Cooling of a viscous liquid using a half-coil jacket

Y Pydisetty & N S Jayakumar
Department of Chemical Engineering, D D Institute of Technology, Nadiad 387 001, India

Cooling of a hot viscous glycerine liquid using water as coolant flowing through a half-coil jacket has been studied experimentally in a batch stirred system to investigate the individual as well as the overall time-variant heat transfer coefficients.

In an indirect heat transfer system jackets or coils are used for heating/cooling or temperature control of a chemical reaction. Coils are more suitable than jackets since coil geometry offers higher heat transfer rates and ensure close temperature control. There are some reports on heat transfer in a limpet coil, also known as half-coil. An equation has been derived for heat transfer efficiency of a limpet coil comparing it with a jacketed vessel for both unsteady state and steady state conditions. The present experiment has been carried out using a half-coil jacket since the data available in such a unit is little.

The experimental set-up and coil geometry used in the present study are shown in Fig. 1, while Table 1 shows the experimental conditions. The experiments were conducted by varying the flow rate of the cooling water and the initial temperature of hot glycerine. The transient temperature data on the vessel ($T_h$) as well as on the coil side ($T_c$) are shown in Fig. 2.

Results and discussion—The temperature vs time data for fixed time intervals obtained from Fig. 2 have been used to calculate the corresponding logarithmic mean temperature difference, shown in Table 2. The individual and overall heat transfer coefficients have been calculated from heat balances assuming negligible heat losses from the vessel lid, bottom and side, uniform temperature throughout the bath inside the vessel, steady flow of coolant throughout the experiment and constant wall temperature throughout the wall thickness. Heat balance on bath side gives:

\[- V \rho_h C_{ph} \frac{dT_h}{dt} = h_i A_i (\Delta T) ln \]

Heat balance on coil side gives:

\[ q_{pc} C_{pc} (T_c - T_e) = h_o A_o (\Delta T) ln \]

$h_i$, obtained from Eq. (1) is corrected for viscosity by wall temperature calculations. The logarithmic mean temperature difference, $\Delta T_{lm}$, is given by

\[ \Delta T_{lm} = \frac{1}{2} \ln \left( \frac{T_{h0} - T_i}{T_c - T_e} \right) \]

Table 1—Experimental conditions of the present study

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volume of hot glycerine, $V$</td>
<td>$400 \times 10^{-6}$ m$^3$</td>
</tr>
<tr>
<td>Flow rate of cooling water, $q$</td>
<td>$1.67-6.00 \times 10^6$ m$^3$/s</td>
</tr>
<tr>
<td>Initial temperature of hot glycerine, $T_{hi}$</td>
<td>45-96°C</td>
</tr>
<tr>
<td>Thickness of the vessel, $x_w$</td>
<td>0.0015 m</td>
</tr>
<tr>
<td>Length of the limpet coil, $L$</td>
<td>1.525 m</td>
</tr>
</tbody>
</table>

![Fig. 1 - Experimental set-up and coil geometry](image1)

![Fig. 2 - Experimental transient temperature data](image2)
The properties $\rho_c$ and $C_{pc}$ are taken at the mean temperature of $T_c$ and maximum $T_e$ for any experiment. The properties $\rho_h$ and $C_{ph}$ are taken at the bath temperature $T_h$ at any time, $t$. In the present study, as a viscous liquid was used inside the vessel, the wall temperature was found to be different from the bath temperature. Hence, the inside heat transfer film coefficient is modified for viscosity correction by multiplying with a factor of $(\mu_h/\mu_{hw})^{0.14}$. The transport property, $\mu_{hw}$ is taken at the wall temperature, $T_w$ which is obtained from the heat balance between the wall and hot liquid, given as

$$h_i A_i (T_h - T_w) = \frac{K_w A_w}{x_w} (T_w - T_e)$$

For estimation of wall temperature, it is assumed that $A_w$ equals to $A_o$. Rearranging the terms in Eq. (4) the wall temperature, $T_w$ is obtained as

$$T_w = \frac{\left( \frac{h_i x_w}{K_w} T_h + \left( \frac{D_{eq} L}{\pi D_i H} \right)^{0.5} T_e \right)}{\left( \frac{h_i x_w}{K_w} + \left( \frac{D_{eq} L}{\pi D_i H} \right)^{0.5} \right)}$$

