Control of a robotic manipulator—A model following approach

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A model following approach has been used to produce a consistently good dynamic performance in a manipulator, whose effectiveness is illustrated through the simulated responses of a two degree of freedom plant with variable inertia and fluctuating payload. The model is provided with the reference input and produces the "ideal response" that sets the standard to be followed by the plant which runs totally on the basis of the differences of its states from those of the model. More eruditely, the plant is constrained by the model to emulate its states. A higher order plant was intentionally made to follow a lower order model to test the effectiveness of the controller. Satisfactory simulation results assure the usefulness of the scheme even under considerable payload fluctuations (±10%), and a 1:8 range of inertia variation that follows a quadratic law. This system is found to have a noteworthy performance robustness despite its simple and cost effective hardware implementation.

Precision and accuracy of modern robotics demand highly efficient manipulators and therefore, it is important that the performance of these devices be enhanced. The conventional methods of control are not always effective in producing a desired performance as in many applications there are changes in system parameters during operation. The changes may arise due to variation of inertia as a result of changing configuration or due to fluctuations in the payload. It is necessary, therefore, to seek more appropriate methods of control.

Often a robot joint drive consists of a smaller inertia connected flexibly to a configuration dependent larger inertia by means of either a belt, chain or a gear drive. Configuration changes of the arm are associated with variation of inertia. It is imperative that in order to maintain a level performance, it shall be necessary to adjust the feedback gains in accordance with this inertia variation. This problem has been resolved by means of software servo control\textsuperscript{1-3}. One non-conventional method\textsuperscript{4} receiving increased attention in recent times—the Model Reference Adaptive Control (MRAC)—has also been tried with success to prove equal to this task\textsuperscript{5-7}. Nevertheless, these methods are not simple and hardware implementation for them are both complex and expensive. Most adaptive control algorithms, developed before 1982 assumed adaptive controllers of the same order as that of the plant. It was believed that a lower order adaptive controller cannot control a higher order plant due to errors resulting from "unmodelled dynamics". Bar-Kana\textsuperscript{8} in 1991 showed that lower order adaptive controllers can also be used to control higher order plants if proper caution is taken on the bounds of stability.

In the present study, a simple version of the Model Reference approach by deleting the idea of adaptation has been investigated. This is the model following control (MFC). In this process a model is first designed, which is a clone of the original system. The parameters of the model are so chosen that its response is considered ideal in terms of the control objectives such as overshoot, settling time, speed of response, etc. The model, running simultaneously with the plant, is given the reference input $R$. States of the model are then compared with the corresponding states of the plant and the errors are fed to the plant in the feed forward path, thus providing the control necessary to constrain the plant states to coincide with the corresponding ones of the model. Further, the plant chosen is a two degree of freedom robot whose shoulder joint is driven by a dc servomotor through a voltage source amplifier. The plant dynamics is of 5th order, while the chosen model is an implicit one of 4th order.

Model Following Control

The optimal control theory has been very well developed and can meet the requirements for complex and high performance control systems. But the theory cannot always be applied successfully since the (quadratic) performance index is often difficult to construct in terms of the real control
One effective way to avoid this difficulty is to use the Linear Model Following Control system (LMFC). LMFC specifies the design objectives through a model which may either be conceptual (implicit), utilized only for the construction of the control law or real (explicit), i.e., a part of the original system. Variation of plant parameters during operation requires the application of Adaptive Control for consistent precision. LMFC system with an explicit model can be extended to Adaptive Model Following Control (AMFC). Here, the system containing a plant and a model is shown in Fig. 1, has three matrices: $K_M$—the model feedback gain, $K_P$—the plant feedback gain and $K_U$—the feedforward gain. $U$ is the controlled input to the plant. The elements of these matrices are so chosen that perfect model following occurs at null initial conditions as indicated by null state errors (displacement & velocity), denoted here by $e$ for any input $R$. However, Adaptive Control is always more complex and expensive. The authors are of the opinion that satisfactory plant response can also be obtained by suitable choice of a model in the simple LMFC scheme even under considerable amount of parameter variations and this is on the basis of studies made earlier with wide range of inertia variations (1:8), where disturbance is also imposed on the plant in terms of fluctuating payload (±10%). Encouraging results were obtained to justify this contention. Here, the general LMFC proposal is applied to a simple configuration. Though the requirement for the state errors to be null for any input may not be satisfied, especially at the earlier stages of response, still very good end results can be obtained by suitable design of the model. The block diagram representation of the scheme is given in Fig. 2. This scheme utilizes a model, which can be considered ideal in terms of the control objectives mentioned earlier. The model runs simultaneously with the plant and exerts the requisite control to force the plant states to coincide with those of the former, i.e., to produce null state errors. Further, in this paper, the reference input is applied only to the model. Clearly, the plant is made to run under a control which is proportional to the state errors. It is immaterial whether the model is implicit or explicit. The advantage in this scheme is that the model can have constant parameters. It is noteworthy that use of the model enables the determination of errors in all the states of the plant which is otherwise unavailable if the input $R$ is used directly for comparison.

