Analysis of coupling characteristics of rectangular core optical waveguide coupler

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A theoretical formulism based on the boundary matching technique has been proposed to study the systematic of an asymmetric waveguide coupler. Same formulism is applicable to a symmetric coupler. The equivalent parallel coupling length to account for the coupling in the two S-shaped regions in the input and the output of the coupler is obtained from the coupler geometry and is dealt with more accurately and sensitively in our calculations. No fitting parameter is required for our calculation. The calculated characteristics of the asymmetric coupler are very close to those of experimental ones. Our model can aid in the design of diffused channel single mode rectangular core optical waveguide couplers.

[Keywords: Asymmetric couplers, Broadband couplers, Boundary matching technique]
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1 Introduction

Single mode waveguides with rectangular cross-section are the building blocks of the integrated optical devices having wide range of application in optical communication area. Among these the optical directional coupler is perhaps the most important and is used to fabricate low loss optical switches, high-speed modulators, polarization converters and wavelength filters. Hence, it is quite important to study the coupling properties of such a directional coupler for their efficient design and performance. The wavelength sensitivity of an optical waveguide directional coupler is one of the most important parameters as far as the performance of the communication system is concerned. The coupler bandwidth must accommodate the shift of the laser source central wavelength or the spectral broadening of the laser when modulated. Furthermore, for switched multi-wavelength communication systems utilizing amplifiers with gain bandwidth of the order of 25 nm we require switches that have bandwidth at least that large.

Channel waveguides and couplers have been treated with a variety of theoretical techniques1-3,8-11. The Marcatili2 technique is the most important and is used extensively. Kumar et al.3 have modified the analysis of Marcatili2 by including the effect of corner regions through the first order perturbation theory. In this paper, we have greatly improved the technique of Kumar et al.3 and made it applicable to the symmetric as well as the asymmetric waveguide coupler. In our formulism, we have incorporated a technique properly take into account the S-shaped region in the input and the output of the coupler, this was not studied by Kumar et al.3. The importance of the S-shaped region is crucial as seen in the practical design of the broadband couplers by Takagi et al.4-6. They have used Beam Propagation Method (BPM) technique for obtaining the systematic of the coupler. We would like to mention that the solution given by BPM emphasize the beam characteristics rather than the modal properties of the field. A useful method in designing a coupler is the one, which extracts the coupler performance and the field’s characteristics from the coupler geometry. That is, it should compare the propagation constant, the length of the parallel coupling region for transfer of power, coupling coefficient κ, coupling efficiency η and the modal fields for the combined set of profile with few mathematical simplifications and approximations.

Here, we propose a simple approximation along with a set of trial field for describing the propagation characteristics of a single mode rectangular waveguide. The trial field enables us to define equivalent waveguiding structure consisting of slab waveguides. Using the equivalent guiding structure, we have obtained, in a simple way, the two fundamental vector modes E11 and E21 of the original wave guiding structure. This structure greatly simplifies the analysis of composite guiding structure such as a, as asymmetric obtained of the optical

2 Theory

The asymmetric coupler. According to the obtained coupling length L by

\[ \eta = \frac{\kappa^2}{\delta^2} \]

Here κ and δ are

\[ \kappa = \sqrt{1 - \frac{\delta^2}{\delta_0^2}} \]

Fig. 1 (b)
such as a directional coupler, both symmetric as well as asymmetric configuration. The theoretical results obtained by us predict the experimental behaviour\(^6\) of the optical waveguide coupler very nicely.

### Theory

The cross-sectional view of the symmetric and the asymmetric coupler is as shown in the Fig. 1(a and b). According to the couple mode theory\(^2\) for a coupler formed of two parallel loss-less guides of interaction length \(L\) the power coupled into the guide II is given by

\[
\eta = \frac{K^2}{\delta^2} \sin^2(\delta \cdot L) \tag{1}
\]