The corrected inside heat transfer film coefficient, $h'_i$ is given as

$$h'_i = h_i (\mu_h/\mu_{hw})^{0.14} \quad \ldots (6)$$

The experimental overall heat transfer coefficient based on outside area, $U_{oe}$ is calculated using the following equation

$$\frac{1}{U_{oe} A_0} = \frac{1}{h'_i A_i} + \frac{x_w}{K_w A_w} + \frac{1}{h_0 A_0}$$

where $A_w$ is the logarithmic mean area of the wall. Table 2 shows the values of $h'_i$, $h_0$ and $U_{oe}$. As the present work involves batch heat transfer study, the time-averaged experimental overall heat transfer coefficient, $\bar{U}_{oe}$ is calculated for each experiment as follows

$$\bar{U}_{oe} = \frac{\sum_{i=1}^{M} U_{oe} t_i (\Delta t_i)}{\sum_{i=1}^{M} t_i (\Delta t_i)}$$

In all the above calculations an equivalent diameter for coil side, $D_{eq}$ has been defined using hydraulic mean radius, $r_{H}$ as follows

$$D_{eq} = 4 r_{H} = \left( \frac{\pi}{\pi + 2} \right) d$$

Experimental Nusselt number is related to Dean number as

$$Nu = 18.2 (De)^{-0.5}$$

### Table 2—Results on heat transfer film coefficients

<table>
<thead>
<tr>
<th>Expt. No.</th>
<th>$q\times10^{-6}$ m$^3$/s; $T_w=64.0^\circ$C; $T_h=27.0^\circ$C; $T_e=32.0^\circ$C</th>
<th>$t$, s</th>
<th>$(\Delta T)_m$, $^\circ$C</th>
<th>$T_w$, $^\circ$C</th>
<th>$h'_i$, W/m$^2$K</th>
<th>$h_0$, W/m$^2$K</th>
<th>$U_{oe}$, W/m$^2$K</th>
</tr>
</thead>
<tbody>
<tr>
<td>E-1</td>
<td>6.0</td>
<td>120</td>
<td>47.73</td>
<td>58.9</td>
<td>204.13</td>
<td>199.30</td>
<td>114.40</td>
</tr>
<tr>
<td>E-2</td>
<td>8.0</td>
<td>240</td>
<td>38.89</td>
<td>53.0</td>
<td>197.97</td>
<td>171.95</td>
<td>104.02</td>
</tr>
<tr>
<td>E-3</td>
<td>10.0</td>
<td>360</td>
<td>32.29</td>
<td>47.7</td>
<td>169.58</td>
<td>143.36</td>
<td>89.20</td>
</tr>
<tr>
<td>E-5</td>
<td>14.0</td>
<td>480</td>
<td>27.60</td>
<td>45.1</td>
<td>167.89</td>
<td>111.85</td>
<td>76.55</td>
</tr>
<tr>
<td>E-6</td>
<td>16.0</td>
<td>600</td>
<td>23.50</td>
<td>43.0</td>
<td>174.68</td>
<td>87.57</td>
<td>64.34</td>
</tr>
</tbody>
</table>