When the plant and model are in exact correspondence, it is expected that the plant should be able to emulate the ideal states of the model. The displacement and the velocity error vectors are given as:

$$e = q_m - q_p \text{ and } \dot{e} = \dot{q}_m - \dot{q}_p$$

These errors are provided with suitable weights by means of error gain vectors $A_p$ and $A_D$ for producing effective control. For single input, vector $B$ in the feedback line of the model is given by:

$$B = \{0 \ 1\}$$

Here, the control is simplified to the extent of selecting only the gain vectors $A_p$ and $A_D$ in accordance with the performance expected from the manipulator.

**Model Formulation**

Fig. 3 shows the schematic diagram of the robot manipulator being modelled after the shoulder joint. This is a two degree of freedom system with lumped parameters. A smaller inertia is connected with a larger (variable) one by means of a flexible belt. This is a follow up of the studies of Seto et al. and Mukhopadhyay & Sengupta. Schematic diagram of the model is given in Fig. 4. Here a rotational model with constant inertia is chosen. The
The model is so designed that its response is slower than that of the plant and its natural frequencies are made close to those of the variable inertia plant at its higher inertia setting. We shall consider that the state vectors $q_m$ and $q_m'$ are ideal at any instant.

Here a digital simulation is made with an implicit model providing ideal response norms for the plant. Also the plant or the robot joint is driven by a linear dc servomotor, which is powered by a voltage source. Such a voltage source could be a cost effective analog device like a power operational amplifier. The model in this paper assumes a motor driven by a current source. It is to be noted that for the given specifications, the plant is of 5th order and is being made to follow a 4th order model.

**Stability Analysis**

The model (current source driven motor) dynamics are represented by the following equations:

$$J_m \ddot{q}_m + C_m \dot{q}_m + K_m q_m = K_{em} (R - Bq_m) \quad \ldots (1)$$

where

$$J_m = \begin{bmatrix} J_m & 0 \\ 0 & J_{m2} \end{bmatrix}$$
$$C_m = \begin{bmatrix} C_m + C_{m1} r_{m1}^2 & -C_{m1} r_{m1} r_{m2} \\ -C_{m1} r_{m2} r_{m1} & C_m + C_{m2} r_{m2}^2 \end{bmatrix}$$
$$K_m = \begin{bmatrix} K_{m1} r_{m1}^2 & -K_{m1} r_{m1} r_{m2} \\ -K_{m1} r_{m2} r_{m1} & K_{m2} r_{m2}^2 \end{bmatrix}$$
$$K_{em} = [K_{em'}]^T$$

For the model following plant, the equation for a voltage source driven motor is

$$V - E_b = L \frac{di}{dt} + R_a \cdot i$$

from which,

$$i = \frac{1}{(R_a + Ls)}(V - E_b) = \mathscr{L}(V - E_b) \quad \ldots (2)$$

where $s$ is the operator $d/dt$, $E_b$ is the back emf produced in the motor, $K_{e1}$, $q_{p1}$, and $\mathscr{L}$ is a linear exponential lag operator given by:

$$\mathscr{L} = \frac{1}{(R_a + Ls)}$$

voltage applied to the motor (from the voltage source) is given by:

$$V = K_e(A_{p1} e_1 + A_{p2} e_2 + A_{d1} e_1 + A_{d2} e_2) \quad \ldots (3)$$

Eqs (2) and (3) combine with others to form the plant equation as follows:

$$J_p \ddot{q}_p + J_p \dot{q}_p + C_p \dot{q}_p + K_p \mathscr{L} K_{e2} q_p + K_p q_p + T_e = K_{ep} \mathscr{L} (A_p q_m + A_D q_m) \quad \ldots (4)$$

where,

$$C_p = \begin{bmatrix} C_{p1} + C_{p1} r_{p1}^2 + K_{ep} \mathscr{L} A_{d1} & K_{ep} \mathscr{L} A_{d2} - C_{p1} r_{p1} r_{p2} \\ -C_{p1} r_{p1} r_{p2} & K_{ep} \mathscr{L} A_{d2} \end{bmatrix}$$
$$K_p = \begin{bmatrix} K_{p1} r_{p1}^2 + K_{ep} \mathscr{L} A_{p1} & K_{ep} \mathscr{L} A_{p2} - K_{p1} r_{p1} r_{p2} \\ -K_{p1} r_{p1} r_{p2} & K_{p2} r_{p2}^2 \end{bmatrix}$$

$J_p = \begin{bmatrix} J_{p1} & 0 \\ 0 & J_{p2} \end{bmatrix}$; $J_f = \begin{bmatrix} 0 & 0 \\ 0 & J_{f2} \end{bmatrix}$; $K_b = \begin{bmatrix} K_b & 0 \\ 0 & 0 \end{bmatrix}$

$T_e = [0 \ T_e]^T$; $q_p = [q_{p1} \ q_{p2}]^T$

$A_p = [A_{p1} \ A_{p2}]$; $A_D = [A_{d1} \ A_{d2}]$

From Eq. (4) the characteristic equation for the plant is derived as

$$b_5 s^5 + b_4 s^4 + b_3 s^3 + b_2 s^2 + b_1 s + b_0 = 0 \quad \ldots (5)$$

where,

$$b_5 = J_{p1} J_{p2} L$$
$$b_4 = J_{p2} C_{p1} L + J_{p1} C_{p2} r_{p1}^2 L + J_{p1} J_{p2} R_a + J_{p1} C_{p2} r_{p2}^2 L$$
$$b_3 = \ldots$$
$$b_0 = \ldots$$
\[ b_3 = J_{p_2} K_{p_2} r_{p_1}^2 L + J_{p_2} C_{p_1} R_a + J_{p_2} C_{p_1} r_{p_2}^2 L + J_{p_2} C_{p_1} R_a + K_i K_b J_{p_2} \]
\[ + K_{e_1} A_{d_1} J_{p_2} + C_{p_1} C_{p_2} R_{a_1} + J_{p_2} C_{p_1} R_{a_1} + K_i K_b J_{p_2} \]
\[ + J_{p_2} L K_{p_2} r_{p_2}^2 \]
\[ b_2 = J_{p_2} K_{p_2} r_{p_1}^2 R_a + K_{e_1} A_{d_1} J_{p_2} + C_{p_1} C_{p_2} r_{p_2}^2 R_a \]
\[ + K_i K_b C_{p_2} r_{p_2}^2 + J_{p_2} K_{p_2} r_{p_2}^2 R_a + K_{e_1} A_{d_1} C_{p_2} r_{p_2}^2 \]
\[ + K_{e_1} A_{d_2} C_{p_2} r_{p_2} + K_{p_2} C_{p_2} L \]
\[ b_1 = K_{e_1} A_{d_1} C_{p_2} r_{p_2}^2 + K_{p_2} C_{p_2} r_{p_2} + K_{e_1} A_{d_2} C_{p_2} r_{p_2} \]
\[ + K_{e_1} A_{d_2} C_{r_1} r_{p_1} r_{p_2} \]
\[ b_0 = K_{e_1} A_{d_1} K_{p_2} r_{p_2} + K_{e_1} A_{d_2} K_{p_2} r_{p_1} r_{p_2} \]

The conditions for the system to be stable are obtained by the application of Routh's criteria as follows:

\[ b_5 > 0, b_4 > 0, b_3 > 0, b_2 > 0, b_1 > 0, b_0 > 0 \quad \ldots (6) \]
\[ b_4 b_3 > b_5 b_2 \quad \ldots (7) \]
\[ b_2 > (b_4 b_1 - b_5 b_0)/(b_4 b_3 - b_5 b_2) \quad \ldots (8) \]
\[ b_4 b_1 > b_2 b_0 + [b_0 (b_4 b_3 - b_5 b_2)]/(b_2 (b_4 b_3 - b_5 b_2)) \]
\[ - b_3 (b_1 b_4 - 4 b_5 b_0)] \quad \ldots (9) \]

Eqs (6)-(9) provide only the stability ranges for the gains of the system: \( A_{p_1}, A_{p_2}, A_{d_1} \) and \( A_{d_2} \). But to visualize the system response and its stability it is necessary to study the root loci of the system by varying the gain vectors \( A_F \) and \( A_D \).