Here \(K\) and \(\delta\) are given by:

\[
K = \frac{\beta_s - \beta_A}{2} = \sqrt{\left(\frac{\beta_s - \beta_A}{2}\right)^2 + \kappa^2} \tag{2}
\]

\[
\delta = \frac{\beta_s - \beta_A}{2} = \sqrt{\left(\frac{\beta_s - \beta_A}{2}\right)^2 + \kappa^2} \tag{3}
\]

where \(\beta_1\) and \(\beta_2\) are the propagation constants of the uncoupled waveguides I and II, respectively. \(\beta_s\) and \(\beta_A\) are the symmetric and anti-symmetric propagation constant of the coupled guides I and II. The amplitude term \((K^2/\delta^2)\) represents the maximum power transfer and the phase term \((\delta \cdot L; L\) being the parallel coupling length), specifies the location of the position of the maximum power with respect to wavelength. If the two guides I and II are identical, so that, \(\beta_1 = \beta_2\), we have \((K^2/\delta^2 = 1)\) and \(\delta = \kappa\) in Eq.1 Fig. 1(a), the coupling ratio will reach 100% at a particular wavelength. However, if the waveguides are not identical, so that \(\beta_1 \neq \beta_2\) \((K^2/\delta^2 < 1)\), the maximum coupling ratio cannot be 100%, therefore complete power transfer cannot take place. However, it is observed that decrease in the power transfer results in the broadening of the coupling efficiency response with respect to wavelength. The condition \(\beta_1 \neq \beta_2\) can be easily realized by varying the width of one of the guides [Fig. 1(b)].

Firstly, we determine \(\beta_1\) and \(\beta_2\) of the individual guide with core refractive index \(n_1\) (=1.455) surrounded by a material of refractive index \(n_s\) (1.4514, which is 0.25% less than \(n_1\)), as shown in the Fig. 2 (a and b). We assume that the it is a weakly guiding structure i.e. the cladding refractive index \(n_s\) is about 0.25% less than the core refractive index \(n_1\).

The guide shown in the Fig. 2(b) is the perturbed form of the actual waveguide shown in Fig. 2(a) because we choose the refractive index profile as:

\[
n^2(x,y) = n^2(x) + n^2(y) - n_1^2 \tag{5}
\]

where \(n_1^2(x) = n_1^2 \quad x < -a,\)

\(= n_1^2 \quad -a < x < a,\)

\(= n_s^2 \quad x > a,\)

and

\[
\delta = \frac{\beta_s - \beta_A}{2} = \sqrt{\left(\frac{\beta_s - \beta_A}{2}\right)^2 + \kappa^2} \tag{3}
\]

...
order perturbation theory. The refractive index profile given in Eqs (5 and 6) enables us to solve the following scalar wave equation by the method of separation of variables.

\[
\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \left( \kappa_o^2 n^2(x, y) - \beta^2 \right) \right) \psi(x, y) = 0
\]

where \( \kappa_o = 2\pi/\lambda \) is the wave number and \( \beta \) is the propagation constant. Eq. (7) separates into the following two equations:

\[
\left( \frac{d^2}{dx^2} + \kappa_o^2 n^2(x) - \beta_x^2 \right) X(x) = 0 \quad \ldots(8)
\]

\[
\left( \frac{d^2}{dy^2} + \kappa_o^2 n^2(y) - \beta_y^2 \right) Y(y) = 0 \quad \ldots(9)
\]

with

\[ \psi(x, y) = X(x)Y(y) \]

and \( \beta_x \) and \( \beta_y \) represent the separation constants and are related to propagation constant \( \beta \) as

\[ \beta^2 = \beta_x^2 + \beta_y^2 - \kappa_o^2 n_s^2 \quad \ldots(10) \]

These solutions are given as

\[ X(x) = AB_1 \exp(\nu_1 x) \quad x < -a \]

\[ = AB_2 \cos(\nu_2 x) + AB_3 \sin(\nu_2 x) \quad -a < x < a \]

\[ = AB_4 \exp(-\nu_1 x) \quad x > a \]

and

\[ Y(y) = C_1 \exp(-\nu_1 y) \quad y > b \]

\[ = C_2 \cos(\nu_2 y) + C_3 \sin(\nu_2 y) \quad -b < y < b \]

\[ = C_4 \exp(\nu_1 y) \quad y < -b \]

with

\[ \nu_1 = \sqrt{\left( \beta^2 - \kappa_o^2 n_s^2 \right)} \quad \ldots(13) \]

\[ \nu_2 = \sqrt{\left( \kappa_o^2 n_s^2 - \beta^2 \right)} \quad \ldots(14) \]

where \( \beta^* \) is \( \beta_x \) for Eq. (11) and \( \beta_y \) for Eq. (12).
Using the computer program whose flow chart is given in Appendix A the ten arbitrary constants in Eqs (11 and 12) (i.e. $A_i B_i$ and $C_i$; $i = 1$ to 4, along with two propagation constants $\beta_1$ and $\beta_2$) are determined by solving the ten simultaneous nonlinear equations obtained from the following boundary conditions:

1. $\psi(x,y)$ and $\partial \psi(x,y)/\partial x$ continuous at $x = \pm a$; 
2. $\psi(x,y)$ and $\partial \psi(x,y)/\partial y$ continuous at $y = \pm b$; 
3. The wave function given in Eqs (11 and 12) are normalized:

$$\int_{-\infty}^{\infty} |X(x)|^2 dx = 1; \int_{-\infty}^{\infty} |Y(y)|^2 dy = 1$$

The infinite integration limit is considered explicitly by using Gamma function, whereas using Gauss quadrature routine with 96 points does the finite integration. The complete eigen wave function along with the propagation constant $\beta$ is determined.

Finally the correction in the propagation constant $\beta$ (as discussed in Appendix B) due to the difference in the refractive index profile in the edge regions is applied using the first order perturbation technique. The corrected value of the propagation constant is:

$$\beta^2 = \beta_0^2 + \kappa^2 (n_{r}^2 - n_{s}^2) \gamma_1$$

which is given in Eq. B.6.

The complete wave function and the corrected propagation constants $\beta_1$ and $\beta_2$ for the individual guides in isolation are determined. In a similar way to determine the symmetric and the anti-symmetric propagation constant of the coupler formed by the two waveguides we consider the profile as shown in Fig. 3(a). The guide shown in the Fig. 3(b) is the perturbed form of the actual waveguide shown in Fig. 3(a) because we choose the refractive index profile as in Eq. (5). Here

$$\begin{align*}
n_1(x) &= n_s^2 \quad x > d + 2a_z, \\
n_2(x) &= n_s^2 \quad -(d + 2a_z) < x < d, \\
n_3(x) &= n_s^2 \quad d < x < d + 2a_z,
\end{align*}$$

Fig. 3 — Profile shown in Fig. 3(b) matches the profile shown in Fig. 3(a)

\begin{align*}
n^2(y) &= n_s^2 \quad y < -b, \\
n^2(y) &= n_s^2 \quad -b < y < b, \\
n^2(y) &= n_s^2 \quad y > b
\end{align*}$$

where also the above profile is shown in Fig. 3(b) matches the actual profile shown in Fig. 3(a) everywhere except in the corner region where it differs by $(n_2^2 - n_1^2)$, and is very small. The correction in the propagation constant due to this can be taken into account by using the first order perturbation theory.
Since symmetric refractive index profile along x-direction leads to two solutions of the Eq. (8), namely symmetric ($X_S$) and anti-symmetric ($X_A$) while in the Y direction we have only one solution and is described by Eq. 12.

These X solutions are given as

$$X_s(x) = A_1 \cdot \exp(v_1x) \quad x < -(d+2a_1)$$

$$= A_2 \cdot \cos(v_2x) - A_3 \cdot \sin(v_2x) \quad -(d+2a_1) < x < -d$$

$$= A_4 \cdot \exp(v_1x) + A_5 \cdot \exp(v_1x) \quad -d < x < d$$

$$= A_6 \cdot \cos(v_2x) + A_7 \cdot \sin(v_2x) \quad d < x < d+2a_2$$

$$= A_8 \cdot \exp(-v_1x) \quad x > d + 2a_2 \quad \ldots \quad (18)$$

and

$$X_A(x) = -B_1 \cdot \exp(v_1x) \quad x < -(d+2a_1)$$

$$= -B_2 \cdot \cos(v_2x) + B_3 \cdot \sin(v_2x) \quad -(d+2a_1) < x < -d$$

$$= B_4 \cdot \exp(v_1x) - B_5 \cdot \exp(v_1x) \quad -d < x < d$$

$$= B_6 \cdot \cos(v_2x) + B_7 \cdot \sin(v_2x) \quad d < x < d+2a_2$$

$$= B_8 \cdot \exp(-v_1x) \quad x > d + 2a_2 \quad \ldots \quad (19)$$

Y($y$) solutions are same as in Eq. (12) and here $v_1$, $v_2$ are as given in the Eqs (13 and 14), where $\beta^*$ now is $\beta_S$ for Eq. (18), $\beta_A$ for Eq. (19) and $\beta_y$ for Eq. (12).

