Model development for the filter effluent quality

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A new mathematical model has been developed to predict the effluent quality of horizontal filters which provide a longer filter runs resulting in backwash water economy in comparison to conventional co-current rapid filters. The predictions closely match the observed data which justifies the robustness of the presented model. Advance effluent quality predictions can greatly benefit a filter operator in optimising filter backwashing.

A filter run is terminated when its headloss, rate, or effluent quality fall below their permissible levels. In practice, however, the termination is done for filter backwashings at 24 h intervals for convenience. In conventional rapid filters, its top layers get clogged and contribute to filter headloss. Thus, the entire filter media depth does not contribute to capture the influent turbidity. In a counter-current system (influent passing from coarser to finer media), the turbidity capture is more and uniform throughout the filter depth. As a result, the filter can run for longer times without much deterioration in head-loss, rate, and effluent quality. This system requires a judicious choice of multi-media that can arrange itself in the required fashion (i.e. finer sized media at bottom and coarser sized media at top) after each backwash operation. This arrangement with the same media is easily possible in horizontal or radial filters made in portable small sizes wherein the individual and differently sized media layers can be taken out for washing and replacement. Appropriate number of such small package units with 25 per cent standby units can be employed at a water treatment plant, and the individual units can be backwashed in a rotational order.

The present study is made on declining rate horizontal filters operating on the counter-current mode, and a model is developed to predict the effluent quality which generally dominates as a filter termination criteria in the said operational mode. An advance prediction of the filter effluent quality would benefit the filter operator immensely and would result in an economy of backwash water use due to optimised filter cleaning intervals. The non-eutrophied waters can be subjected to primary sedimentation before their filtration, for economic considerations. To resemble such situations, bentonite clay was used in this study to generate turbidity in the filter influent.

Theoretical Model Development

The difficulty of determining or predicting the specific deposit ($Q_d$, expressed as the volume of the deposit of impurities per unit filter volume) had been a constraint in the use of the various models for the prediction of the filter effluent quality. Bhargava and Pandey had presented a simple though approximate technique to determine the specific deposit. The technique however, required the termination of the filter run.

According to Iwasaki, the rate of removal of suspended solids from the influent per unit depth of the filter bed is assumed to be proportional to the local concentration of the suspended solids. This is mathematically expressed as,

$$-\frac{\partial C_s}{\partial L} = \lambda \cdot \partial L \quad \quad \quad (1a)$$

or

$$-\frac{\partial C_s}{C} = \lambda \cdot \partial L \quad \quad \quad (1b)$$

In Eq. (1), $C_s$ is the concentration of the suspended solids in the influent at any time $t$ and at any depth $L$ in the filter; $\lambda$ is the filter coefficient at any time $t$ and at any depth $L$ and is assumed to maintain over a depth of lamina $\partial L$. $\lambda$ varies with $L$ and $t$. Ives suggested an expression for $\lambda$ which is shown in Eq. (2).

$$\lambda = \lambda_0 \left[ 1 + \frac{\Omega_d}{1 - \varepsilon_0} \right] \left[ 1 - \frac{\Omega_d}{\varepsilon_0} \right]^* \left[ 1 - \frac{d}{(\Omega_d)_s} \right]^* \quad \quad \quad (2)$$
In Eq. (2), $x$, $y$, $z$ are exponent constants; $\Omega_d$ is the amount of specific deposit at any time at a particular depth; $(\Omega_d)_u$ is the ultimate value of $\Omega_d$ at a particular depth when the filter becomes almost ineffective (i.e. no particle capture takes place) at that depth and the suspended solids concentration tends to equal their influent concentration, and this event takes place at some time denoted by $t_u$ when the rate of filtration would be minimum; $e_o$ is the porosity of the clear filter bed; and $\lambda_o$ is the filter coefficient of the clean filter bed and remains constant over the total filter depth $L_T$.

Substitution of Eq. (2) in Eq. (1b), yields on integration,

$$\left[- \frac{\partial C_{ss}}{C_{ss}} \right] = \int \left\{ \lambda_0 \left[ 1 + \frac{\Omega_d}{1 - e_o} \right]^y \left[ 1 - \frac{\Omega_d}{1 - e_o} \right]^z \right\} \partial L + K \quad \ldots (3)$$

or

$$- \ln \left( \frac{C_{ss}}{C_{ss}} \right) = \int \left\{ \lambda_0 \left[ 1 + \frac{\Omega_d}{1 - e_o} \right]^y \left[ 1 - \frac{\Omega_d}{1 - e_o} \right]^z \right\} \partial L + K \quad \ldots (3a)$$

In Eqs (3) and (3a), $K$ is the constant of integration. To evaluate $K$, the boundary conditions at the influent end of the filter bed (i.e. $L=0$, $C_{ss}=(C_{ss})_0$ and $\Omega_d=0$) are substituted in Eq. (3a). This results in,

$$K = - \ln \left( C_{ss} \right)_0 \quad \ldots (3b)$$

In Eq. (3b), $(C_{ss})_0$ denotes the $C_{ss}$ value at $L=0$ (just the influent end of the filter).

