Adaptive optics and its applications

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The adaptive optics (AO) system combines technologies that enable corrections in real time for the deleterious effects of the atmosphere allowing terrestrial telescopes to achieve their near diffraction-limit. Such a system introduces controllable counter wavefront distortions, which both spatially and temporally follow that of the atmosphere. AO system has advantages over post-detection image restoration techniques that are limited by noise and recovers near diffraction-limited images and improves the point source sensitivity. This system may become standard tool for the new generation large optical imaging systems. Development of such a technique was made possible due to the significant improvements in technological innovation over the past few decades. After a brief introduction on the deleterious effects of the atmosphere on the image, this article discusses in detail about required components in order to develop the AO system and its applications in various fields, particularly in observational astronomy.

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1 Introduction

The twinkling of stars in the night sky has been viewed with awe and admiration by the mankind from times immemorial. Ever since the invention of telescope, the phenomena of atmospheric turbulence, which cause the twinkling of stars, has frustrated the astronomers who need to look into deep space to unravel the secrets of the universe. The turbulence is because the various layers of the atmosphere have different refractive indices depending on the temperature and pressure at that height. In such situation the resolution of conventional astrophotography with large telescopes is not limited by the diffraction on the finite aperture, but with the size of the atmospheric turbulence cell. Typically the size of a turbulence bubble is 10 cm. Diffraction on the bubbles causes an angular blur of \( \frac{\lambda D}{2 \pi} \approx 500 \text{ nm}/10 \text{ cm} = \text{an arc second} \), in which \( \lambda \) is the wavelength of the observation. This is the maximum resolution that can be obtained irrespective of aperture size of the telescope. The higher resolution can be achieved inspite of the image degradation by such cells either by using a passive method called speckle imaging techniques or by using a direct method known as adaptive optics. An alternative arrangement, however, was made by deploying Hubble space telescope above the atmosphere in order to make sharp images, but its size is small compared to a dozen 4-10 metre class telescopes operating on the ground. Consistent efforts of scientists, spread over two decades, have led to the development of the AO technology for compensating atmospheric turbulence effects by introducing an optical imaging system in the light path, which senses the perturbations and compensates in real time.

2 Diffraction Limited Resolution

An idealized astrophysical source of monochromatic radiation, propagating in the absence of the atmosphere, is known as plane wave having uniform magnitude and phase across the telescope aperture. All telescopes have inherent limitation to their angular resolution due to the diffraction of light at the telescope's aperture. When a continuum of wave components pass through an aperture, the superposition of these components result in a pattern of constructive and destructive interference. For astronomical instruments, the incoming light is approximately a plane wave since the source of the light is so far away. In this far-field limit, Fraunhofer diffraction occurs and the pattern projected onto the focal plane of the telescope will have little resemblance to the aperture.
Let us examine how does the diffraction pattern of a star look like and what is the limit of a telescope's resolution. For wavefunction $\Psi$, at point $P$, in the imaging plane, the intensity is given by:

$$I(\theta) = |\Psi|^2 = I_0 \left[ 2J_1(u) / u \right]^2$$  \hspace{1cm} (1)

and

$$u = ka \theta$$  \hspace{1cm} (2)

where $k = 2\pi/\lambda$ is the wave number, $a$ is aperture size, and $\theta$ is the angle between telescope axis and range for the point $P$.

This intensity pattern of constructive and destructive interference rings is known as the Airy diffraction pattern (see Figure 1). 84% of the total intensity is located within the central circle or the Airy disk. The dark destructive interference rings occur at the minimum of $J_1(u)$, where $u = 3.83, 7.02 \ldots$ or $\theta = 0.61\lambda/a, 1.12\lambda/a, \ldots$.

The limit for the telescope's resolution is set by the diffraction at the aperture of the telescope. For a point source, like a star, the resulting image is a Airy pattern. The Rayleigh criterion for resolution of two point sources is that the central maximum of one image lies at the first minimum of the second image. Therefore the limit of the angular resolution is,

$$\Delta \theta = 1.22 \lambda / D$$  \hspace{1cm} (3)

The Airy disk has angular radius $\Delta \theta$, so the radius of the central disk is,

$$\rho = \lambda f / D$$  \hspace{1cm} (4)

where $f$ is the focal length.

3 Atmospheric Turbulence

Diffraction limited resolution is an ideal condition for the image which never happens in real life mainly due to atmospheric turbulence that is considered to be highly turbulent medium. Heating of the Earth's atmosphere by solar radiation causes turbulent motions. According to the Kolmogrov's theory of fluid turbulence, when the kinetic energy of the air motions at a given length-scale is larger than the energy dissipated as heat by viscosity of the air at the same scale the kinetic energy of large scale motions would be transferred to smaller and smaller motions; motions at small scales would be statistically isotropic; at the small scales, viscous dissipation would dominate the break-up process. During day time, large warm packets of air closer to the ground move up due to buoyancy and initiate convection causing the turbulence near the ground. They dissipate their kinetic energy continuously and randomly into smaller and smaller packets of air, each having unique temperature. These packets are called eddies. Convection changes with isolation and disappears during night time. However, horizontal circulation of air starts. Kolmogrov law represents the distribution of the turbule sizes, from millimeters to meters, with lifetimes varying from milliseconds to seconds. An important property of eddies is that they exist in a variety of length-scales and their distribution is random. There exists an upper limit, $L_o$, decided by the process that generates turbulence and a lower limit, $l_o$, decided by the size at which viscous dissipation overtakes the break-up process.

The refractive index values fluctuate with time due to fluctuations in temperature and pressure and are random in nature. The refractive index fluctuations follow a power law with large eddies having greater power. To a first order approximation, the refractive index of air is related to the pressure and temperature as,

$$n = 7.9 \times 10^{-2} \frac{P}{T} + 1$$  \hspace{1cm} (5)

where $P$ and $T$ are given in units of atmosphere and kelvin respectively.
The telescope is a key element in transforming the data from the image plane of the telescope into the image plane of the PSF.

