Numerical solution of unsteady flow of a radiating and chemically reacting fluid with time-dependent suction

S Raji Reddy & K Srihari*
Department of Mathematics, Mahathama Gandhi Institute of Technology, Gandipet, Hyderabad
*E-mail: kotagirisrihari@yahoo.com

Received 14 May 2008; revised 29 August 2008; accepted 20 October 2008

The numerical solution of unsteady laminar, boundary layer flow of a viscous incompressible, electrically conducting fluid along a semi-infinite vertical plate, in the presence of thermal and concentration buoyancy effects has been obtained, using implicit finite difference method for velocity, temperature and concentration fields. The results for the velocity, temperature shear stress, Nusselt number and Sherwood-numbers have been obtained and discussed for different flow parameters such as $Sc$, $K_r$, $Gr$, $Gm$, $NR$ and $Pr$. It has been found that an increase in the chemical reaction leads to decrease in the velocity and concentration boundary layer, but, an increase in the thermal radiation increases the velocity and temperature boundary layer. These results are found to be in good agreement with the previous results.

Keywords: Buoyancy effects, Radiative flux, Implicit finite difference technique

1 Introduction

Heat flow and mass transfer over a vertical porous plate with variable suction and heat absorption/generation have been studied by many workers. Chem$^1$ studied heat and mass transfer with variable wall temperature and concentration. Perdikis and Rapti$^2$ studied the unsteady MHD flow in the presence of radiation. Rahman and Sattar$^3$ analyzed the MHD convective flow of a micro polar fluid past a continuously moving vertical porous plate in the presence of heat generation/absorption. Chaudhary and Jain$^4$ employed a perturbation technique to study the effect of radiation on mixed convection flow of a magneto-micro polar fluid past a vertical porous plate through a porous medium with variable permeability in slip-flow regime. Recently, the effect of thermal radiation, time-dependent suction and chemical reaction on the two-dimensional flow of an incompressible Boussinesq fluid, applying a perturbation technique to coupled non-linear partial differential equations has been studied$^7$.

In the present paper, the investigation is confined to the numerical solution of the work carried out by Prakash and Ogulu$^7$. An implicit finite difference method has been employed to solve the problem, because it is more economical from computational viewpoint. The results obtained are in good agreement with the results of previous works.

2 Theory

2.1 Formulation of the problem

An unsteady two-dimensional, laminar, boundary layer flow of a viscous, incompressible, electrically conducting fluid along a semi-infinite vertical plate, in the presence of thermal and concentration buoyancy effects has been considered. The $x'$-axis taken along the plate in the vertically upward direction and $y'$-axis normal to it. Further, due to the semi-infinite plane surface assumption, the flow variables are functions of normal distance $y'$ and $t'$ only. A time dependent suction velocity is assumed normal to the plate. Now, under the usual Boussinesq’s approximation, the governing boundary layer equations are:

Continuity
\[
\frac{\partial v'}{\partial y'} = 0 \quad \ldots (1)
\]

Linear momentum
\[
\frac{\partial u'}{\partial t'} + v' \frac{\partial u'}{\partial y'} = \nu \frac{\partial^2 u'}{\partial y'^2} + g \beta (T - T_\infty) + g \beta' (C - C_\infty) \quad \ldots (2)
\]

Energy
\[
\frac{\partial T'}{\partial t'} + v' \frac{\partial T'}{\partial y'} = \nu \frac{\partial^2 T'}{\partial y'^2} - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y'} \quad \ldots (3)
\]
Mass transfer

\[
\frac{\partial C}{\partial t'} + \frac{\partial C}{\partial y'} = v \frac{\partial^2 C}{\partial y'^2} - k^2_C \frac{C}{y'^2} \quad \ldots (4)
\]

By using the Rosseland approximation\(^5\), the radiative flux \(q_r\) can be written as:

\[
q_r = \frac{4\sigma T^4}{3k^*} \quad \ldots (5)
\]

It has been assumed that the temperature differences within the flow are sufficiently small and \(T^4\) may be expressed as a linear function of the temperature \(T\). This is accomplished by expanding \(T^4\) in a Taylor series about the free stream temperature \(T_\infty\), as follows:

\[
(T^4)_{\infty} = (T^4) + \epsilon \frac{\partial T^4}{\partial T} \frac{\partial T}{\partial y} + \ldots (6)
\]

After substitution of Eqs (5) and (6) in Eq. (3), the energy equation transformed to

\[
\frac{\partial T}{\partial t'} + \frac{\partial T}{\partial y'} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y'^2} - \frac{16\sigma T^3}{3k^*} \frac{\partial^2 T}{\partial y'^2} \quad \ldots (7)
\]

