Steady state characteristics of finite porous oil journal bearings of arbitrary wall thickness

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Received 13 February 1995; accepted 4 August 1995

The aim of the present work is to study theoretically the effect of wall thickness on steady state characteristics viz., load parameter, friction parameter, oil flow rate and attitude angle of porous oil bearings of arbitrary wall thickness. In the analysis slip flow has been considered at the bearing-film interface. The results indicate that load parameter decreases and friction parameter increases with wall thickness of bearings, but attitude angle and end flow are marginally affected.

Now-a-days oil-filled porous bearings are extensively used as hydrodynamic bearings because of their self-acting nature and relative economy. The manufacturing process of such bearings has been made easy by the powder metallurgy process.

A pioneer work in establishing a mathematical model to investigate hydrodynamic lubrication of porous bearings has been done. Reports are also available on porous bearings using various approximations. The concept of tangential velocity slip and a mathematical criterion called Beavers-Joseph criterion to account for slip have been discussed.

Porous bearings have been analysed taking into account the Beavers-Joseph criterion. Results were obtained for axially undefined bearings with slip and for finite bearings with different slip conditions. An analytical solution and design data for starved porous bearings and finite porous bearing with \( H/R \leq 0.2 \) are available. Theoretical investigations to study the dynamics of finite porous bearings with slip and steady state characteristics of finite hydrostatic bearings were reported. Experimental investigation to study the mechanism of lubrication in porous journal bearings was also published. Results are also available for the effect of variable permeability and radial clearance on the steady state performance of porous oil journal bearings without slip.

Considering the tangential velocity slip model, in the present paper, a theoretical analysis of the static characteristics of finite porous journal bearings of arbitrary wall thickness has been carried out. Full Beavers-Joseph slip model has been assumed to account for the slip at the bearing-film interface and the governing differential equations are solved numerically employing finite difference method with successive over-relaxation scheme. The effect of \( H/R \) ratio on the static characteristics is reported in the form of graphs.

Analysis

Fig. 1 shows schematically a nonporous journal of radius \( R \) rotating with a uniform tangential velocity \( U \) within an anisotropic porous bearing of wall thickness \( H \) which is force fitted in a solid metal sleeve. It is assumed that the pores of the bearing are impregnated with an incompressible Newtonian fluid of the constant viscosity \( \eta \). Since \( H/R \) is arbitrary, Cartesian coordinate system is not adequate to describe the flow in the porous matrix and, therefore, the cylindrical coordinate system is used. However, in the analysis of the fluid film in the bearing clearance space, Cartesian coordinate system may be used as the curvature of the film is very small.
The governing steady state equations of flow in the porous matrix and in the bearing clearance can be written in nondimensional form as

\[ \frac{\partial^2 \bar{p}}{\partial \theta^2} + \frac{\partial^2 \bar{p}}{\partial \xi^2} + \frac{\partial \bar{p}}{\partial r} + (D/L)^2 K_z \frac{\partial^2 \bar{p}}{\partial z^2} = 0 \quad \ldots (1) \]

\[ \frac{\partial}{\partial \theta} \left[ \bar{h}^3 (1 + \xi_\theta) \frac{\partial \bar{p}}{\partial \theta} \right] + (D/L)^2 \frac{\partial \bar{p}}{\partial \xi} \]

\[ \times \left[ \bar{h}^3 (1 + \xi_z) \frac{\partial \bar{p}}{\partial z} \right] - \rho \beta (H/R) \left[ \frac{\partial \bar{p}}{\partial r} \right] = 0 \quad \ldots (2) \]

where

\[ \xi_{\theta, z} = 3(2a + \alpha_{\theta, z} \bar{h}) /[h \alpha_{\theta, z} (1 + h \alpha_{\theta, z})] \]

\[ \xi_{0, \theta} = (1 + h \alpha_{\theta, z})^{-1} \]

The boundary conditions are as follows:

In the porous matrix:

\[ \frac{\partial \bar{p}}{\partial \xi} (\theta, 1 + H/R, \xi) = 0 \]

\[ \frac{\partial \bar{p}}{\partial \xi} (\theta, r, 0) = 0 \]

\[ \bar{p}'(\theta, r, \pm 1) = 0 \]

\[ \bar{p}(0, r, z) = \bar{p}'(\theta_1, r, z) = 0 \]

\[ \theta_1 \text{ is the angular coordinate beyond which the pressure is negative and taken to be zero during computation.} \]

In the film region:

\[ \frac{\partial \bar{p}}{\partial \xi} (\theta, 0) = 0 \]

\[ \bar{p}(\theta, \pm 1) = 0 \]

\[ \bar{p}(\theta, z) = 0 \]

\[ \left( \frac{\partial \bar{p}}{\partial z} \right)_{\theta=\theta_2} = 0, \bar{p} = 0, \theta_2 < \theta < 2\pi \]

\[ \theta_2 \text{ is the angular coordinate at which the film cavitates.} \]

In the bearing film interface:

\[ \bar{p}'(\theta, 1, \xi) = \bar{p}(\theta, \xi) \]

Pressure field in the nondimensional form can be obtained by the simultaneous solution of Eqs (1) and (2) with the above boundary conditions. Solution of the above equations satisfying the boundary conditions is accomplished by using finite difference method with successive over-relaxation scheme.

**Load carrying capacity**—Load carrying capacity of the bearing can be obtained by integrating the pressure round the circumference and along the total length

Thus

\[ \bar{W}_y = -2 \int_0^1 \int_0^{\xi_\theta} \bar{p} \cos \theta \, d\theta \, d\xi \]

\[ \bar{W}_x = 2 \int_0^1 \int_0^{\xi_\theta} \bar{p} \sin \theta \, d\theta \, d\xi \]

where \( \bar{W}_x \) and \( \bar{W}_y \) are components of nondimensional load (load parameter) along \( x \) and \( y \) directions respectively.

**Load parameter**—

\[ \bar{W} = (\bar{W}_x^2 + \bar{W}_y^2)^{1/2} \quad \ldots (3) \]

Attitude angle

\[ \phi = \tan^{-1}(\bar{W}_y/\bar{W}_x) \quad \ldots (4) \]

**Friction parameter**—Shear stress acting on the journal surface is given by

\[ \tau = \eta \left( \frac{\partial u}{\partial y} \right) = h \quad \ldots (5) \]

and it can be shown that

\[ \tau/(\eta U/c) = \frac{1}{2} \bar{h} \left( \frac{\partial \bar{p}}{\partial \theta} \right) \left( 1 + \frac{1}{3} \xi_\theta \right) + (1 - \xi_{0, \theta})/\bar{h} \quad \ldots (6) \]

The above equation is valid when there is no cavitation. So, the total frictional force in the non-cavitated region is given in the nondimensional form as

\[ \bar{F}_{1s} = 2 \int_0^1 \int_0^{2\pi} \left[ \bar{h}/2 \left( 1 + \frac{1}{3} \xi_\theta \right) \left( \frac{\partial \bar{p}}{\partial \theta} \right) \right] \right) \quad \ldots (7) \]

Friction for the cavitated zone, where there is a mixture of oil and vapour can be computed as

\[ \bar{F}_{1s} = 2 \int_0^1 \int_0^{2\pi} \left[ \bar{h}_2/\bar{h} \right] \left( 1 - \xi_{0, \theta} \right)/\bar{h} \, d\theta \, d\xi \quad \ldots (8) \]

Total nondimensional friction force

\[ \bar{F}_f = \bar{F}_{1s} + \bar{F}_{1s} \quad \ldots (9) \]

and friction parameter

\[ (\eta R/c) = \bar{F}_f/\bar{W} \quad \ldots (10) \]
Conclusion

On the basis of the above study it can be concluded that an increase in thickness parameter \( (H/R) \) reduces the load parameter and increases the friction parameter. A probable physical explanation may be that if the pores of the bearings be conceived to consist of myriads of tortuous passages, then a part of the hydrodynamic pressure head generated by a particular operating condition of the bearing is lost due to viscous friction while flowing through the passages and so the static pressure which is responsible for carrying the load is reduced. This can also be explained from Eq. (2), wherein the contribution of the physical wedge term on the right hand side of the equation is reduced by the second term which is a function of \( (H/R) \). The optimum value of \( (H/R) \) which is a compromise between the metallic bearing and the porous bearing is normally 0.2 considering the strength, amount of oil retained for self acting bearing and ease of manufacturing process. The effect of \( (H/R) \) on attitude angle and end flow is, however, marginal.