Real condition of the atmospheric life mainly is considered to be turbulent. The Earth's surface is turbulent due to its theory of energy, further the energy of the air is statistically independent of the air at the height. These turbulent motions are called as dissipation motions; and horizontal motions are called as statistically independent of the dissipation motions. During day time, these motions lead to the ground break-up of the convection currents and hence, to the refractive index. This index changes in the course of propagation. This can lead to image jitter, intensity fluctuations (scintillations) and image spreading; in aggregate, the effect will blur the image.

3.1 Statistical model

Atmosphere is a non-stationary random process and the seeing conditions evolve with time, therefore, one also need to know the statistics of their evolution, mean value and standard deviation for a given atmosphere. Environment parameters, viz., thermal gradients, humidity fluctuations and wind shears produce atmospheric turbulence. The wavefront of an image will change in the course of propagation. This translates into image jitter, intensity fluctuations (scintillations) and image spreading; in aggregate, the effect will blur the image.

From Eq. (5), the phase variations, expressed in terms of the optical path difference (OPD), caused by a cell of length $L$, would be:

$$OPD(waves) = 7.9x\frac{P}{\lambda T^2} L \Delta T \approx 2L \Delta T \quad \ldots (6)$$

for visible wavelengths at standard temperature and pressure.

A small temperature fluctuation of one tenth of a degree would thus generate strong wavefront perturbations over a propagation distance of a few hundred metres. Naturally occurring variations in temperature ($<$1 °C) cause random changes in the wind velocity, which we view as turbulent motion in the atmosphere. Further, the changes in temperature give rise to small changes in atmospheric density, and hence, to the refractive index. This index changes of the order of $10^{-6}$ and can accumulate. The cumulative effect can cause significant inhomogeneities in the index profile of the atmosphere. Environment parameters, viz., thermal gradients, humidity fluctuations and wind shears produce atmospheric turbulence. The wavefront of an image will change in the course of propagation. This translates into image jitter, intensity fluctuations (scintillations) and image spreading; in aggregate, the effect will blur the image.

The value of the initial subrange, $l_0 < \rho < L_0$, in which $\rho$ is the vector between the two points of interest, would be different at various locations at the site.

Diffraction and refraction of the wavefront by atmospheric inhomogeneities cause the most severe time varying effects. The temporal structure function is simply obtained by substituting $|v|\tau$ for $\rho$ in Eq. (7), in which $v$ is the velocity of the wind and $\tau$ is time constant. The refractive index is fairly wavelength independent from red to thermal infrared, therefore, the above quantity is wavelength independent.

3.2 Turbulence and wind profile models

The spatial and temporal characteristics of field data of wavefront distortions convey essential information. However, the numerical evaluation of the critical parameters requires the knowledge of the $C_n^2$ and wind profiles as a function of altitude. Refractive index structure constant $C_n^2$ is a measure of the strength of the turbulence. It is not constant but keeps varying with seasons, daily and hourly. It also varies with geographic location and with altitude and can be perturbed very easily by artificial means such as aircraft. Its unit is $m^{-2/3}$. Since most of the above parameters are directly or indirectly related with $C_n^2$, the following are the models used to model $C_n^2$.

The most widely preferred two models are:

- SLC-Day model
- Hufnagel-Valley model
- 21 model
- 21 model

The SLC-Day turbulence model has two free parameters, $A$ and $W$. The first is normally set to $1.7 \times 10^{-14} m^{-2/3}$, while the second, which represents the average wind speed, can be adjusted to achieve the desired low-altitude shape. The most common value for $W$ is 21 m/s, which yields an expression referred as the HV-21 model. $C_n^2$ variations with altitude are shown in the Figure (2a).

SLC-Day turbulence model is described as,

$$C_n^2(h) = 4.008 \times 10^{-14} h^{1.504} \quad 0 \text{ m} < h < 19 \text{ m}$$

$$= 1.300 \times 10^{-15} \quad 19 \text{ m} < h < 230 \text{ m}$$

$$= 6.352 \times 10^{-7} h^{-2.966} \quad 850 \text{ m} < h < 7,000 \text{ m}$$

$$= 6.209 \times 10^{-16} h^{6.6229} \quad 7,000 \text{ m} < h < 20,000 \text{ m} \quad \ldots (8)$$

The Hufnagel-Valley turbulence model is described as,

$$C_n^2(h) = 5.94 \times 10^{-33} \left(\frac{W}{27}\right)^2 h^{10} \exp\left(-h/1000\right) + 2.7 \times 10^{-16} \exp\left(-h/1500\right) + A \exp\left(-h/100\right) \quad \ldots (9)$$

$$\Delta$$

where $\Delta$ is the vector between the two points of interest, would be different at various locations at the site.
4 Critical Atmospheric Parameters

The perturbations in the wavefront produce effects similar to optical aberrations in the telescope and thus degrade the image quality. When a very small aperture is used, a small portion of the wavefront is intercepted and the phase of the wavefront is uniform over the aperture. If the amplitudes of the small scales corrugation of the wavefront are much smaller than the wavelength of the light, the instantaneous image of a star is sharp and resembles the classical diffraction pattern (see Figure 1). But as the wind moves the eddies past the aperture, the tilt of the intercepted wavefront changes. This change in tilt causes random motion of the stars image at the focal plane. As the aperture size increases, there is decrease in the sharpness and amplitude of the motion. When a large aperture is used, the amplitude of the random variation of phase across the intercepted wavefront is larger. This leads to the blurring of the image. The image motion and blurring together are referred to as atmospheric seeing. Knowledge of the following atmospheric parameters is essential to any system designer and needs to be characterized accurately. All these parameters are very much dependent on the turbulence strength constant ($C_n^2$) and the wind velocity.