**Practical Example**

A practical example, similar to the one of Seto et al. is considered to illustrate the different aspects of the MFC scheme for a voltage source driven joint motor.

The plant is assumed to have the following parameters:

\[ K_p = 5 \times 10^5 \text{ N/m}; \quad C_p = 2100 \text{ N s/m}; \]
\[ C_{p_1} = 0.1 \text{ N ms/rad}; \quad K_{e_1} = K_1; \quad K_t = 125 \text{ NmV/rad A}; \]

where,

\[ K_1 = \text{(Voltage source) Amplifier gain} = 100 \text{ V/rad}; \]
\[ \text{Output swing of the voltage source amplifier} = \pm 15 \text{ V}; \]
\[ K_t = \text{Motor torque constant} = 1.25 \text{ N m/A}; \]
\[ r_{p_1} = 9.55 \text{ mm}; \quad r_{p_2} = 38.2 \text{ mm}; \quad J_{p_1} = 0.0018 \text{ kg m}^2; \]
\[ J_{p_2} = 0.0089 \text{ kg m}^2 \text{ to } 0.071 \text{ kg m}^2; \]
\[ L = 0.003 \text{ H}; \quad R_a = 0.5 \text{ Ohm}; \]
\[ K_b = \text{Back emf constant of motor} = 0.15 \text{ Vs/rad}. \]

Proper design of a reference model is the first important step in MFC scheme. This is done by adjusting its parameters, such that its natural frequencies closely match those of the plant at its higher inertia setting. An implicit model is chosen and for the sake of convenience, it is built to 1:1 scale with the plant. The larger inertia of the model is made equal to the maximum value of higher inertia of the plant.

The parameters of the model are:

\[ C_m = 2100 \text{ N s/m}; \quad C_{m_1} = 0.4 \text{ N ms/rad}; \]
\[ C_{m_2} = 3 \text{ N ms/rad}; \]
\[ K_m = 5 \times 10^5 \text{ N/m}; \quad K_t = 0.65 \text{ N m/A}; \]
\[ K_1 = \text{(Current source) Amplifier} \text{ gain} = 100 \text{ A/rad}; \]
\[ r_{m_1} = 9.55 \text{ mm}; \quad r_{m_2} = 38.2 \text{ mm}; \]
\[ J_{m_1} = 0.0018 \text{ kg m}^2; \quad J_{m_2} = 0.071 \text{ kg m}^2. \]

Main objective of the MFC scheme is to choose suitable gains for the computed errors to produce the requisite control action, irrespective of the plant parameter variations and impart reasonable performance robustness to the system. To achieve this, the root loci of the plant are obtained from its characteristic equation, by varying one gain at a time and keeping the others fixed at some nominal values, taken as \( A_{p_1} = A_{p_2} = 1; \quad A_{d_1} = A_{d_2} = 0.001. \)

Higher inertia of the plant is kept at the geometric mean (0.0251 kg m\(^2\)) of its minimum and maximum values for the purpose of this computation.

The root loci are useful in selecting the starting values of the gains on the basis of the margin of stability. Based on these values, extensive search is made for the optimum gains to ensure minimum deviation between the model and plant responses for both increasing and decreasing plant inertia. The choice of optimum gains is made by defining a quadratic performance index \( Z \) as follows:

\[ Z = \int_0^T [w_1 \cdot \dot{e}_2^2 + w_2 \cdot \dot{e}_2^2 + w_3 \cdot i^2] \, dt \]

where \( e_2 = q_{m_2} - q_{p_2} \) and \( i \) is motor current in the energy term. \( T \) is chosen to be 300 ms and the
weight factors are taken as \( w_1 = 10000; w_2 = 1 \) and \( w_3 = 0.1 \), where maximum emphasis is given to the position error.