Nomenclature

- \( C = \) Radial clearance
- \( D = \) Shaft/bearing diameter
- \( e = \) Bearing eccentricity
- \( F = \) Shear force on the journal surface
- \( F' = \) nondimensional shear force on journal surface,
- \( 2CF/\eta URL \)

End flow—End flow from the bearing consists of two parts, i.e. flow from the clearance space, and flow from the open ends of the bush. Flow from the open ends is usually very small compared with clearance space, and hence only flow from the clearance space has been calculated. It can be shown that the nondimensional flow from the clearance space is given by

\[
\bar{Q} = -\frac{1}{6} \int_0^{2\pi} \bar{h}^2 (1 + \xi_1) \left( \frac{\partial \bar{p}}{\partial \bar{z}} \right)_{\bar{z}=1} \, \text{d}\xi_1 \quad \ldots (11)
\]

All the integrations in the above analysis have been carried out numerically using Simpson’s one-third rule.

Results and Discussion

Results have been obtained for isotropic bearings with different \( H/R \) ratios, while keeping other parameters same. These are shown in Figs 2 and 3. It can be seen from Fig. 2 that load parameter \( \bar{W} \) falls with increase in thickness ratio. But for lower value of \( \beta \), the change in load parameter with thickness ratio is too insignificant. The reduction of load parameter with \( H/R \) is very prominent at higher values of \( \beta \).

Friction parameter \( (fR/C) \) is found to increase with \( H/R \) as shown in Fig. 3. It changes very insignificantly with \( H/R \) at lower values of \( \beta \). However, at higher values of \( \beta \), the change is quite noteworthy.

Attitude angle is found to increase with wall thickness parameter \( H/R \); but this increase is so small that it cannot be shown in figure. Effect of \( H/R \) on end flow has been found to be too meagre to report.
f = friction coefficient
H = thickness of the wall of porous bearing
h = local film thickness
\( h = \) nondimensional film thickness, \( h/C \)
\( h_1 = \) film thickness at the beginning of the cavitated zone
\( h_2 = \) nondimensional film thickness at the beginning of the cavitated zone, \( h_2/C \)

\( K_\theta, K_z = \) anisotropic factors in \( \theta \) and \( Z \) direction, respectively, \( k_\theta/k_\phi, k_z/k_z \)
\( k_\phi, k_\theta, k_z = \) permeability coefficients along \( \theta, r \) and \( Z \) directions, respectively.

p = local fluid pressure above ambient
\( \bar{p} = \) nondimensional pressure, \( pC^2/\eta UR \)
\( p' = \) pressure of fluid in porous matrix
\( \bar{p}' = \) nondimensional pressure of fluid in porous matrix, \( p'C^2/\eta UR \)

Q = end flow from the bearing
\( \bar{Q} = \) nondimensional end flow, \( QL2UR^2C \)

R = shaft/bearing radius
r, \( \theta, Z = \) cylindrical coordinate system
\( \bar{r}, \bar{Z} = \) dimensionless coordinate, \( r/R, 2Z/L \)

U = surface velocity of the journal
W = load capacity
\( \bar{W} = \) nondimensional load parameter, \( 2C^2W/\eta UR^2L \)

\( \theta_1 = \) angular extent of uncavitated film
\( \alpha = \) slip coefficient
\( \beta = \) bearing feeding parameter, \( 12k,R^3/C^3H \)
\( \sigma = \) isotropic permeability factor
\( \sigma_\theta, \sigma_\phi, \sigma_z = \) permeability factors in \( \theta, r \) and \( z \) directions, respectively, \( C/k_\theta, C/k_\phi, C/k_z \)
\( \phi = \) attitude angle
\( \varepsilon = \) eccentricity ratio
\( \tau = \) shear stress on journal

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References