4.1 Atmospheric coherence length

Atmospheric coherence length $r_0$, widely known as Fried’s parameter, is defined as the transverse aperture through which the beam can be transmitted with optical phase distortion mean square value within 1 rad. It is a function of the turbulence strength constant $C_n^2$, wavelength of the light, altitude, slant angle of beam direction and of course, path length. Seeing angle through atmosphere is inversely proportional to the Fried’s parameter and is given by,

$$r_0 = 3.0(C_n^2k^2)^{3/5} \quad \text{...(11)}$$

It is important to note that the quality of seeing is characterised by Fried’s parameter, i.e.,

$$\theta_s = 0.976\lambda/r_0 \quad \text{...(12)}$$

in which $\theta_s$ is the seeing disk that determines the image quality.

The seeing disk is defined as the full width at half maximum (FWHM) of a Gaussian function fitted to a histogram of image position in arc sec. The seeing fluctuates on all time scales down to minutes and seconds. At a given site, $r_0$ varies dramatically night to night. It can be a factor 2 better than the median or vice versa. Figure 3 depicts the night time variations of $r_0$ on 28-29 March 1991 at the Cassegrain focus of the 2.34m Vainu Bappu Telescope (VBP). It is found that average observed $r_0$ is higher during the later part of the night than the earlier part.

4.2 Atmospheric time constant

Changes in the refractive index in different portions of the aperture result to the phase changes in the value of the aperture function. Turbulence can be thought as a spatial (spatial frequency) and temporal (temporal frequency) point spread function (PSF). The image aberration is given by:

$$\tau_0 = 0$$

The integral exposure is then given for short-exposure (usually atmospheric dispersion) and long-exposure (dispersionless exposure).

4.3 Isoplanatic Point

Turbulence is statistically independent over a given direction; therefore $\theta_s$ denotes the isoplanatic angular offset.

The statistical independence of atmospheric turbulence over a given direction:
... (11)

Figure 3 - Night time variations of $r_0$ at the 2.34 m VBT site, Kavalure, India, on 28-29 March, 1991 (Ref. 9). The solid line curve is for the zenith distance corrected value, while the dotted curve is for the uncorrected value.

![Graph showing night time variations of $r_0$.]
crescent shaped pattern. Wind slewing causes the beam to take on a characteristic crescent shaped pattern.

5 Adaptive Optics

For long range and high resolution imaging it is essential to compensate for atmospheric turbulence in real time. Adaptive optics is the only technology available currently for real-time correction of atmosphere turbulence. Optical phase conjugation is the method used for compensation of atmosphere induced aberrations. It is a multi-disciplinary subject and a late entry among the list of current technologies. In recent years, the technology and practice of adaptive optics have become, if not common place, at least well-known in the defence and astronomical communities. The purpose of AO system is to (i) sense the wavefront perturbations, and (ii) compensate them in real time\(^1,12\).

Figure 4(a) depicts the plane wavefront that generated at the laboratory with a laser source offering zero volt to the tip-tilt mirror. While Figure 4(b) depicts the wavefront tilt measured with same source after applying 1 V by computer to the said mirror\(^3\). These images are grabbed by a CMOS imager based Shack-Hartman (SH) sensor\(^4\). These plane and tilted wavefront (Figure 4 (a)) Plane wavefront (b) Tilted wavefront recorded at the laboratory (Courtesy: V. Chinnappan). resemble to the wavefront arriving to a detector from a distant star before and after passing through the turbulence of the atmosphere respectively. The laboratory experiment shows only tilt as a major error, while in the case of atmosphere, the wavefront tilts have complicated contours. Nevertheless, a reverse situation can be created by employing the AO systems in order to improve the throughput of the large telescope.

There is no single inventor of adaptive optics. The technology has evolved over years due to contribution of numerous scientists and engineers. In 1953, Babock\(^5\) proposed scheme to correct for the rapidly changing atmospheric seeing effects.

Although his suggestions have been thoroughly researched by the US military, only since the mid-1980s Babock's ideas were developed for the astronomical use.

For unresolved sources, adaptive optics attempts to put as many photons in as small an image area as possible, thus enhancing the image contrast against the sky background thereby improving the resolution, and allowing better interferometric imaging with telescope array. For resolved sources, the improved resolution extends imaging to fainter and more complex objects. "Active" optics is a technique similar to adaptive optics, but its purpose is to correct for wavefront distortions caused by the relatively slow mechanical, thermal and optical effects in the telescope itself. These corrections are made at frequencies \(\sim 1\) Hz.

The main components of an AO system are: telescope, combination of flexible mirrors such as tilt mirror (TTM), deformable mirror (DM) whose surface can be electronically controlled in real time to create a conjugate surface enabling to compensate the wavefront distortion, wavefront sensor (WFS), wavefront phase error computation\(^12,16-18\), and a laser source to generate an artificial guide star (beacon). A typical adaptive optics imaging system (AOIS) is illustrated in Figure 5.

Performance of such imaging system is close to the diffraction limit of the input aperture and can only be achieved in the limit of:

- The tip-tilt angles are not restricted by the mechanical structure of the observatory.
- The tilts of the TTM and DM are stable for at least one rotation of the field.
- A laser guidance system maintains the stability of the beam through the atmosphere.

5.1 Telescope

Non-zero coherence length, atmospheric turbulence and long range propagation of the signal can reduce the signal to noise ratio. The best case scenario would be the paraboloidal mirror telescope with a large mirror and low transmission. The diffraction limit of the array is difficult to improve beyond the best case scenario by making smaller mirrors due to cost and maintenance of the large structure. Therefore, a combination of slow and fast correction of the wavefront is required. The most common application of adaptive optics is to reduce the effects of seeing on images taken with large telescopes.
The angular separation between the turbulence probe and the target object should be smaller than the isoplanatic angle.

