A generalized correlation of nucleate pool boiling heat transfer

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Following the concept of transient heat condition and latent heat transport mechanisms to operate simultaneously to describe heat transfer rate during nucleate pool boiling of liquids, and the expressions of nucleation site density, bubble departure diameter and bubble emission frequency, a correlation for boiling of liquids under subatmospheric pressures has been developed. The correlation is free from the term accounting the effect of surface characteristics of the heating surface. The values of heat transfer rate predicted from correlation agree well with the experimentally-determined values for the boiling of various liquids from heating surfaces of differing characteristics employed in different investigations. The resulting correlation is capable to examine the consistency of experimental data of nucleate pool boiling of liquids under subatmospheric pressures.

Nucleate boiling heat transfer is a surface phenomenon. All the events in the process of heat transfer from heating surface to the boiling liquids: the initiation of vapour bubbles on the heating surface, their growth and detachment to travel in the pool of liquids are greatly influenced by the characteristics of the heating surfaces. Its measurement in terms of the size, shape and population distribution of the nucleation sites on the heating surface is highly improbable. Therefore, almost all the correlations, available in literature, are empirical or semi-empirical in nature. They contain invariably a term which takes into account the surface-liquid combination of the investigation. Due to this reason, no correlation seems to be capable to generalize the experimental data of nucleate pool boiling of various liquids conducted on heating surfaces of differing surface characteristics. This paper attempts to discuss a correlation of nucleate pool boiling heat transfer which is free from heating surface characteristics and hence can generalize the experimental data of various investigators employing heating surfaces of different characteristics at atmospheric and subatmospheric pressures.

Analytical Model

The complexity of nucleate boiling makes it impossible to explain the phenomenon by a single mechanism. As a matter of fact, it may be considered to include several mechanisms operating simultaneously. In the region of isolated vapour bubbles, heat flow from the heating surface to saturated liquids is the combination of (i) transient conduction to and the subsequent replacement of the superheated liquid layer adjacent to the heating surface, (ii) latent heat transport, and (iii) the natural and forced convection at areas of the heating surface outside the area of influence of each bubble. The contribution by each of the mechanisms depends on the value of the heat flux, and the pressure. For high values of heat flux, and subatmospheric pressure conditions, the area outside the influence of each vapour bubble becomes small to render the contribution by the natural and forced convection to be insignificant. Therefore, the following analysis considers the first two mechanisms responsible for heat transfer during boiling of liquids from heating surfaces at subatmospheric pressures.

Heat transfer by transient conduction to and subsequent replacement of superheated liquid layer adjacent to the heating surface is given by Mikic and Rohsenow

\[ Q = 2 \sqrt{\pi} \left( \sqrt{K_\rho L C_i} \right) \sqrt{f} D_0^2 \Delta t_w N \]  

(1)

Rallis and Jawurek have developed the following latent heat transport model to calculate heat transfer rate:

\[ Q = \frac{\pi}{6} D_3 \rho \lambda f N \]  

(2)

Eqs (1) and (2) assume vapour bubble to be spherical in shape. Combination of Eqs (1) and (2)
results the following expressions for the heat transfer rate:

\[ Q = [K' 2\pi (K'\rho c) \sqrt{D_b} \Delta t_w + K'' (\pi \frac{D_b}{6} \rho \lambda f)] N \]  

... (3)

Where \( K' \) and \( K'' \) represent the fraction of heat transfer contributed due to transient conduction and due to the latent heat transport, respectively. Since physico-thermal properties of the liquid are the functions of pressure, the values of \( K' \) and \( K'' \) also depend upon the pressure.

### Number of Nucleation Sites

The number of nucleation sites can be determined by the following equation developed by Brown:

\[ N = C_0 \left( \frac{r_i}{r_c} \right)^m \]  

... (4)

Where \( C_0 \) and \( m \) are the constants characterizing the heating surface and \( r_i \) is the radius of the nucleation sites for which the site density, \( \frac{N}{A} \) is one per unit area.

