Robust control of a pH process

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Model reference non-linear controller (MRNLC) with explicit integral and derivative actions is applied to a pH control of a strong acid-strong base system. The performance of the MRNLC is compared with that of a transformed non-linear controller. The inclusion of integral action makes the controller robust even at a lower value of feedback gain. The effect of derivative time and sampling time on the performance of MRNLC is also evaluated.

Due to the presence of severe non-linearity, control of a pH process is difficult, particularly for a strong acid-strong base system1. The performance of conventional proportional-integral-derivative (PID) controllers is not satisfactory for such systems1. The use of non-linear controllers is a necessity. Non-linear control methods such as non-linear adaptive control2, non-linear transformation method3, generic model control4 and non-linear internal model control5 are applied to the control of a pH process. A reference non-linear controller (MRNLC) model has been proposed for the control of a pH process6. The robustness of the controller under parameter variations are reported. However, a very large value of feedback gain is needed to eliminate the steady-state error. To overcome this problem, an integral action term arbitrarily in the MRNLC equation has been added6. Further, the value of the integral time is selected by trial and error and the controller does not have a derivative action. In the present work, the design of MRNLC is modified to have integral and derivative actions. Simple procedure is given to select the values of the integral and derivative times. It is to be noted that the integral action is required to make the closed-loop system off-set free under process/model parameter uncertainty and/or process disturbances. The derivative action is required to enhance the stability of the closed-loop system.

Model Development

In the system under consideration, a strong acid and a strong base flow separately into a well mixed tank and the flow rate of the strong base is manipulated to have a neutral outlet stream. The system model equations are given by3,6,7

\[ \dot{x} = f(x) + g(x)u + d(t) \]  \( \ldots (1) \)

\[ h(x,y) = 0 \]  \( \ldots (2) \)

where

\[ f(x) = -a_1x \]

\[ g(x) = -a_3(x + a_2) \]

\[ d(t) = a_1d_1 \]

\[ h(x,y) = x + 10^{y-14} - 10^{-y} \]  \( \ldots (3) \)

The control objective is to keep \( y(t) = y_s \) even in the presence of a disturbance in \( d(t) \).

**Design of an MRNLC**

Excellent review of non-linear controllers design are available8-11. The design procedure of an MRNLC is intuitive to the designer than the differential geometry control theory. The reported model reference non-linear controllers do not have explicit integral and derivative actions. Let us assume the reference model in the output \( y \) is given by

\[ y_m = \lambda_m y_m - \lambda_m y_i \]  \( \ldots (4) \)

\[ y_m(t = 0) = y_{m,0} \]

Where \( \lambda_m \) is the eigen value of the reference system and \( y_i \) is the desired final value of \( y \). The error is defined by

\[ e = y_m - y \]  \( \ldots (5) \)

\[ \dot{e} = \dot{y}_m - \dot{y} \]  \( \ldots (6) \)

Combining Eqs (1), (2) and (6), one gets

\[ \dot{e} = \dot{y}_m - Jf - Jgu - Jd. \]  \( \ldots (7) \)
where \( J = \frac{\partial h/\partial x}{\partial h/\partial y} \) \( \ldots (8) \)

Let

\[
y_m - Jf - Jgu - Jd = -k \int_0^t [e + (1/\tau_i)] \cdot \text{ed}t + \tau_D \cdot \dot{e}
\]

Then, one gets

\[
u = (Jg)^{-1} \left[ y_m - Jf - Jd + \left( e + (1/\tau_i) \int_0^t \text{ed}t + \tau_D \cdot \dot{e} \right) \right]
\]

The error dynamics Eq. (7) becomes

\[
\dot{e} = -k \left[ e + (1/\tau_i) \int_0^t \text{ed}t + \tau_D \cdot \dot{e} \right] \ldots (11)
\]

differentiating once with respect to time, one gets

\[
\tau_i \ddot{e} + 2 \tau_D \dot{e} + e = 0 \ldots (12)
\]

where

\[
\tau_i = (1 + k \tau_D)/(k/\tau_i) \ldots (13a)
\]

and

\[
\xi = \tau_i/(2.3) \ldots (13b)
\]

The error system [Eq. (12)] is stable for \( \xi > 0 \). For a second order system, \( \xi \) and \( \tau \) are related to the settling time\( ^2 \) as

\[
t_s = 5 \tau / \xi \ldots (14)
\]

Once \( \xi \) and \( t_s \) of the error system are specified then \( \tau_i \) and \( \tau_D \) are calculated from Eqs (13a) and (13b) as:

\[
\tau_i = 0.4 \xi^2 t_s \ldots (15a)
\]

\[
\tau_D = (0.1 t_s - (1/k)) \ldots (15b)
\]

The value of \( k \) has to be assumed. For \( \tau_D \) to be positive, we have to put a restriction on \( k \) to be

\[
k > 10/\tau_i \ldots (16)
\]

In case the same value of \( k \) and \( \tau_i \) are to be used for both MRNLC with PI action and MRNLC with PID action, then to have a smaller value of \( \tau \) [Eq. (13a)] for MRNLC with PID action than that with PI action, we have to take \( k \) such that

\[
0 < k \tau_D < 1 \ldots (17)
\]

In which case \( \tau_D \) can be negative.

