Finite element analysis of laminar convection heat transfer and flow of the fluid bounded by V-corrugated vertical plates of different corrugation frequencies

Mohammad Ali & M. Nawsher Ali
Department of Mechanical Engineering, Bangladesh Institute of Technology, Khulna 9203, Bangladesh
Received 27 December 1993; accepted 5 May 1994

A study on steady-state natural convection heat transfer and flow characteristics of the fluid in a horizontal duct with V-corrugated vertical walls has been carried out using control volume based Finite Element Method. The effects of corrugation frequency and Grashof numbers on local Nusselt number, total heat flux, vertical velocity and temperature distributions at the horizontal mid-plane have been investigated. Corrugation frequency from 1 to 3 and Grashof number from $10^3$ to $10^5$ have been considered in this study. The results are compared with the available information on natural convection and fluid flow.

Natural convection is one of the important phenomena in everyday engineering problem. It is of particular importance in designing nuclear reactors, solar collectors and in many other design problems where heat transfer is occurring by natural convection. Heat transfer by natural convection depends on the convection currents developed by thermal expansion of the fluid particles. Further, the development of the flow is influenced by the shape of the heat transfer surfaces. Therefore, the investigation for different shapes of the heat transfer surfaces is necessary to ensure the efficient performance of the various heat transfer equipments.

There are several studies on convective heat transfer with corrugated walls. In these studies, however, the heat transfer has been considered only from a lower hot corrugated plate to an upper cold flat plate, both being horizontally placed. No research has, so far, been carried out on convective heat transfer with vertical hot and cold corrugated plates. Chinnapa carried out an experimental investigation on natural convection heat transfer from a horizontal lower hot V-corrugated plate to an upper cold flat plate. He collected data for a range of Grashof numbers from $10^4$ to $10^6$. The author noticed a change in the flow pattern at $Gr=8 \times 10^4$, which he concluded as a transition point from laminar to turbulent flow. Randall et al. studied local and average heat transfer coefficients for natural convection between a V-corrugated plate ($60^\circ$ V-angle) and a parallel flat plate to find the temperature distribution in the enclosed air space. From this temperature distribution they used the wall temperature gradient to estimate the local heat transfer coefficient. Local values of heat transfer coefficient were investigated over the entire V-corrugated surface area. The author recommended a correlation of the average heat flux which presents 10% higher value than that for parallel flat plates.

Zhong et al. carried out a finite-difference study to determine the effects of variable properties on the temperature and velocity fields and the heat transfer rate in a differentially heated, two-dimensional square enclosure. Nayak and Cheny considered the problem of free and forced convection in a fully developed laminar steady flow through vertical ducts under the conditions of constant heat flux and uniform peripheral wall temperature. Chenoweth and Paolucci obtained steady-state, two-dimensional results from the transient Navier-Stokes equations given for laminar convective motion of a gas in an enclosed vertical slot with large horizontal temperature differences.

System Description
The problem schematic is shown in Fig. 1. The top and bottom walls of the enclosure are insulated and the left and right vertical walls are V-corrugated. The left and right walls are kept at constant temperature. The temperature of the left wall is $T_L$ and that of the right wall is $T_R$, where $T_L > T_R$. The characteristic length of the square enclosure is $L$. The origin of the X-Y coordinate
system is located at the left bottom corner of the cavity.

**Basic Equations**

The Navier-Stokes equations\(^6\) for two-dimensional, incompressible flow with constant properties in cartesian coordinates can be written as follows:

**Continuity equation,**
\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \ldots (1)
\]

**x-momentum equation,**
\[
\frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + v \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + s^x \quad \ldots (2)
\]

**y-momentum equation,**
\[
\frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + v \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + s^y \quad \ldots (3)
\]

In the above equations, \(u\) and \(v\) represent the velocity components in the \(x\) and \(y\) directions respectively and \(p\) is the pressure. The source terms \(s^x\) and \(s^y\) consider the other body and surface forces in the \(x\) and \(y\) directions respectively and \(v\) is the kinematic viscosity.
By differentiating Eqs (2) and (3) with respect to \( y \) and \( x \) respectively and then subtracting the results of the former from the latter, a single vorticity transport equation can be obtained,

\[
\frac{\partial \omega}{\partial x} + v \frac{\partial \omega}{\partial y} = \nu \left( \frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2} \right) + \left( \frac{\partial S}{\partial x} - \frac{\partial S}{\partial y} \right) \quad (4)
\]

where \( \omega \) is the vorticity, defined as,

\[
\omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \quad (5)
\]

Upon defining the stream function, \( \psi \) as

\[
\frac{\partial \psi}{\partial y} = u \quad (6)
\]

\[
-\frac{\partial \psi}{\partial x} = v \quad (7)
\]

the Poisson equation relating \( \omega \) to \( \psi \) may be obtained by substituting Eqs (6) and (7) into Eq. (5):

\[
\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \omega = 0 \quad (8)
\]

The details for implementing \( \omega \) and \( \psi \) conditions are available in the reference 7.

