Conclusions

This paper proposed a simplified algebraic solution procedure to reexamines a multi-item FPR problem with rework and multi-shipment policy\(^1\). As a result, this study demonstrates that optimal common production cycle time can be derived without using differential calculus. Such a simplified approach is handy solution for practicing managers to solve the real world multi-item FPR problem.

Notations

\( h_1 = \text{unit holding cost} , \quad h_{1i} = \text{holding cost per reworked item} , \quad K_i = \text{production setup cost} , \quad K_{1i} = \text{fixed delivery cost per product} i \text{ per delivery} , \quad C_{2i} = \text{unit shipping cost for product} i , \quad H_{1i} = \text{maximum level of on-hand inventory for} \text{ product} i \text{ when the regular production ends} , \quad H_{2i} = \text{maximum level of on-hand inventory in units for} \text{ product} i \text{ when the rework process ends} , \quad t_{1i} = \text{production uptime for} \text{ product} i , \quad t_{2i} = \text{the reworking time for} \text{ product} i , \quad t_{3i} = \text{the delivery time for} \text{ product} i , \quad T = \text{common production cycle length - the decision variable} , \quad n = \text{number of fixed quantity installments of the finished batch to be delivered to buyers in each cycle} , \quad Q_i = \text{production lot size per cycle for product} i , \quad t_{ui} = \text{fixed interval of time between each installment of} \text{ finished product} i \text{ being delivered during} t_{3i} , \quad I_i(t) = \text{on-hand inventory of perfect quality items for} \text{ product} i \text{ at time} t , \quad I_p(t) = \text{on-hand inventory of defective items for} \text{ product} i \text{ at time} t \).
product \( i \) at time \( t \),

\[ TC(Q_i) = \text{total production-inventory-delivery costs per cycle for product } i. \]

\[ E[TCU(Q_i)] = \text{total expected production-inventory-delivery costs per unit time for } L \text{ products in the proposed system}. \]

\[ E[TCU(T)] = \text{total expected production-inventory-delivery costs per unit time using common production cycle time } T \text{ as decision variable}. \]

References