In case of process/model perfect match with a non-zero initial error and error derivative, the error can be made decay to zero as per the dynamics of the Eq. (12) by using the values of \( k \), \( \tau_i \) and \( \tau_D \) from the Eqs (16), (15a) and (15b). The control law Eq. (10) is used to calculate \( u \) in order to get the desired error dynamics. For the pH problem, the control law Eq. (12) is given by

\[
u = \frac{2.303(-x + 2(10-y)(\lambda_m y - \lambda_m y_1 + \alpha) - a_1 x + a_1 d)}{(a_1 x + a_1 d)}/(a_1 x + a_1 d) \ldots (19)
\]

where

\[
a = k \left[ e + (1/\tau_i) \int_0^t \text{ed}t + \tau_D \cdot \dot{e} \right] \ldots (20)
\]

Results and Discussion

The system parameters values are the same as reported earlier\(^3\). The system Eq. (1) is solved using Euler numerical integration method with an integration step size of \( 10^{-5} \). At \( t = 1.0 \) s, the system is disturbed with a step input in inlet feed acid concentration by 100% of the nominal condition. Let us consider the regulation of pH at 7.0. Let us compare the performance of the present controller with that of the transformed non-linear PID controller\(^3\). The comparison result is shown in Fig. 1. As can be seen from Fig. 1, the performance of MRNLC, even without explicit integral and derivative actions, performs better than the transformed non-linear PID controller\(^3\). The condition given in Fig. 1 corresponds to perfect parameters values.

Let us first consider the MRNLC with integral action [i.e., \( \tau_D = 0 \) in Eq. (20)]. The robustness of

![Fig. 1—Comparison of the present controller with the transformed non-linear PID controller\(^1\) [sampling time = 1s; 1—present controller with \( \lambda_m = -5 \) and \( k = 10 \), \( \tau_i = \infty \), \( \tau_D = 0 \); 2—transformed non-linear PID controller\(^1\) with \( k = 100 \), \( \tau_i = 1.3 \) and \( \tau_D = 0 \)]](image)
the present control system for +25% or -25% perturbation in the parameter \(\alpha_3\) is studied. First consider the perturbation on positive side, i.e., the value of \(\alpha_3\) used in the control law is 1.25 times that of the nominal value entering the process. Similarly in the other case, i.e., perturbation on the negative side, the value of \(\alpha_3\) used in the control law is 0.75 times that of the nominal process value. A robust response is obtained on both the occasions. The robustness of the controller is compared with that of the MRNLC with no explicit integral action (i.e., with \(\tau_i = \infty\) and \(\tau_D = 0\)). The comparative performances are given in Fig. 2. A significantly lower value of feedback gain \((k)\) gives a robust response, if integral action is included. If there is no explicit integral action, very larger value of \(k\) (1,000,000) has to be used to make the controller robust.

The effect of changing the value of \(\xi\) (keeping \(\tau_i\) constant) on the controller’s performance is also studied. Three different values (0.5, 0.75 and 1.0) of \(\xi\) are used. Under perfect model parameters, the performance is same for all the values of \(\xi\). However, under parametric perturbations, the response is affected by \(\xi\). It is found that \(\xi = 0.5\) gives a better result. The performance of the controller remains unaffected by settling time \((\tau_i)\) for the ranges studied (from 0.1 to 1 s). However, under the parametric perturbations, settling time has a serious effect and a value of \(\tau_i = 0.1\) is found to give a better result.

