Study of axisymmetric turbulent free mixing layer
Bazle Anwer Gama, M A Taher Ali & A K M Sadrul Islam
Department of Mechanical Engineering, Bangladesh University of Engineering and Technology, Dhaka 1000, Bangladesh

Experiments have been carried out on axisymmetric free turbulent mixing or shear layer issuing from a circular air jet. The present results are compared with reported results. In the present study, initial momentum thickness and Reynolds number are taken as characteristic identifiers of initial condition. The Reynolds number and initial momentum thickness have less effect on mean flow characteristics but cause significant shifts in the virtual origin of the mixing layer. The self preserving variable based on momentum thickness is found to be the best and the variable based on the half radii is equally good for the mixing layer.

A free jet is formed (Fig. 1) when a fluid flows through an orifice or a nozzle into a stagnant infinite reservoir of the same fluid. As the jet proceeds it mixes with the surrounding fluid generating turbulence and the jet itself grows thicker. In the first region, known as the flow development region, turbulence generated at the shear layer progresses towards the centreline of the jet, creating a wedge like region of undiminished mean velocity. This wedge is known as the potential core and is surrounded by a mixing layer. In the second region, known as the fully developed flow region, the turbulence penetrated to the axis and as a result, the potential core is disappeared. For axisymmetric jets the length of the potential core is about four times the diameter of the nozzle.

In the flow development region, the entrainment of ambient fluid creates a continuous transfer of momentum and energy from the jet fluid to the surrounding fluid and forms a pronounced degree of instability due to intensive shearing of the ambient fluid and is known as mixing layer. The thickness of the mixing layer increases as the jet travels downward from the exit.

Axisymmetric Turbulent Mixing Layer
Shear layer or mixing layer is formed in the near field of a jet flow. The developing zone of jet flow is considered as the mixing layer zone (Fig. 2). There are mainly two types of mixing layer namely plane and axisymmetric mixing layer. The former is due to plane jet flow and the later is due to axisymmetric jet flow. Tollmien analytically solved the equation of motion for plane turbulent shear layer. Goertler also solved the general equations of free shear layer. Abramovich observed the similarity of axisymmetric mixing layer with a 100 mm diameter jet at a velocity of 40 m/s. The similarity of the velocity distribution in the axisymmetric mixing layer has been well sustained by the observations of Albertson et al and Rajaratnam and Pani.

The variation of $Y_{0.10}/D$, $b/D$, $Y_{0.95}/D$ with the non dimensional distance from the nozzle, $X/D$, measured by Rajaratnam and Pani can be described by the following empirical equations.

Eq. (1) indicates that the length of the potential core is about 5$D$, where $D$ is the diameter of the nozzle. The experimental results of Rajaratnam and Pani have
\[
\frac{Y_{0.95}}{D} = 0.475 - 0.097 \frac{X}{D} 
\]
\[
\frac{b}{D} = 0.05 + 0.111 \frac{X}{D} 
\]
\[
\frac{Y_{0.10}}{D} = 0.535 + 0.158 \frac{X}{D} 
\]

shown that \(\alpha_1\), the angle of the inner edge of the mixing layer, is about 5.7°, whereas \(\alpha_2\), the angle of the outer edge, is about 9°.

Moore⁵ studied the role of shear layer instability waves in jet exhaust noise and found that for unexcited jet the mean velocity for a conical nozzle increases over the first half-diameter downstream because the nozzle acceleration had not been completed. The potential core extends to approximately 5 diameters downstream, where the mean velocity begins to fall gradually. Moore⁵ found the self preservation profiles in close agreement with theoretical computations of Michalke⁶ than that of Chan⁴ while describing the self preserving variable as \((Y - Y_w)/X\).

Husain and Hussain⁷ investigated the axisymmetric mixing layer and effect of initial condition on it. They found that the shear layer achieved self preservation in the range \(1.5 < X/D < 4.0\) for both laminar and turbulent layers.

Self Preservation Profiles

It is a common practice to plot velocity against self preserving or similarity variables. The self preserving independent variable was defined in many different ways by different authors to express the flow to be self-preserving. Moore⁵ used \(\eta_1 = (Y - Y_{w})/X\) as the self preserving variable for expressing the flow to be self preserving in the developing region. Hussain and Clark⁸ used \(\eta_2 = (Y - Y_{0.50})/\theta_{0.10}\) as the self preserving variable whereas Schlichting⁹ and Azim and Islam¹⁰ used \(\eta_3 = (Y - Y_{0.5})/X\) as the self preserving variable for developing region.