- The spacing between the control elements on the DM should be well matched to the turbulence coherence length.

- A sufficiently high update rate should be maintained i.e. less than inverse of the coherence time.

5.1 Telescope

Normally Cassegrain type telescope used for transmitting the beacon as well as receiving the optical signal for the WFS and imaging camera. For long range AOIS, beacon laser is needed to improve the signal-to-noise ratio (SNR) for the wavefront signal. There are two major components of the telescope: primary mirror and secondary mirror. The best combination of the mirrors is primary as parabolic and secondary as hyperbolic. These optics need to be supported in some suitable structure to maintain alignment with each other. The support structures are indeed an engineering issue. In addition to supporting the optics, the structure also required to be capable of tracking astronomical objects as they move across the sky because of the rotation of the earth.

5.2 Steering/tip-tilt mirrors

Fast steering mirrors (FSM) are effectively used in active and adaptive optics for various dynamic applications such as precision scanning, tracking, pointing laser beam and image stabilization. FSM, as shown in Figure 6, is a mirror mounted to a flexure support system that may be tilted fast about its axis independent of the natural frequency of the spring/mass system in order to direct an image in x-y plane.

In adaptive optics the fast steering mirror is used as one of the two main phase correctors for beam or image stabilization by correcting beam jitter and wander introduced by atmospheric turbulence as well as thermal and mechanical vibrations of optical components. Steering mirrors with high bandwidth operation can be electronically controlled to tilt around two orthogonal axes (tip-tilt movements) independently. Beam wander is the first order wavefront aberration that limits the beam stabilization and pointing accuracy onto the distant targets. Two, three and four actuator based steering mirrors are generally designed to cater for the dynamic application in mind with appropriate dynamic range, tilt resolution and frequency bandwidth. The simplest design is the two-axis tilting mirror with two PZT actuator stacks pushing the tilt platform/mirror substrate at 90 degrees around the central pivot. However, the two-axis tilt mirror suffers from the thermal instabilities and cross-talk between the tilting axes at high frequencies.

5.3 Deformable mirrors

AO system requires novel devices to implement the phase shift operation necessary for wavefront control.
The phase of the wavefront can be controlled either by changing the propagation velocity or the optical path length. Refractive index varying devices such as SLMs and other ferroelectric or electro-optic crystal devices have been used with limited success to implement phase control. Frequency response and amplitude limitations have been limiting factors for these devices. Reflective surface modifying devices such as segmented mirrors and continuous surface DMs are very successful in several high-end applications. In such Deformable mirrors, two kinds of piezo actuators are used namely as Stacked and Bimorph actuators.

5.3.1 Discrete Stacked Actuator deformable mirror

Stacked actuator DM contains a thin deformable facesheet mirror on a two-dimensional array of electrostrictive stacked actuators supported by rugged baseplates as shown in Figure 7. In some cases actuators are not produced individually, but rather a multi-layer wafer of piezo-ceramic is separated into individual actuators. When some voltage $V_i$ is applied to the $i$-th actuator, the shape of DM is described by the influence function $r_i(x, y)$ multiplied by $V_i$. It resembles a bell-shaped (or Gaussian) function for DMs with continuous facesheet (there is, however, some cross-talk between the actuators, typically 15%). When all actuators are driven, the shape of the DM is equal to

$$r(x, y) = \sum_i V_i r_i(x, y) \quad \ldots (17)$$

A multi-channel high-voltage amplifier must have a short response time, despite a high capacitive load of DM electrodes. For high bandwidth applications such DMs are preferred and further it could be easily cooled. Both zonal and modal wavefront reconstruction techniques can be applied with stacked actuator DMs.

5.3.2 Bimorph deformable mirror (BDM)

A bimorph mirror is made from two thin layers of materials bonded together. One layer is a piezoelectric material such as PZT and the other is the optical surface, made from glass, Mo or Si or both pieces may be PZT material, with the outer surface between the two layers and acts as a common electrode. When a voltage is applied to an electrode, one layer contracts and the opposite layer expands, which produces a local bending much like that of a bimetal strip. The local curvature being proportional to voltage, these DMs are called curvature mirrors. The PZT electrodes need not be contiguous. The geometry of electrodes in BDM as shown in Figure 8 is radial-circular, to match best the circular telescope apertures with central obscuration. In this way, for a given number of electrodes (i.e. a given number of controlled parameters) BDMs reach the highest degree of turbulence compensation, better than segmented DMs. Mirror surface Poisson solution of the applied voltage at a particular point. BDM very well suits with the curvature type wavefront sensor. Modal wavefront reconstructor is preferred with BDM control. There is no such simple thing as influence functions for bimorph DMs. The surface shape as a function of applied voltages must be found from a solution of Poisson equation which describes deformation of a thin plate under a force applied to it. The boundary conditions must be specified as well to solve this equation. In fact, these DMs are made larger than the beam size, and an outer ring of electrodes is used to define the boundary conditions-slopes at the beam periphery. The mechanical mountings it must be fixed in grooves.

5.3.3 Micro-structured membrane (MM)

The membrane consists of two stretched layers. Applying a voltage results in a change of the membrane shape. Figure 9 shows parts (i) and (ii) of a micro-structured membrane, and both parts have the same shape. The boundary conditions are specified in Figure 9 and condition (i): a supported layer of electrodes is used to define the boundary conditions.
mounting of a bimorph DM is delicate: on one hand, it must be left to deform, on the other hand it must be fixed in the optical system. Typically, 3 V-shaped grooves at the edges are used.

