Application of Backward Light Scattering Interference Patterns for Calculating the Refractive Indices of Some Polymeric Fibres

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An experimental method to determine the refractive index of polymeric fibres is described. The method is based on an analysis of the back-scattered light of a laser beam impinging on the fibre axis. The mean parallel refractive indices, calculated by two different formulae, are in agreement with those published earlier but obtained with different techniques, namely interferometric and Becke-line methods. The results are also in good agreement with the Wilkes equation published recently.

Keywords: Backward light scattering method, Nylon fibre, Polyethylene fibre, Polymeric fibre, Polypropylene fibre, Refractive index

1 Introduction
Light scattering methods for determining refractive indices have been investigated extensively\(^1\)\(^-\)\(^6\). The backward light scattering method is especially useful because this does away with complex data-processing procedures. The technique is also advantageous because of its non-contacting nature.

2 Theoretical Consideration and Experimental Procedure
The procedure is based on the technique described by Presby\(^1\). The technique involves the use of a CW He-Ne laser as a source of light impinging on the fibre and observation of the light scattered backward by the fibre. A CW He-Ne laser manufactured by Leybold Heracus Koln, West Germany, was used. Its characteristics were: radiant power, 1 mW (approx.); TEM\(_{00}\); and wavelength of the unpolarized emitted light \(\lambda\), 632.8 nm. The pattern appears on a viewing screen placed in front of the fibre as bright line interference fringes oriented perpendicularly to the fibre axis, and symmetric about a small aperture in the screen (through which the laser beam passes) [Fig. 1]. The line exhibits a sharp cut-off in brightness at a distance \(L\) from the aperture. The distance \(L\) from the centre of the patterns to the cut-off point is related to the distance \(h\) between the fibre and the screen. The scattering angle \(\Phi\) is defined as:

\[
\Phi = \tan^{-1}\left(\frac{L}{h}\right)
\]  \(\ldots (1)\)

From a geometric optics analysis of Eq. (1) Presby showed that the relation between \(\Phi\) and the fibre refractive index \(n\) as:

\[
\Phi = 4\sin^{-1}\left[\frac{2}{n\sqrt{3}}\left(1 - \frac{n^2}{4}\right)^{\frac{1}{2}}\right] - 2\sin^{-1}\left[\frac{2}{\sqrt{3}}\left(1 - \frac{n^2}{4}\right)^{\frac{1}{2}}\right]
\]  \(\ldots (2)\)

By measuring \(L\) and \(h\), the refractive indices were calculated from Eqs (1) and (2).

Wilkes\(^7\) showed that Eq. (2) is not the simplest for such a calculation. A series of rays incident upon one-half of the section of a fibre is shown in Fig. 2. By the laws of reflection and Snell's law of refraction:

\[
i_0 = i_1\]  \(\ldots (3)\)

\[
n_0 \sin i_0 = n \sin r\]  \(\ldots (4)\)

Fig. 1—Set-up to observe back-scattering light [(1) Continuous He-Ne laser of wavelength 632.8 nm; (2) Optical fibre; (3) Viewing screen; (4) Incident radiation; (5) Back-scattered radiation; and (6) Camera]
Presby obtained the total deviation $\theta$ of the ray by:

$$\theta = \pi + 2i - 4r$$  \hspace{1cm} \ldots (5)$$

By putting $d\theta/di=0$ and combining with Snell's law, the minimum deviation is given by:

$$\cos \phi_m = \left( \frac{n^2 - 1}{3} \right)^{\frac{1}{2}}$$  \hspace{1cm} \ldots (6)$$

By using the Eqs (4), (5) and (6), it is possible to derive the following relation between $\Phi$ and $n$:

$$\Phi = 2 \sin^{-1} \left[ \frac{1}{n} \left( \frac{4 - n^2}{3} \right)^{3/2} \right]$$  \hspace{1cm} \ldots (7)$$

An iterative solution to this equation is much easier to obtain than to Eq. (2), but it is perhaps more significant than that from Eq. (7). One can derive a cubic equation in $n^2$:

$$n^6 + 3[(1 - 9 \cos \phi)/2] n^4 + 48 n^2 - 64 = 0$$  \hspace{1cm} \ldots (8)$$

This equation can be solved by the standard algorithm without resorting to a numerical iteration technique. Eqs (2), (7) and (8) are not valid if $n > 2$, but this is unlikely to occur in the case of most fibres of interest.