The average of two sets of gains for both increasing and decreasing inertia is then forwarded to give a final set of gains as a compromise for both cases. These are obtained as:

\[
A_{p_1} = 0.08; A_{p_2} = 5.37; A_{d_1} = 0.011; A_{d_2} = 0.05.
\]

Final gains thus obtained are utilized to draw the root loci for the plant by varying the gains in turn, keeping others fixed at the above calculated values. These are shown in Figs 5-8 depicting clearly the stability ranges of the plant for all the gains. The operating points, marked with \( \Delta \), indicate that this choice produces the near-highest margin of stability as judged by their distance in the left half plane from the vertical (imaginary) axis.

**Simulation Results**

Control systems often find themselves in difficulty while handling the problem of parameter variations during operation. This is given major importance here while simulating the system equations. Inertia \( J_{p_2} \) of the output rotor of the plant is made to vary within a range as wide as 0.0089 kg m\(^2\) to 0.071 kg m\(^2\) (1:8 range). The output inertia of the plant is assumed to increase or decrease according to the following law:

\[
J_{p_2} = J_{p_2} \text{ [initial]} + J_{p_2} \cdot t, \quad \text{where} \quad J_{p_2} = a + bt.
\]

This law presumes a cylindrical robot with uniform radial motion where the slewing at the shoulder joint is controlled by the proposed servo-system. Further, when the inertia \( J_{p_2} \) increases, \( J_{p_2} \) is assumed to rise uniformly from 0.12 kg m\(^2\)/s to 0.21 kg m\(^2\)/s in 300 ms time. For decreasing \( J_{p_2} \), \( J_{p_2} \) is assumed to be attenuated uniformly from -0.12 kg m\(^2\)/s to -0.21 kg m\(^2\)/s in 300 ms. Additionally a sinusoidally fluctuating disturbance of magnitude \( T_e = 14 + 1.4 \sin \omega t \) is imposed on the system at near resonant frequencies. For increasing
Load: \( T_e = 14 + 1.4 \sin 500t \) (a) SSC control

Fig. 9—Transient response of loaded plant with varying \( J_{p2} \) (uniformly increasing)

\[ \text{Fig. 9—Transient response of loaded plant with varying } J_{p2} \text{ (uniformly increasing)} \]

\[ \text{Fig. 10—Transient response of loaded plant with varying } J_{p2} \text{ (uniformly decreasing)} \]

Fig. 10 gives the transient response of the system with decreasing inertia, other conditions being similar to the previous one. Again the plant response shows remarkable coincidence with the ideal states of the model after tolerable departures within the first 120 ms and the residual oscillations die down at around 200 ms. Current required and energy expenditure are again in agreement with those noted for an increasing rate of inertia.

Figs 9 and 10 document quantitatively the level performance provided by the present scheme under very wide parameter variations. Note that between 100 ms and 200 ms \( J_{p2} \) increases from 0.024 kg m\(^2\) to 0.045 kg m\(^2\) in Fig. 9 while the same falls from 0.056 kg m\(^2\) to 0.035 kg m\(^2\) in Fig. 10 and the shape of the response remains quite uniform upto 200 ms (truncated here). The performance robustness is therefore noteworthy.

Fig. 11 shows a comparison of the step response obtained by the present MFC control with the SSC control as proposed by Seto et al. In the SSC the inertia is not varying continuously but is fixed at two representative values \( J_{p2} = 0.0089 \text{ kg m}^2 \) and \( J_{p2} = 0.071 \text{ kg m}^2 \). The response of the proposed MFC is much superior to that of the SSC even with continuous \( J_{p2} \) variation.

\[ \text{Fig. 11—Comparison of step response for SSC and MFC} \]

\[ \text{Fig. 11 shows a comparison of the step response obtained by the present MFC control with the SSC control as proposed by Seto et al.} \]

Conclusion

Parameter variation of a system during operation is considered to have the most damaging effect on its dynamic characteristics. A number of methods have been used with success to mitigate this effect. One of these methods, the Model Following Control, is experimented here with some modification in that the reference input is applied only to the model.

As mentioned earlier, the output inertia \( J_{p2} \) is modelled after the shoulder joint of a cylindrical robot with uniform radial motion at the end effector where the inertia variation (over a 1:8 range) is a quadratic function of time. For increasing inertia,
Further, the joint has been subjected to a sinusoidally fluctuating load torque disturbance with a ±10% pulsation at frequencies close to resonance.