5.3.3 Micro-machined DM

The micro-machined deformable mirror (MMDM) consists of a thin flexible reflective membrane stretched over an array of electrostatic actuators. Applying voltages to these actuators, individual responses superimpose to form the necessary optical figure, can locally deflect electrically grained conductive membrane. The mirror consists of two parts (i) the die with the flexible mirror membrane and (ii) the actuator structure. A low stressed nitride membrane forms the active part of MMDM as shown in Figure 9. In order to make the membrane reflective and conductive, the etched side is coated with the thin layer of evaporated metal, usually aluminium or gold. Reflective membranes, fabricated with this technology have a good optical quality. Assembly of the reflective membrane with the actuator structure should ensure a good uniformity of the air gap so that no additional stress or deformations are transmitted onto the mirror chip. All components of a MMDM except the reflective membrane can be implemented using PCB technology. Hexagonal actuators are connected to conducting tracks on the back side of the PCB by means of vias (metalized holes). These holes reduce the air damping, extending the linear range of the frequency response of a micro machined mirror to at least 1kHz, which is much better than for similar devices mounted over plane silicon dies.

Influence function is primarily determined by the relative stiffness of actuators and face sheet. If actuators are very stiff compared to the face sheet, the clamped-clamped is more appropriate and vice-versa if the relative stiffness are reversed. Stiffer actuator structures reduce inter-actuator coupling but require high central voltages. A more practical approach is to reduce the stiffness of the face sheet material by...
reducing its thickness and/or elastic modulus and by increasing the inter actuator spacing. Figure 10 depicts the test patterns of MMDM at different conditions. In order to estimate the influence function of the deformable mirror actuators, a Veeco interferometer has been used. During testing and characterization of DM, voltages are applied to single and multiple actuators of the deformable mirror. Interferograms for following cases were recorded (see Figure 10):

(a) absence of any voltage to its actuators, shows astigmatism shape; (b) all actuators are applied with equal voltage, shows spherical shape; (c) one of the adjacent actuators to the central actuator is applied voltage, shows comatic shape; (d) central actuator is applied voltage, shows defocus shape.

5.3.4 Liquid crystal DM

Wavefront correction in AO is generally achieved by keeping the refractive index constant and tuning the actual path length with a mirror. An optically equivalent alternative is to fix the actual path length and tune the refractive index. This could be achieved using many different optical materials; a particularly convenient class of which is liquid crystals because they can be made into closely packed arrays of pixels which may be controlled with low voltages.

Electrically addressed NLCs are generally used for the wavefront correction in conventional AO system, whereas optically addressed SLMs are also being used to develop an unconventional AO with all optical correction schemes. NLCs are having lower frame rates so that it is not so appropriate for the atmospheric compensation under strong turbulent conditions. Second type of LCs i.e., ferroelectric LCs are optically addressed in which the wave plates whose retardance is fixed but optical axis can be electrically switched between two states. Phase only modulation with a retarder whose axis is switchable is more complicated than with one whose retardance can be varied. The simplest method involves sandwiching a FLC whose retardance is half a wave in between two fixed quarter wave plates. FLCs have the advantage that they can be switched at kHz frame rates, but the obvious disadvantage that they are bistable. The use of binary algorithms in WF correction is the simplest approach to develop closed loop control. The basic wave front correction algorithm is: whenever the WF error is greater than λ/2, then correction of λ/2 is applied.

5.4 Multichannel DM driver electronics

Electronics for the actuator system are the most complex, and by far the most expensive part of the system itself, typically accounting for the 2/3 of its cost. In an extreme example, the first 2000 channel mirror built had approximately 125 electronic components per control channel just for the driver. These drivers are incredibly safe but so complex as to be unreliable. Main component of the single channel driver electronics is high voltage operational amplifier. Additionally, there are requirements of a feedback loop which limits the available current and shuts the driver down in case of the actuator fails or short circuit. This prevents damaging the mirror by power dissipation in the actuator. Also required is a voltage driver, frequently with an A/D converter on the O/p to provide the main system computer with moment to moment information on the status of each corrector channel. Today analog inputs are generally insufficient since most wavefront controllers are digital, so each channel has its own D/A converters for the input. The actuator is a low loss capacitor which must be charged and discharged at the operating rate, typically up to 1 kHz. The required peak current, I_{peak} is derived as

I_{peak} = 2 \pi f CV \tag{18}

In which C is the capacitance of the actuator and V the control voltage of the stroke.

Thus, the peak power rating of the channel is

P_{peak} = V^2 / 2C

Thus, electrical circuit design and peak power rating of each channel is critical. In fact reliable power dissipation for the crystal used is an assumption, and 1/4 of the total power rating is safely allowed.

5.5 Jitter sensors

A position sensitive photoelectric detector (PSD) is attached to each channel with lasers for precise characterization of the manufactured components and to check calibration. High quality, outstanding linearity for simple operation is required. An output spot is attached to lasers often with lasers and attached to each PSD with lasers. The spot is being intercepted by a spot on the output and the output is being recorded by a spot on the output. Thus, the output is recorded by a spot on the output.

Fig. 10 — Test patterns of MM DM (a) Astigmatic (b) Spherical (c) Comatic (d) Defocus
sandwiching the input signal between the cells. The cells have the same active sensing area of 77 \mu m \times 100 \mu m in WFS. The output signal, and the host computer then processes the signal. The computer and software perform basic calculations of the position and power of the monitored beam. Let A, B, C, D are the four quadrants respectively, and R is the radius of the incident beam illuminating the detector. The beam position is calculated using the following formulas:

\[ X = \frac{(B+D)-(A+C)}{A+B+C+D} \]

and

\[ Y = \frac{(A+B)-(C+D)}{A+B+C+D} \]

where \( P \) (total power) = A+B+C+D.

The output position is displayed as a fractional number or as a percentage figure, where the percentage represents the fraction of beam movement relative to the X or Y direction as shown in Fig. 11b.