Substituting of Eq. (3b), in Eq. (3a) yields Eq. (3c) on simplification.

$$\ln \left( \frac{(C_{ss})_0}{C_{ss}} \right) = \int \left\{ \lambda_0 \left[ 1 + \frac{\Omega_d}{1 - e_o} \right]^y \left[ 1 - \frac{\Omega_d}{1 - e_o} \right]^z \right\} \partial L \quad \ldots (3c)$$

Eq. (3c) is not simple to integrate because $\Omega_d$ varies with $L$ and $t$. It has been reported that $1/(\Omega_d)_u$ varied linearly with $L$ as shown by Eq. (4).

$$\frac{1}{(\Omega_d)_u} = a + bL \quad \ldots (4)$$

Bhargava and Pandey$^2$ reported the respective values of the constants $a$ and $b$ in Eq. (4), in the 62-205 and 0.37-10.7 ranges for declining rate horizontal filters operated in the counter current mode. The $[\Omega_d/(\Omega_d)_u]$ ratio is assumed to vary with $(t/t_u)$ in functional form as shown in Eq. (5).

$$\frac{\Omega_d}{(\Omega_d)_u} = \left( \frac{t}{t_u} \right)^p \quad \ldots (5)$$

where $p$ is a constant exponent.

Substituting Eqs (4) and (5) in Eq. (3c) would yield Eq. (6).

$$\ln \left( \frac{(C_{ss})_0}{C_{ss}} \right) = \int \left\{ \lambda_0 \left[ 1 + \frac{1}{(1 - e_o)} \cdot \frac{1}{(a + bL)} \cdot \left( \frac{t}{t_u} \right)^p \right] \right\} \partial L$$

$$\times \left[ 1 - \frac{1}{(a + bL)} \cdot \left( \frac{t}{t_u} \right)^b \frac{1}{e_o} \right]$$

$$\times \left[ 1 - \left( \frac{t}{t_u} \right)^p \right] \partial L \quad \ldots (6)$$

On the basis of the reported$^2$ values of $a$ and $b$, the product of the terms having exponents $y$ and $z$ in Eq. (6) become very close to 1 irrespective of the values of $y$ and $z$. Hence, the terms having exponents $y$ and $z$ can safely be removed from Eq. (6). Thus Eq. (6) simplifies to Eq. (7).

$$\ln \left( \frac{(C_{ss})_0}{C_{ss}} \right) = \int \left\{ \lambda_0 \left[ 1 - \left( \frac{t}{t_u} \right)^p \right] \right\} \partial L$$

$$= \lambda_0 \left[ 1 - \left( \frac{t}{t_u} \right)^p \right] \cdot L \quad \ldots (7)$$

In order to evaluate $\lambda_0$ ($\lambda$ of clean filter bed, i.e., when $t=0$), boundary conditions are applied at the effluent end of the filter where $L=L_T$ and $C_{ss}=[(C_{ss})_0]_e$. $[(C_{ss})_0]$ denotes the suspended solids concentration in the effluent at the start of the filter run (when $t=0$). Applying these boundary conditions in Eq. (7), results in,

$$\lambda_0 = \frac{L}{L_T} \ln \left( \frac{(C_{ss})_0}{[(C_{ss})_0]} \right) \quad \ldots (8)$$

Substituting the value of $\lambda_0$ from Eq. (8) in Eq. (7), we get,

$$\ln \left( \frac{(C_{ss})_0}{C_{ss}} \right) = \left[ 1 - \left( \frac{t}{t_u} \right)^p \right] \cdot \frac{L}{L_T} \ln \left( \frac{(C_{ss})_0}{[(C_{ss})_0]} \right) \quad \ldots (9)$$

or
Eq. (10a) can be used to predict $C_{ss}$ (i.e. the effluent suspended solids concentration at any $L$ and $t$) provided that $[[C_{ss}]_0]$ (the effluent concentration at the start of the filter run), $L_T$ (total filter depth), $t_u$ (the time of filter termination when $(\Omega_{q}^*)$ value is approached), and the exponents $p$ and $x$ are known. At least four measurements of $C_{ss}$ at different $t$ values will enable the determination of $[[C_{ss}]_0]$, $p$, $x$ and $t_u$, after which the above model (Eq. (10)) can be used to know the variation of $C_{ss}$ with $t$. Similarly, the time of filter termination based on a desired effluent quality can be determined which shall be of great help to the filter operator for an economical backwash interval.