3 Results and Discussion
Typical back-scattered patterns of the fibres of poly-
Table 1—Principal Parallel Refractive Indices of Polyethylene, Polypropylene and Bicomponent (Nylon 6 and 66) Fibres

<table>
<thead>
<tr>
<th>Fibre</th>
<th>Becke-line&lt;sup&gt;a&lt;/sup&gt;</th>
<th>Fizeau method&lt;sup&gt;b&lt;/sup&gt;</th>
<th>Back-scattering method&lt;sup&gt;c&lt;/sup&gt;</th>
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<tbody>
<tr>
<td></td>
<td>Wavelength</td>
<td>n&lt;sup&gt;&quot;&lt;/sup&gt;</td>
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<tr>
<td>Polyethylene</td>
<td>546.1</td>
<td>1.534-1.539</td>
<td>632.8</td>
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<tr>
<td></td>
<td></td>
<td>1.5269-1.5319</td>
<td>1.5253</td>
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<tr>
<td></td>
<td></td>
<td>1.5253</td>
<td>1.5267</td>
</tr>
<tr>
<td>Polypropylene</td>
<td>546.1</td>
<td>1.528-1.531</td>
<td>632.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.528-1.530</td>
<td>1.5252</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.5252</td>
<td>1.5271</td>
</tr>
<tr>
<td>Bicomponent (Nylon 6 and 66)</td>
<td>546.1</td>
<td>1.578-1.581</td>
<td>632.8</td>
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<td></td>
<td>1.5708-1.5849</td>
<td>1.5737</td>
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<td>1.5737</td>
<td>1.5747</td>
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<sup>a</sup>Values have an accuracy of ± 0.0005.
<sup>b</sup>Values under favourable conditions have an accuracy of ± 0.0001.
<sup>c</sup>Values have an accuracy of ± 0.0003.

Note: 25 Samples were subjected to back-scattering technique to get the mean values given.

propylene, polyethylene and the bicomponent Nylon (6 and 66) are shown in Fig. 3. By measuring \( L \) and \( h \), we could calculate the refractive indices from Eqs (2) and (8) of Presby<sup>1</sup> and Wilkes<sup>7</sup> (Table 1). The mean calculated values of \( n''_e \) for the three polymeric fibres are more in agreement with those obtained by interferometric and the Becke-line methods<sup>8-10</sup> by Wilkes formula than with those by Presby formulas.

In the case of an unpolarized beam, \( n'_e \) is the only measurable value. The results cannot be interpreted on the basis of geometric optics alone, as the use of the non-polarized laser beam, owing to the emergence of two back-scattered patterns corresponding to the ordinary and the extraordinary rays and their superposition, could give rise to diffuse patterns, as has been obtained in the present work. Also, the anisotropy of the fibres leads to comparatively weak patterns.

There are some limitations encountered in the back-scattering technique in the case of polymeric fibres. These are:

1. The complete back-scattered fringe pattern is localized in the range \( \Phi = \pm 20^\circ \).
2. Some discrepancies may arise owing to the incomplete circularity of some fibres and to the drawing and spinning processes.
3. Light polarized parallel to the axis of the fibre must be used for these measurements, because calculation of the combined Fresnel coefficients shows that light polarized perpendicular to the axis of the fibre has a minimum irradiance when it emerges from the fibre at angles very close to \( \Phi_n \).

A back-scattering pattern method provides a quick and easy means of measuring the refractive index without an interferometer for measurement. Also, it is easier than the Becke-line method, which needs the preparation of a series of liquids with refractive indices increasing in steps. The Becke-line method measures the refractive index at the fibre surface. Interferometric measurements of the refractive index across the fibre have shown that Becke-line refractive indices are not confined to the fibre surface but may occur at any point on the fibre radius<sup>11</sup>.

Clearly, any method or any formula used for calculating refractive indices and other optical parameters has its own merits, and the preference for one method or one formula over another is likely to be determined by the ease of application.

We therefore conclude that back-scattering is a very promising technique to investigate the polymeric fibre properties of circular cross-section. Its potential, however, has not yet been fully explored.

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References