5.6 Wavefront sensor

The problem of measuring wave-front distortions is common to optics (e.g. in the fabrication and control of telescope mirrors), and typically is solved with the help of interferometers. Why do not use standard laser interferometers in AO Wavefront Sensors (WFSS)? First, an AO system must use the light of stars passing through the turbulent atmosphere to measure the wavefronts, hence use incoherent (and sometimes non-point) sources. Even the laser guide stars are not coherent enough to work in typical interferometers. WFS must work on white-light interferometers. WFS must work on white-light incoherent sources. Second, the interference fringes are chromatic. We cannot afford to filter the stellar light, because we want to use faint stars. WFS must use the photons very efficiently. Third, interferometers have an intrinsic phase ambiguity of \( 2\pi \), whereas atmospheric phase distortions exceed \( 2\pi \), typically. The WFS must be linear over the full range of atmospheric distortions. There are algorithms to "unwrap" the phase and to remove this ambiguity, but they are slow, while atmospheric turbulence evolves fast, on a millisecond time scale: WFS must be fast.

5.6.1 Shack Hartmann wavefront sensor

It is based upon the classical Hartmann test. The major advantage of this sensor is its high optical efficiency, white light capability and operation with continuous or pulsed light sources. It has a high spatial and temporal resolution, large dynamic range.

Thus, the peak power consumption, \( P_{\text{peak}} \) can be deduced as,

\[ P_{\text{peak}} = \sqrt{2} V_{\text{max}} I_{\text{peak}} \]

Thus, each driver is a linear power amplifier with peak rating of 1-10 W per channel. Certainly, every channel is not operating at its full rating all the time. In fact reasonable thumb rule is that average power dissipation in a driver package is obtained on the assumption that each channel is operating at \( 1/3 \) \( V_{\text{max}} \) and \( 1/4 \) of \( f_{\text{max}} \).

5.5 Jitter sensor (Quadrant Detector)

A position-sensing detector (PSD) is a photoelectric device that converts an incident light spot into continuous position data. Many industrial manufacturers and laboratories around the world use PSDs in their daily work. PSDs are able to characterize lasers and align optical systems during the manufacturing process. When used in conjunction with lasers they can be used for industrial alignment, calibration, and analysis of machinery. It provides outstanding resolution, fast response, excellent linearity for a wide range of light intensities and simple operating circuits. In order to measure the X and Y positions from the PSD, four electrodes are attached to the detector and an algorithm then processes the four currents generated by photo absorption.

The Quadrant detector is a uniform disk of silicon with two gaps across its surface as shown in Fig. 11(a). For optimum performance and resolution, the spot size should be as small as possible, while being bigger than the gap between the cells. Typically, the gap is of the order of 10-30 \( \mu \)m and the active sensing area is 77 or 100 mm\(^2\) (depending on the exact model). When illuminated, the cells generate an output signal proportional to the magnitude of the illumination. It is the electronic card, which digitizes the output signal, and the host computer then

Fig. 11 — (a) Quadrant detector, (b) Beam wanders relative to the X or Y direction
and no $2\pi$ ambiguities. All these reasons make Shack Hartmann Sensor more preferred over other kind of phase measuring sensors.

In the Shack Hartmann Wavefront Sensor (SHWS), the entire wavefront is divided into a large number of samples, called sub-apertures by a two dimensional lenslet array and forms an array of spots (Fig. 12). A relay lens re-images these arrays of focal spots onto a High Frame rate CCD camera. Wavefront measurement by SHWS is based on the measurements of local slopes of a distorted wavefront $\partial \phi / \partial n$ relative to a reference plane wavefront. The local slope is proportional to shift of the spot center $\Delta S$. By measuring these small shifts, the local gradient of the wavefront is measured by the CCD camera, which is interfaced to a fast transfer rate frame grabber for almost real time data acquisition and data analysis. By integrating these measurements over the beam aperture, the wavefront or phase distribution of the beam can be determined. In particular the space-beam width product, $M^2$, can be obtained in single measurement. The intensity and phase information can be used in concert with information about other elements in the optical train to predict the beam size, shape, phase and other characteristics anywhere in the optical train. Moreover, it also provides the magnitude of various Zernike coefficients to quantify the different wavefront aberrations prevailing in the wavefront. Fig. 9 shows schematically the SHWS principle.

The displacement of the spot center $(x'_c, y'_c)$ within the sub-aperture with respect to a reference position $(x'^{0}_c, y'^{0}_c)$ is measured. Local gradient of the wavefront $\Phi(x,y)$ is obtained according to

$$\frac{\partial \phi}{\partial x} = \frac{1}{f} S_x, \quad \frac{\partial \phi}{\partial y} = \frac{1}{f} S_y \quad \quad \quad \text{(21)}$$

where $S_x = x_c - x_n$, $S_y = y_c - y_n$, $f$ is the focal length of the lenslets.

### 5.6.2 Curvature wavefront sensor

The curvature wavefront sensor measures the intensity $I_1$ in an intrafocal plane and the intensity $I_2$ in an extrafocal plane as shown in Fig. 13 and compares these intensities to determine the curvature of the wavefront. As the normalised difference, $(I_1 - I_2)/(I_1 + I_2)$, is used for the comparison, and $I_1$ and $I_2$ are measured simultaneously. The sensor is not susceptible to the non-uniform illumination due to scintillation.

$$S = \frac{I_1 - I_2}{I_1 + I_2} = \frac{F(F-1)}{I} \left\{ \left[ \frac{\partial W(x,y)}{\partial n} \delta_x - P(x,y) \nabla^2 W(x,y) \right] \right\}$$

where first term with the curly bracket, is the wavefront derivative in the outward direction perpendicular to pupil edge, $F$ the focal length of the telescope and $P(x,y) = 1$ within pupil and 0 outside. The type of detectors required for CS must have almost zero read out noise and very short integration time. Either APD or specially optimised CCD/CMOS is used for this purpose.