Assuming that in the declining rate filtration the effluent discharge filtration rate $q$ at any time $t$ has the maximum rate $q_{max}$ (occurring at time $t=0$, the beginning of the filter run) which declines to a small rate say $q_{min}$ at $t=t_u$. Since $L_0$ can be assumed to be proportional to $[q/(q_{max} - q_{min})]$, the following expression results,

$$q = \alpha \cdot \left[1 - \left(\frac{t}{t_u}\right)^p\right]^x \quad \cdots (11)$$

or

$$q = (q_{max} - q_{min}) \alpha \left[1 - \left(\frac{t}{t_u}\right)^p\right]^x \quad \cdots (11a)$$

In Eq. (11a), $\alpha$ is the constant of proportionality or,

$$q = \beta \left[1 - \left(\frac{t}{t_u}\right)^p\right]^x \quad \cdots (11b)$$

In Eq. (11b), $\beta = \alpha(q_{max} - q_{min})$ and will be a constant for a given (also type of) filter. It is evident from Eq. (11b) that as $t$ approaches $t_u$, $q$ approaches zero, as expected. If at least four $q$ values are measured at the different $t$ values, the values of $t_u$, $p$ and $x$ can be determined from Eq. (11b). These known values of $t_u$, $p$ and $x$ can then be used in Eq. (10a) to determine the variation of $C_{ss}$ with $t$ provided $[[C_{ss}]_0]$ is known. The value of $[[C_{ss}]_0]$ can be measured in the beginning of the filter run or it can also be evaluated from Eq. (11b) if one value of $C_{ss}$ is measured at a known time $t[p, x$ and $t_u$ values are known].

Results and Discussion

The testing of the presented model was done through data collected on declining rate operated horizontal filter models (with counter-current operation mode) for which the values of the exponents $x$ and $p$ were 1.67 and 1.0, respectively. The details of the two data sets are shown in Fig. 1 which contains the predicted plots (based on Eq. (10a)) of the relative effluent turbidity variation with time (i.e. the plots of $C_s/[C_{ss}]_0$ versus $t$) as well as the observation points. It is apparently seen from Fig. 1 that the observed data points are almost in total agreement with their respective predicted values, and thus, the robustness of the presented model is established. At a few later points, the observed effluent turbidity is insignificantly less than their predicted values probably due to some effluent turbidity getting entrapped in the gravel bed at the effluent end. At the start of a run, the effluent turbidity is slightly in excess of the predicted values due to flushing out of some residual turbidity that was left in the filter, during its earlier backwashing. This is apparent from the fact that the effluent turbidity was almost stabilized soon after, as per the prediction. The time for such effluent turbidity stabilization, however, depends on the filtration rate such that a higher filtration rate would flush out the residual turbidities (from the filter bed) faster. The
model shows that the effluent turbidity variation would very much depend on the ratio of the influent turbidity to the initial expected effluent turbidity which depends on the filter media specifications and the type of the turbidity causing material. Fig. 1 can be used to determine the filter termination time at any desired end effluent turbidity.

Conclusions

Theoretical models have been evolved for the prediction of the effluent turbidity during the operation of a filter. The effectiveness of such models have been tested with the data collected during the actual operation of the laboratory filter models, and the observed data are seen to fit very well with their respective predicted values in a horizontal declining rate filter based on the counter-current mode of operation. The model is most useful to the operator who can know in advance when to terminate the filter run (based on the effluent quality criteria) for doing a backwash.

Nomenclature

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\begin{align*}
L & = \text{length (depth) at any point in the filter bed measured from the influent end of the filter} \\
L_T & = \text{total length (depth) of the filter media bed (measured from the influent end of the effluent end of the filter)} \\
p & = \text{a constant exponent} \\
q & = \text{rate of filtration at any instant} \\
q_{\text{max}} & = \text{maximum rate of filtration} \\
q_{\text{min}} & = \text{minimum rate of filtration} \\
t & = \text{time of filter run} \\
t_* & = \text{time of filter run at termination of the filter run, (when the filter becomes ineffective)} \\
\alpha, \beta, \gamma, \delta & = \text{exponential constant in the Ives filter efficiency model} \\
\alpha & = \text{a constant of proportionality} \\
\beta & = \text{a constant, for a given filter} \\
\lambda & = \text{filter coefficient at any instant, which denotes filter efficiency} \\
\lambda_0 & = \lambda \text{ of the clean filter bed} \\
\Omega_d & = \text{specific deposit (volume of the impurities deposited in a unit filter volume) in the filter at any depth and time} \\
(\Omega_d)_u & = \text{ultimate value of } \Omega_d \text{ at a particular depth when the filter becomes almost ineffective (i.e. no particle capture taking place) at that depth and the suspended solids concentration in the effluent at that depth tends to equal the influent concentration, and this occurs at time } t_*. \\
\end{align*}
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References