Integration of continuity (Eq. (1) for variable suction velocity normal to the plate gives

\[
v' = -V_0 \left(1 + \epsilon A e^{nV'}\right) \quad \ldots (8)
\]

where \(A\) is the suction parameter and \(\epsilon A\) is less than unity. Here \(V_0\) is mean suction velocity, which is a non-zero positive constant and the minus sign indicates that the suction is towards the plate. It is now convenient to introduce the following dimensionless parameters:

\[
\begin{align*}
\frac{u}{U_0} = \frac{u'}{U_0}, & \quad \frac{w}{U_0} = \frac{w'}{U_0}, & \quad \frac{t}{U_0^2} = \frac{t'}{U_0^2}, & \quad Sc = \frac{v}{D}, \\
Pr = \frac{\rho c_p v}{k}, & \quad \frac{T}{T_w}, & \quad \frac{C}{C_w}, & \quad \frac{C}{C_w}
\end{align*}
\]

\[
Gr = \frac{g\beta v(T - T_\infty)}{U_0^3}, \quad Gm = \frac{g\beta^* v(C - C_\infty)}{U_0^3} \quad \ldots (9)
\]

\[
k^2 = \frac{k^2 v}{U_0^2}, \quad N_r = \frac{16\sigma T^3}{3k^*}, \quad n = \frac{vn'}{U_0^2}.
\]

With the help of non-dimensional quantities (Eq. (9), Eqs (2), (4) and (7) are reduced to the following dimensionless forms:

\[
\begin{align*}
\frac{\partial u}{\partial t} - \frac{(1 + \epsilon A e^{nV'})}{\rho c_p} \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y'^2} + Gr\theta + Gm\phi \\
\frac{\partial \theta}{\partial t} - \frac{(1 + \epsilon A e^{nV'})}{Pr} \frac{\partial \theta}{\partial y} = \frac{1 + N_r}{Pr} \frac{\partial^2 \theta}{\partial y'^2} \\
\frac{\partial \phi}{\partial t} - \frac{(1 + \epsilon A e^{nV'})}{Sc} \frac{\partial \phi}{\partial y} = \frac{1}{Sc} \frac{\partial^2 \phi}{\partial y'^2} - k^2 \phi
\end{align*}
\]

with the boundary conditions

\[
\begin{align*}
u = 1, & \quad \theta = 1 + \epsilon e^{nV'}, \quad \phi = 1 + \epsilon e^{nV'} \quad \text{at } y = 0 \\
u \to U_\infty, & \quad \theta \to 0, \quad \phi \to 0 \quad \text{as } y \to \infty
\end{align*}
\]

2.2 Method of solution

By applying Crank-Nicholson method on governing Eqs (10) to (12), following system of equations are obtained:

\[
\begin{align*}
-A_{ij} + r_{ij} - \frac{1}{2} (1 + r) u_{ij} - \frac{1}{2} u_{ij}^{i+1} = A_{ij} \\
-P_{ij} r_{ij} + (1 + P_r) \theta_{ij}^{i+1} - \frac{1}{2} \theta_{ij}^{i+1} = B_{ij} \\
-P_{ij} r_{ij} + (1 + P_r) \phi_{ij}^{i+1} - \frac{1}{2} \phi_{ij}^{i+1} = C_{ij}
\end{align*}
\]

where

\[
\begin{align*}
A_{ij} = \frac{r_{ij}^2}{2} \theta_{ij}^{i+1} - (P_r r - 1 + r) u_{ij} + \left( P_r r + \frac{r}{2} \right) u_{ij}^{i+1} + Gr k \theta_{ij}^{i+1} + Gm k \phi_{ij}^{i+1} \\
B_{ij} = \frac{P_r r}{2} \theta_{ij}^{i+1} + (1 - P_r r - P_r r) \theta_{ij}^{i+1} + \left( P_r r + \frac{P_r r}{2} \right) \theta_{ij}^{i+1}
\end{align*}
\]
\[
C_1^i = \frac{r}{2Sc} \phi_{i+1}^j + \left(1 + P_i rh - \frac{r}{Sc} - k_i k \right) \phi_i^j \\
+ \left( \frac{r}{2Sc} - P_i rh \right) \phi_{i+1}^j \quad \ldots (17)
\]

\[
P_i = 1 + Ae^{n_1}, \quad P_2 = \left(1 + \frac{N_R}{Pr} \right)
\]

here \(r = k/h^2\) and \(h, k\) are mesh sizes along \(y\) and time direction respectively. Index \(i\) refers to space and \(j\) for time. In Eqs (14) to (16), taking \(i = 1(1)\ n\) and using the boundary conditions Eq. (13), following tri-diagonal system of equations are obtained:

\[
A_i X_i = B_i \quad i = 1(1)3 \quad \ldots (18)
\]

where \(A_i\) are tri-diagonal matrices of order \(n\) and \(X_i, B_i\) are column matrices having \(n\)-components. The above system of equations have been solved by Gauss-seidel iteration method, for velocity, temperature and concentration. Also numerical solutions for these equations are obtained, using C-Program. In order to prove the convergence of finite difference scheme, the computation is carried out for slightly changed values of \(h\) and \(k\), running same C-Program. Negligible change is observed in the values of \(u, \theta\) and \(\phi\) and also after each cycle of iteration the convergence checking is performed, i.e. \(|u^{n+1} - u^n| < 10^{-8}\) is satisfied at all points. Thus, it is concluded that the finite difference scheme is convergent and stable.