The twenty-three arbitrary constants in Eqs (18, 19 and 12) (i.e. $A_i$, $B_i$, and $C_j$; $i$ = 1 to 8, $j$ = 1 to 4 along with three propagation constants $\beta_S$, $\beta_A$ and $\beta_y$) are determined by solving the twenty-three simultaneous non-linear equations obtained from the following boundary conditions.

(i) $\psi(x,y)$ and $\partial \psi(x,y)/\partial x$ continuous at $x = \pm d$, $x = d + 2a_2$ and $x = -(d+2a_1)$. (ii) $n^2(x,y)$ $\psi(x,y)$ and $\partial \psi(x,y)/\partial y$ continuous at $y = \pm b$. (iii) The wave function given in Eqs (18, 19, and 12) are normalized:

$$\int_{-\infty}^{\infty} |X_{ss}(x)|^2 \, dx = 1; \int_{-\infty}^{\infty} |X_{sa}(x)|^2 \, dx = 1; \int_{-\infty}^{\infty} |Y(y)|^2 \, dy = 1$$

The complete wave function along with $\beta_S$, and $\beta_A$ is determined.

$$\beta_S = \sqrt{\left(\beta_{sa}^2 + \beta_y^2 - \kappa^2 n_1^2\right)} \quad \ldots \quad (20)$$

$$\beta_A = \sqrt{\left(\beta_{sa}^2 + \beta_y^2 - \kappa^2 n_i^2\right)} \quad \ldots \quad (21)$$

The correction in the propagation constant due to the difference in the refractive index profile in the edge regions is applied:

$$\beta_c^2 = \beta^2 + \kappa^2 (n_1^2 - n_2^2) \gamma_2 \quad \ldots \quad (22)$$

with $\gamma_2$ given in appendix B Eq. B7.

3 Results and Discussion

The numerical calculations to obtain the various propagation characteristics of practical importance for a coupler have been carried out. The results obtained by our boundary value technique are compared with those reported by other researchers as well as with experimental data. Fig. 4 shows the variation of optimum coupling length $L_c$ against separation distance $2d$ for the two fixed wavelengths 1.3 and 1.5 \textmu m. Here the dimensions and the refractive index profile of the coupler shown in Fig. 3(b) are chosen similar to that of Takagi et al. (i.e. $2a_1 = 10 \text{ \textmu m}$, good agreement). This comparison further confirms the accuracy of our technique. Further comparison with the experimental data shows the excellent agreement between the numerical and experimental results.

\section*{3.1 Incremental Couplers}

A practical coupler may have one or more regions of an increased or decreased refractive index. The coupling length may be written as

$$L = L_0 + \Delta L \quad \ldots \quad (23)$$

where $L$ is the total coupling length, $L_0$ is the coupling length in the straight region of the coupler, $\Delta L$ is the coupling length in the region of increased refractive index, and $\Delta L$ is the coupling length in the region of decreased refractive index.

The coupling length $L_c$ may be written as

$$L_c = \frac{\kappa^2}{\delta^2} \sin^{-1} \left( \frac{\kappa}{\delta} \right) \quad \ldots \quad (24)$$

where $\kappa$ is the coupling constant and $\delta$ is the coupling length.

Fig. 4 - Plot of $L_c$ with separation $2d$ between the guides for the symmetric coupler parameters $10 \times 8 \text{ \textmu m}$, $n_i = 1.455$, $n_1 = 1.491\text{ at } 1.3$ and $1.5$ wavelength.