Eq. (4) is transformed into the following equation by the use of Laplace equation and Clausius-Clapeyron equation for subatmospheric pressure conditions, i.e., \( \rho_v << \rho_1 \)

\[ N = C_0 r_i^m \left( \frac{\lambda \rho_i \Delta t_w}{2 \sigma T_s} \right)^m \]  

... (5)

### Bubble Emission Frequency

The bubble emission frequency for the boiling of liquids under subatmospheric pressure is obtained by the following equation due to Hatton and Hall:

\[ f = \frac{3}{\pi \alpha} \left( \frac{16 \lambda \sigma T_s}{(\lambda \rho_v)^2 D_b D_c} \right)^2 \]  

... (6)

### Bubble Departure Diameter

Following equation, developed by Cole and Rohsenow, is used to calculate bubble departure diameter for the boiling of liquids under subatmospheric pressures:

\[ D_b = C g^{\frac{2}{5}} \left( \frac{\sigma}{g(\rho_o - \rho_v)} \right)^{0.5} \]  

... (7)

Where \( C \) is a constant whose value is \( 1.5 \times 10^{-4} \) for organic liquids and \( 4.6 \times 10^{-4} \) for distilled water.

Using the values of number of nucleation sites, \( N \); bubble emission frequency, \( f \); and the bubble departure diameter, \( D_b \) from the respective Eqs (5)-(7) into Eq. (3), the following expression for boiling heat transfer rate at subatmospheric pressures is obtained:

\[ q = \left[ \sqrt{3} K' + K'' \right] \frac{8 C_o r_i^m \frac{\rho_1^{1.75} C_i^{2.25} K_i T_i^{1.25 - m}}{2^{m} 8^{0.5} 8^{2.25 - m} \rho^{2.25 - m} \sigma^{m - 0.5}} \right] \times \Delta t_w^{m + 2} \]  

... (8)

The value of exponent, \( m \) in Eq. (8) is determined by the relationship between wall superheat, \( \Delta t_w \) and the heat flux, \( q \) for the boiling of liquids under subatmospheric pressures. For this purpose, the experimental data of wall superheat have been plotted against heat flux for the boiling of distilled water with pressure as a parameter in Fig. 1. This plot reveals the following noteworthy features:

1. For a given value of pressure, wall superheat, \( \Delta t_w \) increases with rise in heat flux, \( q \) according to the relationship, \( \Delta t_w \sim q^{0.3} \)

2. The value of wall superheat, decreases with the increase in pressure, at a given heat flux.

These features have been consistently observed by other investigators and also for the boiling of distilled water and other liquids at atmospheric conditions.
and subatmospheric pressure. Therefore, the relationship, \( \Delta t_w \propto q^{0.3} \) is employed to calculate the value of exponent, \( m \). It is found to be 1.33. On substituting the value of \( m \) in the Eq. (8) the following expression is obtained:

\[
q = \left[ (3K' + K'') \right] \frac{8C_0r_1^{\frac{1}{3}}C}{2^{\frac{1}{1.33}} S^{0.5}} \left[ \frac{\rho_1^{0.17} C_1^{0.25} K_1}{T_0^{0.08} \kappa_0^{0.92} \rho_0^{0.92} \sigma^{0.83}} \right] \times \Delta t_w^{3.33}
\]

or

\[
\Delta t_w = M[\rho_1^{0.525} C_1^{0.675} K_1^{0.3} T_0^{0.024} \kappa_0^{0.276}] \\
\times \rho_0^{0.276} \sigma^{0.249} q^{0.3}
\]

Where

\[
M = \left( \frac{(3K' + K'')}{2^{\frac{1}{1.33}} S^{0.5}} \right)^{0.3}
\]

The determination of \( M \) requires the knowledge of the terms \( K', K'', C_1, \) and \( r_1 \). Since \( C_1 \) and \( r_1 \) depend upon the heating surface characteristics whose estimation is highly improbable, the analytical determination of the term, \( M \) is most unlikely. Therefore, Eq. (9) cannot be employed in its present form to calculate wall superheat. However, this difficulty can be overcome by adopting the following methodology:

\( M \) can be considered to be a lumped parameter of pressure and heating surface characteristics, as represented by the following equation:

\[
M = F_1(p) F_2(Csf)
\]