2.3 Skin-friction, Nusselt number and Sherwood number

The Skin-friction, Nusselt number and Sherwood number have been obtained as follows:

\[
\text{Skin-friction } \tau = \frac{\partial u}{\partial y}|_{y=0}, \quad \text{Nu} = \frac{\partial \theta}{\partial y}|_{y=0} \quad \text{and} \quad \text{Sh} = \frac{\partial \phi}{\partial y}|_{y=0} \quad \ldots (19)
\]

3 Results and Discussion

The effects of \(Sc, K_r, Pr, N_R\) on velocity field \(u\) have been shown in the Figs 1 and 2, respectively. It is observed from these figures that an increase in \(Sc\) and \(Pr\) decreases the velocity when the plate is cooled by the free convection currents \((Gr>0)\). Further, it is interesting to note that the fluid velocity decreases in the presence of concentration buoyance effect \((K_r)\) while it increases in the presence of thermal buoyance effect \((N_R)\). Also it is observed that the velocity is maximum when \(Pr = 0.71\).

Figure 3 shows the effect of \(Pr, N_R, Sc\) and \(K_r\) on velocity field \(u\), for heating of the plate. From this, it is seen that an increase in \(Pr\) and \(Sc\), increases the velocity of the fluid when the plate is heated by free-convection currents \((Gr < 0)\). Further, it is observed that the fluid velocity increases in the presence of
concentration buoyance effect while it decreases in the presence of thermal buoyance effect. It is also observed that these results are reverse for the case of cooled plate. Figure 4 shows that an increase in the free-convection parameter (Gr) increases the velocity of the fluid.

Figures 5 and 6 show the temperature and concentration profiles, respectively. It is found that an increase in the thermal radiation leads to decrease in the temperature boundary layer, but an increase in the rate of chemical reaction leads to decrease in the concentration boundary layer. It is also found that an increase in the Pr and Sc decreases the temperature and concentration boundary layer, respectively.

The numerical calculations have been done to understand the physical aspect of the problem. The numerical values of the skin friction, Nusselt number and Sherwood number have been presented in Tables 1-3. From these tables it has been observed that an increase in Gr, Gm and N_R leads to an increase in the skin-friction while with an increase in Pr, Sc and K_r it decreases. The skin-friction is maximum for cooling of the plate than in the case of heating of the plate. In the presence of thermal radiation, Nusselt number increases while in the presence of concentration buoyance effect (K_r), Sherwood number decreases. Nusselt and Sherwood numbers decrease with the increase of Pr and Sc. These observations are in good agreement with the results from previous works.

**Nomenclature**

- ρ - Density
- C_p - Specific heat at constant pressure
- ν - Kinematic viscosity
- k - Thermal conductivity
- U - Mean velocity
- Sc - Schmidt number
- T - Temperature
- k_r 2 - Chemical reaction rate constant

| Table 1 — Effects of Gr, Gm, Pr, Sc, K_r, and N_R on Skin-Friction coefficient |
|---|---|---|---|---|---|
| Gr | Gm | Pr | Sc | K_r | N_R | τ |
| 2.0 | 2.0 | 0.71 | 0.24 | 0.5 | 0.5 | 0.487834 |
| 5.0 | 2.0 | 0.71 | 0.24 | 0.5 | 0.5 | 1.596569 |
| 2.0 | 5.0 | 0.71 | 0.24 | 0.5 | 0.5 | 1.851348 |
| 2.0 | 5.0 | 7.0 | 0.24 | 0.5 | 0.5 | 1.319304 |
| 2.0 | 5.0 | 0.71 | 0.60 | 0.5 | 0.5 | 0.237303 |
| 2.0 | 2.0 | 0.71 | 0.60 | 0.5 | 0.5 | 0.443873 |
| 2.0 | 2.0 | 0.71 | 0.24 | 0.5 | 1.0 | 0.567380 |
| +2.0 | +2.0 | 0.71 | 0.24 | 0.5 | 0.5 | -0.990477 |
| +2.0 | +2.0 | 0.71 | 0.24 | 0.5 | 0.5 | -2.808494 |

| Table 2 — Effects of N_R and Pr on Nusselt number |
|---|---|---|
| N_R | Pr | Nu |
| 0.0 | 0.71 | -0.942097 |
| 0.5 | 0.71 | -0.734199 |
| 0.5 | 7.0 | -2.638997 |
| 0.5 | 11.4 | -3.183188 |

| Table 3 — Effects of Sc and K_r on Sherwood number |
|---|---|---|
| Sc | K_r | Sh |
| 0.24 | 0.0 | -0.484627 |
| 0.24 | 0.5 | -0.526215 |
| 0.60 | 0.5 | -0.906969 |
| 0.24 | 1.0 | -0.640454 |
- Small reference parameter $<< 1$

$Pr$ - Prandtl number

$Gr$ - Free convection parameter due to temperature

$Gm$ - Free convection parameter due to concentration

$\chi$ - Darcy number

$A$ - Suction parameter

$n$ - A constant exponential index

$D$ - Molar diffusivity

$NR$ - Thermal radiation parameter

$\beta$ - Coefficient of volumetric thermal expansion of the fluid

$\beta^*$ - Volumetric coefficient of expansion with concentration

References