Fig. 5 - Wave coupling length
Our results are in good agreement with those shown in Fig. 7 of Ref. 5. This comparison strengthens the validity of our technique for designing an asymmetric coupler. Further confirmation of our technique arises from the comparison of the coupling efficiency \( \eta \) against wavelength \( \lambda \) plot for different interaction lengths as shown in Fig. 5. Again the coupler parameters are chosen similar to that of Ref. 8. Our results are nearly similar with those shown in Fig. 2 of Ref. 8.

Incremental parallel coupling length due to S-shaped region

A practical waveguide type of the directional couplers usually have two S-shaped arms as shown in Fig. 6, in the input and the output where the coupling strength gradually increases as we approach the straight parallel region and decrease at the output as we move away from it. To design a coupler for a given coupling ratio this coupling cannot be neglected and has to be taken into account in deciding the length of the straight parallel coupling region. We refer to this coupling in the curvature region at the input and output as a coupling over an incremental parallel coupling length \( \Delta \ell \).

The coupling efficiency for the coupler now can be written as:

\[
\eta = \frac{k^2}{8^2} \sin^2 (\delta (L + \Delta \ell))
\]  

where \( L \) is the length of the parallel coupling region and \( \Delta \ell \) is the incremental parallel coupling length that...

Fig. 5 — Wavelength response of asymmetric coupler for various coupling lengths \( L \) and \( 2a_1 = 8 \mu m, 2a_2 = 6 \mu m, 2b = 8 \mu m, 2d = 3 \mu m, n_i = 1.455, n_s = 1.4514 \)

Fig. 6 — Geometry of the coupler with the two S-shaped arms of the radius \( R \) in the input and the output. \( L \) is the parallel coupling length and \( L_c \) is the length of the curved arm, from the edge that contribute to the coupling accounts for the coupling in the two S-shaped regions (over the length \( L_c \)) in the input and the output. We have to determine \( \Delta \ell \) for designing an asymmetric coupler for maximum efficiency.

The curvature regions are formed by bending the waveguide into an arc of a circle, this introduces losses. These losses depend on the radius of curvature \( R \) of the arc as well as the refractive index difference between the core and the cladding. Smaller is \( R \) the larger is the loss and for a given radius smaller the difference between the refractive index of the core and the cladding larger is the loss. Since we are assuming that the waveguides in our case are weakly guiding so to keep the losses low it is expected that a large radius arc be used. Radius cannot be made arbitrarily large, as this will increase the overall size of the coupler. It is seen that for a SiO\(_2\)-TiO\(_2\) planer waveguide formed on a silicon substrate with a refractive index difference of 0.25% between the core and the cladding, a radius of curvature of the order of 50mm is suitable as the bending losses in this case are less than 0.3dB.

Once the radius is fixed, we determine the phase change \( (\delta \ell \Delta) \) that occurs in the curved region in the direction \( z \). This is estimated by treating the two waveguides as a series of short sections of parallel waveguides of length \( \ell \) with separation between the two guides for each section increasing as we move away from the central parallel region (as a series of steps). This is consistent with the fact that while fabricating curved waveguides or diagonal lines in optical integrated circuits a staircase type of function is utilized which implements the curved portion or the diagonal line as a series of short steps. If there are \( n \) steps then using the above formalism, \( \delta \ell \), for each step is determined and hence the total phase change that occur in the \( n \) steps of length \( \ell \) each is given by...
\[ \Delta \ell = 2 \left( \frac{2L_n}{\pi} \left( \sum \delta_i \right) \right) \quad (i = 1, 2, 3, \ldots, n) \]  

(24)

For a complete power/maximum power transfer from guide I to II, total phase change that is required is \( \pi/2 \) and this happens when the parallel coupling length is equal to the optimum length \( L_\circ \). Using the above formulism the optimum coupling length at a given wavelength is determined and thus the incremental parallel coupling length that accounts for the coupling in the two S-shaped regions in the input and the output for a given guide geometry and at a given wavelength is:

\[ \Delta \ell = 2 \left( \frac{2L_n}{\pi} \left( \sum \delta_i \right) \right) \quad (i = 1, 2, 3, \ldots, n) \]  

(25)