Husain and Zaman¹¹ shows that for plane turbulent mixing layer the momentum thickness, \(\theta\) increases linearly with \(X\) for \(X \geq 300\theta_e\), and \(d\theta/dX\) increases progressively with increasing \(X\) from the origin to \(X = 300\theta_e\). This location for achievement of self preservation was also found by Foss¹², Hussain and Zedan¹³, Husain and Hussain⁷ who also noted a lower slope of \(\theta(X)\) curve for \(X < 300\theta_e\). Hussain and Zedan¹³ showed that this length for achievement of self preservation decreases with increasing values of the Reynolds number, \(R_e\). The asymptotic spread rate \(d\theta/dX\) equals \(V_e/U_e\) (where \(V_e\) is the transverse entrainment velocity) provided that there is a negligible pressure gradient. Hussain and Zaman¹¹ found \(V_e\) equals to 3.2% of the free stream velocity \(U_e\) for plane turbulent mixing layer.

Experimental Procedure

The experiments were carried out in the near field of a 81.4 mm (3.204 inch) circular jet by application of United Sensor (USA) pitot static tube along with Furnace control (UK) pressure transducer and Keithly (USA) data logger for measuring mean axial velocities. The measurements were done in two Reynolds numbers, viz., \(Re_0 = 5.06 \times 10^4\) and \(10.25 \times 10^4\).

In the flow facility (Fig. 3) air flow was created by a 457 mm axial flow fan. Air was first entered into the diffuser through a silencer section, and then through a honeycomb into the first settling chamber. The flow then went through a nozzle, a diffuser, a set of screens (120 and 200 mesh/m), a honeycomb and ultimately exited through a 81.4 mm diameter parabolic profile circular nozzle (Fig. 4) in a laboratory with controlled temperature, humidity and traffic. The axisymmetry of the jet field have been checked before starting the actual measurements.

The pitot tube was traversed with precision with the help of a Mitutoyo (Japan) three coordinate \((X, Y, Z)\) traversing mechanism. In order to minimize the effect of transverse entrainment velocity on zero speed side the integration was terminated at \(Y_{0.10}\) where the value of \(U/U_e\) equals to 0.10.

Results and Discussions

Development of the axisymmetric mixing layer is given in Fig. 5 for Reynolds number
Centerline variation of mean velocity profile is presented in Fig. 7. The solid line shows the computational results of Azim and Islam. The centerline mean velocity remains constant up to $X/D = 3.125$ for the computational results of Azim and Islam. But the centerline velocity remains constant up to $X/D = 3.50$ and 4.25 for $Re = 5.06 \times 10^4$ and $10.25 \times 10^4$, respectively. From this behaviour it can be said that the potential core extended in the downstream direction with the increase of Reynolds number. The decrease rate of centerline velocity of Azim and Islam is found to be nearly same with the present finding for $Re = 5.06 \times 10^4$. The higher decay rate of centerline velocity at lower $Re$ was observed in the figure. This variation in decay rate was also observed by Hussain and Clark and Selim.

In the Figs 8, 9 and 10, the variation of $Y_{0.95}/D$, $(Y_{0.50} - Y_{0.95})/D$ and $Y_{0.10}/D$ with $X/D$, the non-dimensional distance is given for the mixing layer zone. These data are compared with that of Rajaratnam and Pani and Squire and Trouncer. The inner edge of mixing layer, the length scale $b$ and
the outer edge of mixing layer data points show close agreement with that of Rajaratanam and Pani. The outer edge of mixing layer data show a considerable deviation from the prediction of Squire and Trouncer. The effect of Reynolds number on length scale is found insignificant in the inner edge but found significant on the outer edge. This is perhaps for the interaction of outer edge with the ambient still air.

The values of $\alpha_1$ and $\alpha_2$ for $Re_d = 5.06 \times 10^4$ are $4.004^\circ$ and $6.39^\circ$ respectively and the corresponding values for $Re_d = 10.25 \times 10^4$ are $3.95^\circ$ and $5.71^\circ$. This result clearly established the increased jet spread rate for lower Reynolds number.

Streamwise evolution of half radii, defined by $Y_{0.50}/Y_w$ is given in Fig. 11. The result is com-
Fig. 14—Stream wise evolution of entrainment velocity

Fig. 15—Stream wise evolution of \( \theta_{0.10} \)

pared with the computational data of Azim and Islam\(^\text{10}\). The data points are found in close agreement with that of Azim and Islam\(^\text{10}\) up to \( X \approx 5D \) and decays after that. The effect of initial condition and Reynolds number is found to be insignificant.

Figs 12 and 13 show the variation of displacement thickness and momentum thickness with \( X \).

Fig. 14 shows the variation of \( \frac{\Delta \theta}{\Delta X} = \frac{V_e}{U_e} \) with \( X \).