### 5.6.3 Pyramid WFS

Recently, a new kind of wavefront sensor has been proposed which is able to change in a continuous way gain and sampling, thus enabling a better match of the system performances with the actual conditions on the sky. This sensor, named Pyramid Wavefront Sensor, is a novel concept device, as shown in Fig. 14 and it consists of a four-faces optical glass pyramid that behaves like an image splitter. When the tip of the pyramid is placed in the focal plane of the telescope and a reference star is directed on its tip, the beam of
light is then split in four parts. Using a relay lens located behind the pyramid, these four beams are then re-imaged onto a CCD camera, obtaining four images of the telescope pupil. Since the four edges of the pyramid act like a knife-edge (or Foucault) test system, these images contain essential information about the optical aberrations introduced in the beam from the atmosphere. These parameters can be used to recover the astronomical images.

5.7 Wavefront reconstructor/controller

Adaptive optics controls the wavefront, temporally and spatially both, in closed loop: the WFS measures any remaining deviations of the wavefront from ideal and sends the corresponding commands to the DM. This is why small imperfections of DM (like hysteresis or static aberrations) are not very important: they will be corrected automatically, together with atmospheric aberrations. A high bandwidth specialist digital board implements the temporal wavefront control whereas spatial control is achieved by the parallel computing.

This board calculates the aberrations from the wavefront-sensor measurements and generates the actuator commands of the deformable mirror. The calculation must be done fast (within 0.5 to 1 ms), otherwise the state of the atmosphere may have changed rendering the wavefront correction inaccurate. The required computing power needed can exceed several hundred million operations for each set of commands sent to a 250-actuator deformable mirror. Adaptive optic systems could be controlled by zonal or modal methods. In zonal control, each zone or segment of the mirror is controlled independently by wavefront signals that are measured for the sub-aperture corresponding to that zone. In modal control, the wavefront is expressed as the linear combination of modes that best fit the atmospheric perturbations.

Let \( x(t) \) be the input signal (e.g. a coefficient of some Zernike mode) and \( y(t) \) — the signal applied to the DM then the measured error signal by WFS as shown in Fig. 15 is

\[
e(t) = x(t) - y(t) \tag{23}
\]

The error signal must be filtered before applying it to DM, otherwise the servo system would be unstable. In the frequency domain this filter \( G(f) \) is called open-loop transfer function.

5.9 Reference source

One of the major problems applying adaptive optics to astronomical observations is to locate a bright reference object within isoplanatic patch, which is required to measure the wavefront errors. The sources are in most of the cases are too faint, hence their light is not sufficient for the correction. In order to palliate the limitations of low sky coverage, an alternative source is found in the form of introducing the laser guide star source as a reference to measure such errors by means of a wavefront sensor, as well as to map the phase on the reference pupil. A pulsed laser is used to cause a bright compact glow in the upper atmosphere, which serves as the source of measuring the turbulence of the atmosphere. Concerning the flux backscattered by a laser shot, two basic problems: (i) the cone effect which arises due to the parallax between the remote astronomical source and artificial source (located 90 km high in the case Na ID laser), and (ii) the angular anisoplanatic effects are needed to be looked into.

5.10 Error budget

The overall performance of an AO system is estimated by the Strehl ratio, which is determined by the residual wavefront error.

\[
S = \exp(-\sigma_r^2) \tag{24}
\]

where \( \sigma_r^2 \) is residual wavefront varies over the pupil and to first order is made up of the contribution from the fitting error, temporal bandwidth limitation and WFS error. Hence,

\[
\sigma_r^2 = \sigma_{dm}^2 + \sigma_{temp}^2 + \sigma_{wfs}^2 \tag{25}
\]
The fitting error arises from the non-zero spacing between the DM actuators and is given by:

$$\sigma_{\text{fit}}^2 = \alpha \left( \frac{r_0}{r_a} \right)^{5/3}$$  \hspace{1cm} (26)

where \( r_a \) is the actuator spacing referenced to the telescope entrance pupil and \( \alpha \) is a factor related to the geometry of the adaptive device. Its value is 0.4 for a smooth modal influenced function.

The temporal error results from the limited temporal bandwidth of the control system:

$$\sigma_{\text{temp}}^2 = \left( \frac{f_s}{f_{3 \text{db}}} \right)^{5/3}$$  \hspace{1cm} (27)

where \( f_s \) is the Greenwood frequency and \( f_{3 \text{db}} \) is the 3db bandwidth of the AO control system.

Performance of an Adaptive Optic system is greatly affected by the wavefront measurement error of the Shack Hartmann wavefront sensor (SH-WFS). The error associated with the Shack-Hartmann WFS is mainly due to the limitation of the accuracy of the centroid determinations for each sub aperture. Assuming the target is unresolved by each lenslet with the resulting spot near the center of a quad array of the detector it is given by

$$\sigma_{\text{wfs}}^2 = 0.35(\pi/2\text{SNR})^2$$  \hspace{1cm} (28)

where SNR is the radiometric signal to noise ratio of the WFS detector.

5.11 Reduction of the data

Reduction of AO images may be processed with the methods developed for high resolution imaging, speckle interferometry and other image processing algorithms\(^\text{16-18}\). Prior to use such algorithms, the basic operations, namely, dead pixel removal, deblasing or flat fielding, sky or background emission subtraction, suppression of correlated noise, etc., are to be performed. Characterization of AO PSF for anisoplanatic imaging is an important task; variation of point spread function (PSF) limits strongly the deconvolution methods for processing of wide field of view (FOV) images. Fusco et al.\(^\text{28}\) have derived an analytical expression of this PSF degradation in the FOV and applied to a posterior processing of said images; according to them, the technique restored the star parameters with a better precision.