The next step is to fix the length \( z \) of each section. By making several run of the program using different values of \( z \) we find that for \( z \) less than 1.5 \( \mu m \) the change in \( \Delta \ell \) is very small (at the third place of decimal). We have chosen \( z = 1 \) \( \mu m \). Thus for moving a distance \( z \) along the arc of the circle of radius \( R \) the required change in the angle at the center is

\[ \delta \theta = \sin^{-1} \left( \frac{z}{R} \right) \]

(26)

Taking \( \theta = 0 \) at the edge of the central parallel region, we increase the angle \( \theta \) in steps of \( \delta \theta \) and determine the separation between the two guides for each section and then determine the phase change for each section. Summing up the phase change for each section the incremental parallel coupling length is determined as described above. It may be added here that if a staircase function is being used in fabrication the curved portion of the waveguide is then the step size assumed in the staircase function can be taken equal to \( z \) with a minor modification in the program.

The upper limit of the total number of steps i.e. \( n \) is fixed at a point where the coupling coefficient \( K \) between the two guides vanishes. It is worthwhile to mention here that Takagi et al.\(^6\) have chosen a constant value \( \left( L_n = 1.2 \mu m \right) \) away from the edge of the parallel coupling region). In our case \( L_n \) is not a constant value but it varies with \( \lambda \) for a given coupler geometry.

This choice of \( L_n \) makes our formulism more practicable. Thus to calculate the incremental coupling length we take \( z = 1 \) \( \mu m \), we increase the angle in steps of \( \delta \theta \), (starting with \( \delta \theta = 0 \), for this value we calculate \( L_n \)) and determining \( 2d \) the separation for each section. For this value of \( 2d \) we determine \( \delta \) and \( K \). We make a summation of all \( \delta_i \) till for a value of \( 2d \), \( K \) is less than \( 1 \times 10^{-10} \). From this value of \( \delta \) the phase change in the curvature region is determined and hence the incremental coupling length is determined from the Eq. (25). A plot of \( \Delta \ell \) with wavelength is as shown in the Fig. 7.

\[ \Delta \ell = 0.63 \mu m \text{ at } \lambda = 1.3 \mu m, \text{ and is in good} \]
agreement with the experimental data shown in Ref. (5) Finally, we have calculated the variation of \( \Delta \ell \) with radius of curvature \( R \) and is as shown in Fig. 8. The dots on the same plot represent the experimental results as shown by Takagi et al.\(^6\) Fig. 8. Our theoretical results are very close to the experimental data.

Fig. 9 shows the coupling efficiency with wavelength for different parallel coupling lengths corrected for the coupling in the two S-shaped regions in the input and the output. The lines are the calculated theoretical curves obtained due to the above formulism while the points are the experimental data points as given in Fig. 14 of Ref. (5). It can be seen that the theoretical curves in our case match those of Fig. 6 in Ref. (5) and the experimental agreement is very good.

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Fig. 7 — Variation of incremental coupling length \( \Delta \ell \) with wavelength, for the coupler with parameters \( 2a_1 = 8 \mu m, 2a_2 = 6 \mu m, 2b = 8 \mu m, 2d = 3 \mu m, n_f = 1.455, n_i = 1.4514 \)

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Fig. 8 — Variation of coupling efficiency with curvature for a value of \( 2d \) \( = 1.1 \mu m, \) and \( 1.2 \mu m \), with \( 0.25\% \) less than the experimental results. It is noteworthy that even if there is a flatness error of \( 1 \mu m \) in Fig. 9 the theoretical results agree well with the experimental results that for the range of coupler efficiency in the range 1.3-1.7 the coupling is very good.
of 2d we can all δ till from this the region is the differential length Δx with the good

equation of our in the text figure to the


Therefore, our calculated results are improved in comparison to those given in Ref. (5).