The plant in this simulation has no doubt been subjected to a very severe punishment by way of above variations, but its response is quite uniform (Figs 9 and 10). The performance robustness is thus noteworthy. Additionally, there is a high enough speed of response and negligible residual oscillations. The authors' have chosen a voltage source amplifier (linear) which, as mentioned earlier, can be very simply realized by power operational amplifiers that drastically reduce the circuit manufacture cost. Further, in the absence of adaptation, the model can be implemented as an analog circuit with low cost linear micro electronic devices.

The results of simulation are highly encouraging and the arrangement can well compete in performance with other more expensive and complex schemes using adaptation. This proposal is fully practical also from the standpoint of simple hardware implementation. Of further interest is the fact that a reduced order model (4th order) is used to make a higher order plant (5th order) follow the same. It is thus observed that even if the dynamics of the plant be somewhat “neglected” it will still be possible to achieve successful MFC, as with a reduced order model chosen here. The MFC arrangement developed here is perhaps worthy of inclusion in the family of efficient and practical control methods.

Nomenclature

\[ J_p = 0.0089 + J_p t, \]
\[ J_p = 0.12 + 0.29t. \]

Where

\[ J_p = 0.12 + 0.29t. \]

While for decreasing inertia,

\[ J_p = 0.071 + J_p t, \]
\[ J_p = 0.12 + 0.29t. \]

Further, the joint has been subjected to a sinusoidally fluctuating load torque disturbance with a ±10% pulsation at frequencies close to resonance.

\[ K_p = \text{gain vector for the motor cum amplifier (plant)} \]
\[ q_p, q_p = \text{state vectors (plant)} \]
\[ J_p, K_p, C_p = \text{inertia, stiffness and damping matrices respectively (plant)} \]
\[ J_p = \text{rate of inertia variation matrix (plant)} \]
\[ K_p = \text{back emf constant matrix of the servomotor (plant)} \]
\[ J_{m1}, J_{m2} = \text{moment of inertia of the input and output rotors (model), kg m^2} \]
\[ C_m, C_{m1}, C_{m2} = \text{damping constant of the coupler (model), N s/m} \]
\[ J_m = \text{damping constant of the dampers connected with the input and output rotors (model), respectively, N ms/rad} \]
\[ K_m = \text{stiffness of the coupler (model), N/m} \]
\[ r_{m1}, r_{m2} = \text{radius of input and output rotors respectively (model), m} \]
\[ K_m = \text{gain of the motor cum amplifier, N m/A} \]
\[ q_{m1}, q_{m2} = \text{system co-ordinates (model), rad} \]
\[ q_{m1}, q_{m2} = \text{system velocities (model), rad/s} \]
\[ R = \text{reference input to the model} \]
\[ A_p, A_p = \text{position error feedback gains (plant)} \]
\[ A_{p1}, A_{p2} = \text{velocity error feedback gains (plant)} \]
\[ K_m, q_m, q_m = \text{gain vector for motor cum amplifier (model)} \]
\[ J_{m1}, K_m, C_m = \text{inertia, stiffness and damping matrices respectively (model)} \]
\[ B = \text{feedback row vector (model)} \]
\[ e, \dot{e} = \text{error vectors for the system} \]
\[ A_p, A_p = \text{position error feedback gain vector} \]
\[ A_{p1}, A_{p2} = \text{velocity error feedback gain vector} \]
\[ T_p = \text{payload vector associated with the plant} \]
\[ z = \text{performance index} \]
\[ \varepsilon = \text{exponential lag operator, } 1/(R_s + L_s), A/V \]
\[ V = \text{voltage supplied to the motor (plant), V} \]
\[ E_p = \text{back emf produced in the motor (plant), V} \]
\[ R_s = \text{armature resistance of the motor (plant), Ohm} \]
\[ L_p = \text{inductance of the motor armature, H} \]
\[ k_p = \text{current, A} \]
\[ k_m = \text{back emf constant of the motor (plant), V s/rad} \]
\[ k_p = \text{gain of the amplifier} \]
\[ k_2 = \text{motor torque constant, N m/A} \]
\[ t = \text{time, s} \]
\[ T_p = \text{plant torque input} = k_1 t \]

References