The function \( F_2(Csf) \) does not depend upon pressure for a given surface-liquid combination. Hence, the function \( F_2(Csf) \) disappears when the ratio, \( M/M_0 \) is attempted where the value of \( M \) at the reference pressure. An implication of this is that \( \Delta t_w/\Delta t_{w,0} \), a ratio of wall superheat at any pressure to that at the reference pressure, \( P_0 \), becomes independent of surface-liquid combination factor. Therefore, the following equation of \( \Delta t_w/\Delta t_{w,0} \) is obtained from Eq. (9):

\[
\frac{\Delta t_w}{\Delta t_{w,0}} = \left( \frac{M}{M_0} \right)^{-0.525} \left( \frac{C_1}{C_{1,0}} \right)^{-0.675} \left( \frac{K_1}{K_{1,0}} \right)^{-0.3} \\
\times \left( \frac{T_0}{T_{w,0}} \right)^{0.024} \left( \frac{\lambda}{\lambda_0} \right)^{0.276} \left( \frac{\rho_0}{\rho_{v,0}} \right)^{0.276} \left( \frac{\sigma}{\sigma_0} \right)^{0.249} \left( \frac{q}{q_0} \right)^{0.3}
\]

To determine the value of \( M/M_0 \) of Eq. (11), the experimental data for the nucleate pool boiling of benzene on stainless steel cylinder at atmospheric and subatmospheric pressure are used. The values of \( M/M_0 \) are calculated by using the corresponding experimental values of \( \Delta t_w/\Delta t_{w,0} \) and the ratio of other terms appearing in Eq. (11). These values have been plotted against \( P/P_0 \) in Fig. 2. This plot indicates the following relationship:

\[
\left( \frac{M}{M_0} \right) = \left( \frac{P}{P_0} \right)^{-0.589}
\]

Substitution of Eq. (12) into Eq. (11) results the following correlation:

\[
\frac{\Delta t_w}{\Delta t_{w,0}} = \left( \frac{P}{P_0} \right)^{-0.589} \left( \frac{\rho_1}{\rho_{1,0}} \right)^{-0.525} \left( \frac{C_1}{C_{1,0}} \right)^{-0.675} \left( \frac{K_1}{K_{1,0}} \right)^{-0.3} \\
\times \left( \frac{T_0}{T_{w,0}} \right)^{0.024} \left( \frac{\lambda}{\lambda_0} \right)^{0.276} \left( \frac{\rho_0}{\rho_{v,0}} \right)^{0.276} \left( \frac{\sigma}{\sigma_0} \right)^{0.249} \left( \frac{q}{q_0} \right)^{0.3}
\]

Eq. (13) is free from surface-liquid combination factor. Hence, it can be employed to calculate the value of \( \Delta t_w/\Delta t_{w,0} \) for the boiling of various liquids from heating surfaces of differing characteristics at subatmospheric pressure conditions. This equation is also useful for examining the consistency of experimental data obtained on different heating surfaces.

Testing of the Correlation

The correlation, Eq. (13) was tested against the experimental data for the boiling of various liquids from different heating surfaces. Fig. 3 compares the experimental and predicted values of \( \Delta t_w/\Delta t_{w,0} \) using Eq. (13) for the experimental data of Sharma et al. for distilled water, ethanol, metha-
heat for the boiling of liquids under atmospheric and subatmospheric pressures on a given heating surface from the knowledge of wall superheat of the same liquid on the same heating surface at a known value of pressure and heat flux.

2 The correlation provides a procedure to check the consistency of experimental data of nucleate pool boiling heat transfer obtained on heating surfaces having differing surface characteristics under subatmospheric pressures.

Nomenclature

- $A$ = heating surface, area, $m^2$
- $C$ = specific heat, $J/kg K$
- $Cs_l$ = surface liquid combination factor
- $D_b$ = bubble departure diameter, $m$
- $D_n$ = diameter of nucleation site, $m$
- $f$ = bubble emission frequency, $1/s$
- $g$ = acceleration due to gravity, $m/s^2$
- $h$ = heat transfer coefficient, $W/m^2K$
- $K$ = thermal conductivity, $W/mK$
- $N$ = number of nucleation sites
- $P$ = pressure, $N/m^2$
- $Q$ = heat transfer rate, $W$
- $q$ = heat flux, $W/m^2$
- $r_n$ = radius of nucleation site, $m$
- $T$ = temperature, $K$
- $\Delta t_w$ = wall superheat, $(T - T_w)$, $K$
- $\rho$ = density, $kg/m^3$
- $\lambda$ = latent heat of vaporization, $J/kg$
- $\alpha$ = thermal diffusivity, $k/\rho C, m^2/s$
- $\sigma$ = surface tension, $N/m$

Subscripts

- $l$ = liquid
- $o$ = reference
- $v$ = saturation
- $w$ = wall

Dimensionless Group

- $Ja^*$ = modified Jacob Number, $(\rho, C_l T, \rho, \lambda)$

References

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