The data points show a variation from 0.00 to 0.05. The slope of momentum thickness is 0.02916 (from Fig. 13) which necessarily means that the transverse entrainment velocity, \( V_e \), is 2.92% of exit mean velocity which is comparable with the data of Hussain and Zaman\(^\text{11}\) for plane mixing layer.

The stream wise evolution of the shear layer momentum thickness \( \theta_{0.10} \) is shown in Fig. 15.

Where \( \theta_{0.10} = \int_{0}^{y_{0.10}} \frac{U}{U_m} \left( 1 - \frac{U}{U_m} \right) \, dy \); here the inte-

gration is terminated at a location where \( U/U_m = 0.10 \). The solid line shows the similar measurement done by Hussain and Clark\(^8\). By linear extrapolation it shows that the virtual origin is located at \( X/D = -0.483 \) upstream for \( Re = 5.06 \times 10^4 \) and at \( X/D = -0.288 \) upstream for \( Re = 10.25 \times 10^4 \). The location of virtual origin for the measurements of Hussain and Clark\(^8\) was found at \( X/D = -0.24 \) upstream for axisymmetric mixing layer. It can be said that the location of virtual origin moves downstream with the increase in Reynolds number.

Fig. 16 shows the variation of \( \theta/\theta_e \) with \( X/\theta_e \). Data of Hussain and Zaman\(^\text{11}\) shows linearity of \( \theta/\theta_e \) at \( X \approx 30 \theta_e \) and the corresponding value of \( R_\theta \) was 428 for plane mixing layer. Whereas in the present study, the linearity of \( \theta/\theta_e \) for \( Re = 5.06 \times 10^4 \) is found to be at \( X = 95 \theta_e \) and for \( Re = 10.25 \times 10^4 \) at \( X = 140 \theta_e \); with corresponding value of \( R_\theta = 796 \) and 739. The linearity of this non-dimensional momentum thickness is an indication of self-preservation. The above result shows that the achievement of self preservation decreases with increasing values of initial momentum thickness Reynolds number \( R_\theta \). This was also noticed by Hussain and Zedan\(^\text{13}\).

**Self preservation profiles—Mixing layer mode—**

Self preservation profiles of mean velocity against self preserving variables \( \eta_1, \eta_2 \) and \( \eta_3 \) are plotted in the Figs 17-19 for \( Re_d = 10.25 \times 10^4 \). In Fig. 17, the experimental data points show considerable deviations from that of Moore\(^5\). In the Fig. 18, the experimental data points show good consistency with that of Hussain and Clark\(^8\). The computational data of Azim and Islam\(^\text{10}\) also show good consistency with present experimental data points in Fig. 19.
The mixing layer is found to be self preserving from $X/D = 1.5$ to $3.5$ and after that it looses its self preservation. This range of self preservation is perhaps because of the presence of potential core in the shear layer.

**Conclusions**

The major conclusions made from the present experiment are as follows:

1. The length of the potential core increases with increase in Reynolds number.

2. The centerline decay rate and spread rate of jet decreases with the increase in Reynolds number.

3. The virtual origin is located upstream of the nozzle lip for initial turbulent boundary layer and moves downstream with the increase in Reynolds number.

4. The achievement of self-preservation decreases with increasing value of initial momentum thickness Reynolds number, $R_e$.

5. $\eta_2 = (Y - Y_{0.50})/(\theta_{0.10})$ is found to be the best self preserving variable for mixing layer mode.

**Nomenclature**

- $b$ = length scale, $= (Y_{0.50} - Y_{0.95})$
- $D$ = diameter of circular jet
- $H$ = shape factor, equals $b/\theta$
- $R_e$ = Reynolds number, $UD/\nu$
- $U$ = mean velocity in axial direction
- $U_e$ = exit mean velocity
- $U_c$ = centre line mean velocity
- $U_{ce}$ = exit centre line mean velocity
- $U_m$ = maximum mean velocity in axial direction
- $V_e$ = transverse entrainment velocity
- $X$ = co-ordinate in axial direction
- $Y$ = co-ordinate in transverse or radial direction
- $Y_{0.10}$ = radial distance where $U/U_m = 0.10$
- $Y_{0.50}$ = radial distance where $U/U_m = 0.50$
- $Y_{0.95}$ = radial distance where $U/U_m = 0.95$
- $\alpha_1$ = the angle of the inner edge of the mixing layer.
- $\alpha_2$ = the angle of the outer edge of the mixing layer.
- $\delta$ = displacement thickness
- $\theta_1$ = momentum thickness, Exist Momentum Thickness
- $\theta_{0.1}$ = momentum thickness, integration terminated at $Y_{0.10}$
- $\eta$ = self preserving variable
- $\nu$ = kinematic viscosity

**References**

1. Tollmien W. ZAMM, 6 (1926) 468.
2. Goertler H. ZAMM, 22 (1942) 244.