### Table 3—Comparison of experimental and predicted Nusselt number

<table>
<thead>
<tr>
<th>Expt. No.</th>
<th>$U_{oe}$, W/m$^2$K</th>
<th>$De$</th>
<th>$Nu_e$</th>
<th>$Nu_p$</th>
<th>Error, %</th>
<th>% deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>E-1</td>
<td>89.62</td>
<td>435.0</td>
<td>0.914</td>
<td>0.873</td>
<td>77.40</td>
<td>4.5</td>
</tr>
<tr>
<td>E-2</td>
<td>126.55</td>
<td>221.48</td>
<td>1.294</td>
<td>1.223</td>
<td>75.70</td>
<td>5.5</td>
</tr>
<tr>
<td>E-3</td>
<td>156.03</td>
<td>125.64</td>
<td>1.582</td>
<td>1.624</td>
<td>74.90</td>
<td>-2.7</td>
</tr>
<tr>
<td>E-4</td>
<td>105.92</td>
<td>386.65</td>
<td>1.090</td>
<td>0.926</td>
<td>76.50</td>
<td>15.0</td>
</tr>
<tr>
<td>E-5</td>
<td>74.33</td>
<td>389.87</td>
<td>0.765</td>
<td>0.922</td>
<td>78.40</td>
<td>-20.5</td>
</tr>
<tr>
<td>E-6</td>
<td>94.68</td>
<td>274.03</td>
<td>0.975</td>
<td>1.099</td>
<td>77.10</td>
<td>0.6</td>
</tr>
</tbody>
</table>

\[(\Delta T)_m = \frac{(T_h - T_e) - (T_h - T_c)}{\ln \left( \frac{T_h - T_e}{T_h - T_c} \right)} \quad \ldots (3)\]
where Dean number, $De$ is defined as

$$De = \left( \frac{D_{eq} \mu_c v}{\mu_c} \right) \left( \frac{D_{eq}}{D_c} \right)^{0.5}$$

The power for Dean number in Eq. (10) is obtained from a plot of $Nu_e$ versus $De$. Table 3 shows the values of $De$, $U_\infty$, $Nu_e$ and $Nu_p$ which indicates that the experimental and predicted Nusselt number are in good agreement with a standard deviation of 0.104 and a variance of 0.0108. Moreover, the percentage deviation of each experiment is reported in Table 3. The relative heat transfer efficiency of half-coil jacket compared to jacketed vessel, $E$ in the present study is obtained using the following equation

$$E = \frac{1}{3} \left( 2 - \frac{\tan h mL}{mL} \right)$$

where $m^2 = \frac{U_\infty}{Kx_w}$

The values of the efficiency, $E$ (Eq. (12)) are reported in Table 3 with an average efficiency of 77%.

Conclusions—Cooling of a viscous liquid has been studied experimentally. The viscosity correction for inside heat transfer film coefficient has been done using the calculated wall temperature. The experimental Nusselt number is related to Dean number as

$$Nu = 18.2 \ (De)^{-0.5}$$

**Nomenclature**

- $A_i$: inside heat transfer area, m$^2$
- $A'$: cross-sectional area of the limpet, m$^2$
- $A_o$: heat transfer area on coil side, m$^2$
- $C_p$: specific heat of the coolant, J/kg K
- $C_{ph}$: specific heat of the hot liquid, J/kg K
- $h_i$: inside film heat transfer coefficient at any time, t, W/m$^2$ K
- $h_o$: outside film heat transfer coefficient at any time, t, W/m$^2$ K
- $K$: thermal conductivity of the coolant, W/m K
- $K_g$: thermal conductivity of the glass wall, W/m K
- $t_j$: time corresponding to $J$th reading, s
- $(\Delta t)_j$: time interval between $J$th and $J-1$th readings, s
- $T_a$: ambient temperature, °C
- $T_{ci}$: feed temperature of the coolant, °C
- $v$: average velocity of the coolant $(q/A'_c)$, m/s
- $\mu_h$: viscosity of the hot liquid at the bath temperature kg/ms
- $\rho_c$: density of the coolant, kg/m$^3$
- $\rho_h$: density of the hot liquid, kg/m$^3$
- $Nu_e$: experimental Nusselt number ($U_\infty D_{eq}/K$)
- $Nu_p$: predicted Nusselt number (Eq. (10))

**References**

5. Pavlov K F, Romankov P G & Noskov A A, Examples and problems to the course of unit operations of chemical engineering (Mir Publishers, Moscow), 1979, 548, 571.