Under perfect model parameter values, sampling period of up to 1.0 s gives excellent results. But, when there are perturbation in parameters (say +25% or -25% in \(\alpha_3\)), sampling period plays a key role on the controller’s performance. Very small sampling periods have to be used. A sampling period of 0.0001 s or lesser than that only gives good results. This is shown in Figs 3a and 3b. The need for such lower sampling periods for parameter uncertainty was also reported."
Fig. 4a—Effect of derivative mode on sampling period [Different values of $k$ are used in MRNLC with PID and PI actions; 2,3: MRNLC with PI action, $k = 100$, 4,5: MRNLC with PID action, $k = 50$, 2,4: $+25\%$ perturbation in $a_3$, 4,5: $-25\%$ perturbation in $a_3$, other conditions: as in Fig. 3b]

Even a sampling period of 0.0002 s makes the system oscillatory particularly for negative side perturbations ($-25\%$) in $a_3$. However, it gives good results for positive side perturbations.

Effect of derivative mode—In the present simulation study, it is observed that the controller’s performance is not significantly improved by the inclusion of the derivative mode with a sampling period of 0.0001 s. This is shown in Figs 3a and 3b, where the performance of the controller without the derivative mode [i.e., $\tau_D = 0$ in Eq. (20)] is compared with the one with the derivative mode. In Fig. 3a the values of the feedback gain used are different for MRNLC with PI and MRNLC with PID mode. Whereas, in Fig. 3b the same value of feedback gain is used. However, inclusion of the derivative mode permits the use of larger sampling periods. This is shown in Figs 4a and 4b. With the inclusion of the derivative mode, even the sampling period of 0.0002 s can be used. Whereas, only a sampling period of 0.0001 s gives good results in case there is no derivative mode. The same value of $k$ is used for MRNLC with PI and MRNLC with PID actions in Fig. 4b.

The effect of measurement noise is studied by adding a random number (with mean = 0 and standard deviation = 0.1 or 0.2) to the actual value of pH. The corrupted value of pH is used for the calculation of $de/dt$ by backward difference formula. The closed-loop results given in Table 1 show a good response of the proposed controller.

Table 1—Effect of measurement noise in $y$ on the performance of the closed-loop system

\begin{tabular}{|c|c|c|c|}
\hline
S. No & Time & $A$ & $B$ & $C$ \\
\hline
1 & 0.25 & 7.0 & 7.0 & 7.0 \\
2 & 0.50 & 7.0 & 7.0 & 7.0 \\
3 & 0.75 & 7.0 & 7.0 & 7.0 \\
4 & 1.00 & 7.0 & 7.0 & 7.0 \\
5 & 1.25 & 7.0 & 7.006 & 7.004 \\
6 & 1.50 & 7.0 & 6.989 & 6.951 \\
7 & 1.75 & 7.0 & 6.992 & 6.964 \\
8 & 2.00 & 7.0 & 7.008 & 7.013 \\
9 & 2.25 & 7.0 & 7.000 & 7.003 \\
10 & 2.50 & 7.0 & 7.000 & 6.996 \\
11 & 2.75 & 7.0 & 7.002 & 6.989 \\
12 & 3.00 & 7.0 & 7.002 & 7.008 \\
13 & 3.25 & 7.0 & 7.002 & 6.998 \\
14 & 3.50 & 7.0 & 6.980 & 6.914 \\
15 & 3.75 & 7.0 & 7.013 & 7.044 \\
16 & 4.00 & 7.0 & 6.992 & 6.938 \\
17 & 4.25 & 7.0 & 7.014 & 7.069 \\
18 & 4.50 & 7.0 & 6.993 & 6.958 \\
19 & 4.75 & 7.0 & 6.994 & 6.976 \\
20 & 5.00 & 7.0 & 7.008 & 7.043 \\
\hline
\end{tabular}

\begin{itemize}
\item A—without measurement noise in $y$ for control calculation
\item B—with measurement noise in $y$ (mean = 0, standard deviation = 0.1)
\item C—with measurement noise in $y$ (mean = 0, standard deviation = 0.2)
\end{itemize}
Conclusion
The inclusion of an integral action in the model reference non-linear control law makes the closed-loop system robust even at a lower value of feedback gain. A method is given to select the values of the integral and derivative times. The inclusion of derivative mode permits us to use a larger sampling time. The performance of MRNLC is better than that of a transformed non-linear PID controller.

Nomenclature

\[ x = \text{deviation from neutrality} \]
\[ Y = \text{output variable} \]
\[ y_m = \text{reference value for output} \]
\[ y_c = \text{desired new steady-state value for } y \]
\[ y_{m,0} = \text{value of } y_m \text{ at } t = 0 \]

Greek letters

\[ \alpha = \text{defined by Eq. (22)} \]
\[ \lambda_m = \text{eigen value of the error system} \]
\[ \xi = \text{damping coefficient of the error dynamics} \]
\[ \tau = \text{effective time constant of the error system} \]
\[ t_i = \text{integral time, s} \]
\[ t_D = \text{derivative time, s} \]

References