4 Conclusion

In this paper using the boundary value technique we made a detailed theoretical investigation of the coupler with emphasis on the wavelength flattening characteristics. The propagation constant and the modal fields for the combined set of profile are computed with as few mathematical simplifications and approximations as possible. We compare our results with those of the experimental results of Refs (4-6) and find that these are in good agreement. While fabricating curved waveguides or diagonal lines in optical integrated circuits a staircase type of function is utilized which implements the curved portion of the diagonal line as a series of short steps. If a staircase function is being used to fabricate the curved portion of the S-shaped arm of the waveguide then the step size assumed in the staircase function can be taken equal to 2 with a minor modification in our program to determine the equivalent parallel coupling length to account for the coupling in the two S-shaped arms in the input and the output. One point that needs to be mentioned here is the computing time. The computing time to determine the equivalent parallel coupling length for one wavelength is of the order of 5 to 6 min. We are using a simple Pentium based PC at 700 MHz. A higher configuration will reduce the time considerably.

References

Appendix A

n — The number of non-linear equations; ε₁ — The minimum pivot magnitude allowed in the Gauss-Jordan reduction; ε₂ — A small positive number used in convergence test; \( x_{10}, \ldots, x_{n0} \) — The initial guess for given root; \( i_{\text{max}} \) — Maximum number of Newton-Rapson iterations; itcon — Convergence test parameter.

Flow chart of the program to solve the twenty-three non-linear equations to determine the twenty constants and \( \beta_{xs}, \beta_{xa} \) and \( \beta_y \)

The wave equation

\[ \nabla^2 \psi_o + \left( k_o^2 n^2 \right) \psi_o = \psi_{o} \]

where

\[ H_o \psi_o = \beta_o^2 \psi_o \]

In the context difference \( (= n^2) \) can be written as

\[ H \psi = \beta_p^2 \psi \]

where

\[ H = H_o + \varepsilon H_1 \]

\[ \psi = \psi_o + \varepsilon \psi_1 \]

\[ \beta_p^2 = \beta_o^2 + \varepsilon \beta_1^2 \]

and \( H_1 = k_o^2 \)

Substituting we get

\[ H_o \psi_o + \varepsilon H_1 \psi_o = \beta_o^2 \psi_o \]

Multiplying the total area of the propagation

\[ \beta_1^2 = \frac{\iint \psi_o H_1 \psi_o}{\iint \left| \psi_o \right|^2} \]

or
Appendix B

The wave equation can be written as

$$\nabla^2 \psi_o + \left(k_o^2 n^2(x, y) - \beta_o^2 \right) \psi_o = 0$$

where

$$H_o \psi_o = \beta_o^2 \psi_o$$

In the corner regions where the refractive index difference (= n_1^2 - n_2^2) the first order perturbed equation is written as

$$H \psi = \beta^2_{i_p} \psi$$

where

$$H = H_o + \varepsilon H_1$$

$$\psi = \psi_o + \varepsilon \psi_1$$

$$\beta^2_{i_p} = \beta_o^2 + \varepsilon \beta^2_1$$

and

$$H_1 = k_o^2 (n_1^2 - n_2^2)$$

Substituting B3 into B2 and equating terms with \( \varepsilon \) we get

$$H \psi_o + H_1 \psi_o = \beta_o \psi_1 + \beta_1 \psi_o$$

Multiplying both sides by \( \psi_o \) and integrating over the total area of the coupler we get the correction in the propagation constant as

$$\beta^2_i = \frac{\iint \psi_o H \psi_o \, dxdy}{\iint |\psi_o|^2 \, dxdy}$$

or

$$\beta^2_i = H \gamma = k_o^2 (n_1^2 - n_2^2) \gamma$$

where for the single guide

$$\gamma_1 = \frac{T_1 + T_2 + T_3 + T_4}{\iint |\psi(x, y)|^2 \, dxdy}$$

and for the coupler

$$\gamma_2 = \frac{T_1 + T_2 + T_3 + T_4 + T_5 + T_6}{\iint |\psi(x, y)|^2 \, dxdy}$$

where

$$T_1 = \iint_{-d-a}^{d} |\psi(x, y)|^2 \, dxdy; T_2 = \iint_{-d}^{d} |\psi(x, y)|^2 \, dxdy$$

$$T_3 = \iint_{b}^{d} |\psi(x, y)|^2 \, dxdy; T_4 = \iint_{-d}^{d} |\psi(x, y)|^2 \, dxdy$$

$$T_5 = \iint_{d}^{d} |\psi(x, y)|^2 \, dxdy; T_6 = \iint_{a}^{b} |\psi(x, y)|^2 \, dxdy$$