Assuming the properties to be constant other than the density variation in the buoyant forces, the Boussinesq approximation which consists of retaining only the variations of density in the buoyancy terms, may be used on Eq. (4) which results in

\[
\frac{\partial \omega}{\partial x} + v \frac{\partial \omega}{\partial y} = \nu \left( \frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2} \right) + g \beta \frac{\partial T}{\partial x} \quad (9)
\]

The energy transport equation for two-dimensional incompressible flow with constant properties can be written as,

\[
\frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \quad (10)
\]

where \( \alpha \) is the thermal diffusivity of the fluid.

Eqs (4)-(10) can be normalised by introducing the following nondimensional quantities,

\[
X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad U = \frac{uL}{v}, \quad V = \frac{vL}{v}
\]

\[
\Omega = \frac{\omega L^2}{v}, \quad \psi = \frac{\psi}{v}, \quad \theta = \frac{T - T_c}{T_h - T_c}
\]

where \( Gr \) and \( Pr \) are the Grashof and Prandtl numbers respectively and defined as:

\[
Gr = g \beta (T_h - T_c) L^2 / \nu^2
\]

\[
Pr = \nu / \alpha
\]

Here the parameters \( g, \beta \) and \( \alpha \) represent the acceleration due to gravity, the coefficient of thermal expansion and the thermal diffusivity of the fluid respectively.

The boundary conditions of the problem are as follows:

(i) \( U = V = 0 \) at all walls
(ii) \( \psi = 0 \) at all walls
(iii) \( \theta = 1 \) at left wall
(iv) \( \theta = 0 \) at right wall

The calculation domain is first discretized and an example of discretization of the domain is shown in Fig. 2. Following the domain discretization, the integral formulation of the relevant transport equation is imposed on each control
The dimensionless total heat flux at the hot wall,

\[ \frac{q^* L}{k(T_h - T_c)} = -\frac{\partial \theta}{\partial N} \]  

... (13)

Where \( S \) is the dimensionless distance measured along the corrugation of the wall and \( N \) is the dimensionless distance measured normal to same. In Eq. (13) \( q^* \) is the heat flux rate per unit length at the hot wall and \( k \) is the thermal conductivity.

**Results and Discussion**

In this investigation the corrugation amplitude was fixed at 5% of the enclosure height for all runs, where the amplitude "A" is defined as half the horizontal distance measured from the left extremity of the left wall to its right extremity (see Fig. 1). Henceforth, the left and right extremities of the hot wall will be referred to as the "trough" and "peak", respectively. A numerical value \( 10^{-5} \) was considered as the convergence criterion for receiving better accuracy of the results. It may be mentioned that the convergence checking was performed by calculating the residue from the vorticities and coefficients of the relevant nodal equations and comparing it with the convergence criterion. It was found that there was no appreciable change in results for lower value of convergence criterion than the above.

**Effects of corrugation frequency (CF) on total heat flux**—Figs 4 and 5 show the streamline pattern of the fluid flow and isotherm plot into the enclosure for Grashof number \( 10^5 \), respectively and Table 1 shows the effects of CF on total heat flux \( Q \) with different Grashof numbers. This table shows that \( Q \) increases continuously with the increase in CF for \( Gr = 10^3 \) but for \( Gr = 10^4 \) and \( 10^5 \), \( Q \) decreases with increase in CF from 1 to 3. The increase of CF from 1 to 3 leads to a higher value of \( Q \) for \( Gr = 10^3 \), which may be attributed to the enhancement of surface area, but the decrease in \( Q \) for higher \( Gr \) may be explained as the retardation of flow due to increased waviness of the corrugation. This behaviour may also be explained by asserting that at high \( Gr \) the fluid velocity increases near the peaks but drops near the troughs as the boundary layer tends to separate. Thus the fluid fails to maintain close contact near the troughs of the corrugation, resulting in decreased convection heat transfer, whereas for

<table>
<thead>
<tr>
<th>Grashof number</th>
<th>( Q ) for CF = 1</th>
<th>( Q ) for CF = 2</th>
<th>( Q ) for CF = 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 10^3 )</td>
<td>1.126</td>
<td>1.132</td>
<td>1.135</td>
</tr>
<tr>
<td>( 10^4 )</td>
<td>2.295</td>
<td>2.271</td>
<td>2.238</td>
</tr>
<tr>
<td>( 10^5 )</td>
<td>4.837</td>
<td>4.753</td>
<td>4.573</td>
</tr>
</tbody>
</table>

Table 1—The variation of the total heat flux for corrugated walls with different Grashof numbers

---

**Fig. 3**—A typical control volume shown shaded

**Fig. 4**—Streamline plot \( (Gr = 10^5 \ CF = 3) \)
Gr = 10^3, the low vertical velocities thus generated enable the fluid to maintain better contact with the corrugated wall. Thus with increasing CF the corresponding enhancement of heat transfer surface leads to an increase in total heat flux at low Gr, but for the case of high Gr, the lower velocities and consequent decrease in convective heat transfer at the troughs offsets the effect of increased surface area.