6 Adaptive Secondary Mirror

Another way to correct the disturbance in real time is usage of adaptive optics secondary mirror (ASM) that makes relay optics obsolete\(^\text{39}\). The other notable advantages are: (i) enhanced photon throughput that measures the proportion of the light which is transmitted through an optical set-up, (ii) introduction of negligible extra IR emissivity, (iii) causes no extra polarization, and (iv) non-addition of reflective losses\(^\text{30}\). Due to the actuator spacing, the resonant frequency of such a mirror may be lower than the AO bandwidth. The ASM system uses a SH sensor with a array of small lenslets, which adds two extra reflective surfaces to the wavefront sensor optical beam\(^\text{31}\). An f/15 AO secondary with 336 actuators is installed on the 6.5m Telescope of Multi Mirror Telescope (MMT) observatory, Mt. Hopkins, Arizona.

7 Applications of Adaptive Optics

Adaptive optics systems are being used for the military applications. The requirements of AO systems for such applications are different from that of astronomical AO systems. Those defence AO systems requiring a large number of modes, a narrow optical bandwidth, mostly monochromatic, and a high time bandwidth, put less stringent requirement than systems requiring wide optical bandwidth.

AO technology is being applied in medicine by ophthalmologists. It allows for clearer examination of eye's retina leading to an early diagnosis with precision. Construction of fundus camera equipped with such a technology allows one to image a microscopic size of single cell in the living human retina. Liang et al.\(^\text{32}\) have demonstrated that a human eye with adaptive optics correction can resolve fine gratings that are invisible under normal viewing conditions. Roorda and Williams\(^\text{33}\) were able to identify the short, middle, and very long wavelength sensitive cones in the living human eye by combining retinal densitometry with the high resolution images available with adaptive optics.

AO system is being employed routinely at several large reflectors in order to gather new information. These are in the form of (i) imaging of the nucleus\(^\text{16}\) of M31, (ii) surveying of young stars and multiple star systems\(^\text{33}\), (iii) resolving the galactic center\(^\text{36}\), (iv) imaging of Seyfert galaxies, QSO host galaxies\(^\text{37}\), and (v) mapping of the circumstellar environment\(^\text{38}\), etc. The discovery of compact cluster, R136a (HD38268) that was thought to be the most massive star with a solar mass of ~ 2500M, of Doradus nebula in the Large Magellanic Clouds by means of speckle imaging technique\(^\text{39}\) is an excellent achievement in the field of high resolution imaging. In one of the IAU conferences, one full day was spent in discussing about its probability of a black hole\(^\text{40}\). The
observations with adaptive optics system have revealed over 500 stars within the field of view 12.8" x 12.8".

Another most successful application of AO system in astronomy has been in studying of the planetary metereology, for example, Neptune has complex methane clouds that can be discernible. Such a system can also be used for detecting moons around asteroids. Search for planets outside of our solar system is an exciting field in astronomy. The first planet around another star, 51 Peg was inferred from radial velocity measurements of the primary. More than 80 such stars have been identified which appear to have Jupiter like mass planets in orbit. Combination of AO and very long baseline imaging interferometer may in future be able to image the earth like planets. A few brown dwarf around stars have also been imaged using AO systems.

Figure 16 depicts an example of the AO image of θ1 Ori B taken with adaptive secondary mirror at the 6.5 meter MMT. Without AO, this object appears to be two stars, but with AO turned on it is revealed that the lower star is a close binary having separated by 0.1 arcsec; the brighter one is a guide star, and the fainter one slightly to the right (see white arrow) is a very faint companion.

The most fascinating use of AO systems is to get image of the sun, particularly sunspots. Combination of AO and image processing can pull up the sharpest features to make more impressive insights into the sun's surface. AO systems are employed in other branches of physics as well. AO systems are useful for spectroscopic observations, as well as for photon-starved imaging with future very large telescopes, and ground based long baseline optical interferometers.

8 Concluding Remarks

Earth-bound astronomical observations are strongly affected by turbulent air motions in the atmosphere that set severe limit to angular resolution; the deployment of space-bound telescopes may provide the answer, but the size and the cost of such a venture is its shortcoming. Adaptive optics technology has the ability to improve the point source sensitivity; an exact knowledge of points spread function can be derived. It is going to be indispensable equipment to large telescope and will play larger role in the development of the very large telescopes of 30 m class in future. The highest ever high angular resolution images from a single aperture, 10 m Keck II telescope, at near IR wave band have been obtained. Using several deformable mirrors and several guide stars to compensate for turbulence in a 3-D way, multi-conjugate AO system extends the image compensation to large field of view (FOV). Ragazzoni et al. have used this tomography and found advantageous over classical AO approach. At the fall of the next decade (post 2010 A.D.), observations using AO system on new generation extremely large telescopes of 100 m class, will revolutionize in mapping ultra-faint objects like blazer that exhibits the most rapid and the largest amplitude variations of all AGN, exo-planets etc. High-frequency corrections at the secondary mirror are the natural next step in the development of such telescopes to consider the throughput, polarization, IR emissivity, stray light, conjugation, reliability etc.

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References

1 Introduction

Here we introduce Darcy’s law, an important concept in agricultural sciences. Darcy’s law describes the flow of fluid through porous media, such as the soil, which is important for agricultural applications. The law states that the flow rate of water through the soil is directly proportional to the hydraulic gradient and the hydraulic conductivity, which is a measure of the soil’s permeability.

Darcy’s law is given by the equation

\[ Q = -K \frac{dh}{dx} \]

where \( Q \) is the flow rate, \( K \) is the hydraulic conductivity, \( dh \) is the hydraulic gradient, and \( dx \) is the length of the flow path.

Darcy’s law is widely used in the study of groundwater flow, agricultural irrigation, and soil science.

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