Effects of corrugation frequency on local Nusselt number—The decreasing nature in convection heat transfer is evident from Figs 6-8 where it may be observed that the local Nusselt number which is identical to the dimensionless local heat flux attains minimum values at the troughs of the corrugation. It can be noted from these figures that there is a significant increase in local Nusselt number at the peaks of the corrugation. The

![Fig. 5—Isotherm plot (Gr = 10^4 CF = 3)](image)

![Fig. 6—Local Nusselt number distribution on the hot wall (CF = 3)](image)

![Fig. 7—Local Nusselt number distribution on the hot wall (CF = 2)](image)

![Fig. 8—Local Nusselt number distribution on the hot wall (CF = 1)](image)
reason for this is that the peaks cause the fluid to come in contact more intimately with the surface, resulting in large convection heat transfer and consequently the local Nusselt number increases. These figures also show that the peak values of \( Nu \) decrease with increasing vertical distance along the corrugated wall. This may be explained by the fact that the colder fluid collects at the bottom-left corner of the enclosure creating a large temperature gradient with the hot wall, which is the main driving force for heat transfer and as it moves up and receives heat, the temperature gradient decreases, causing the decrease in local Nusselt number.

**Effects of corrugation frequency (CF) on vertical velocity and temperature distribution**—Figs 9 and 10 exhibit the effects of CF on vertical velocity distribution at the horizontal mid-plane for \( Gr=10^5 \) and \( 10^3 \), respectively. Fig. 9 indicates that the peak value of the vertical velocity decreases with increase in CF. This trend can be explained by examining Fig. 11 which indicates that the temperature gradient is lower for higher CF, causing a lower buoyant force and hence a lower vertical velocity. Because of this lower velocity, the strength of convection heat transfer decreases with increasing CF. But in Fig. 10 the vertical velocity increases with increase in CF, which leads to an increase in overall heat transfer.

**Effects of \( Gr \) on total heat flux**—Fig. 12 shows the variation of \( Q \) as a function of \( Gr \) for different CF and portrays the comparison between the results of the present study and Hussain. Here it can be pointed out that for higher values of \( Gr \) the rate of increase of \( Q \) increases with decreasing CF and the different curves “cross” at around \( Gr=10^3 \) indicating a trend reversal which have been discussed earlier.
The overall heat transfer decreases with the increase of corrugation frequency for higher Grashof number but the trend is reversed for lower Grashof number.

2. At low Grashof number the conduction mode prevails and as corrugation frequency increases, total heat flux is enhanced for the enhancement of surface area whereas for high Grashof number the fluid fails to collect heat by transport due to increasing corrugation.

3. The peak values of local Nusselt number decrease with the increase of vertical distance along the corrugated surface and local Nusselt number increases significantly at the peaks and attains minimum values at the troughs of the corrugation.

4. The vertical velocity of the fluid decreases with the increase of corrugation frequency for high Grashof number because of the lower temperature gradient as the boundary layer tends to separate but the trend is reversed for low Grashof number.

5. The decreasing trend of total heat flux may find application in practical situations where heat transfer reduction is desired across large temperature differences by increasing the corrugation of the vertical wall.

**Acknowledgement**

The authors are highly grateful to Computer Centre of Bangladesh University of Engineering and Technology, Bangladesh for providing facilities of the numerical computation.

**Nomenclature**

- \( u \) = horizontal velocity, m/s
- \( v \) = vertical velocity, m/s
- \( U \) = dimensionless horizontal velocity
\( v \) = dimensionless vertical velocity  
\( x \) = horizontal cartesian coordinate, m  
\( y \) = vertical cartesian coordinate, m  
\( X \) = dimensionless horizontal cartesian coordinate  
\( Y \) = dimensionless vertical cartesian coordinate  
\( \mu \) = absolute viscosity, kg/ms  
\( \nu \) = kinematic viscosity, m\(^2\)/s  
\( \rho \) = density, kg/m\(^3\)  
\( \psi \) = stream function, m\(^2\)/s  
\( \Psi \) = dimensionless stream function  
\( \omega \) = vorticity, s\(^{-1}\)  
\( \Omega \) = dimensionless vorticity  
\( \alpha \) = thermal diffusivity, m\(^2\)/s  
\( \beta \) = coefficient of thermal expansion, 1/K  
\( g \) = acceleration due to gravity, m/s\(^2\)  
\( h \) = convective heat transfer coefficient, W/m\(^2\)K  
\( k \) = thermal conductivity, W/m K  
\( Gr \) = Grashof number  
\( Pr \) = Prandtl number  

References  