Optimum design of laminated composite rectangular plates with cutouts using genetic algorithm

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Optimum design of the laminated composite rectangular plate with specified frequencies is attempted in the presence of elliptical cutouts. Orientation of the ellipse with respect to the reference axis, aspect ratio of the cutout, orientation of plies, thickness of plies and material of the plies are used as design parameters with constraints on natural frequencies. As a first step, investigations of free vibration response of composite rectangular plate in the presence of elliptical cutout is studied and some typical results are presented. First-order shear deformation theory (FSDT) is used to account for the transverse shear stresses. Nine noded quadrilateral isoparametric element is used in the finite element formation. Genetic algorithm (GA) is employed to identify the optimum variables. Numerical results are presented for rectangular plates with simply supported edge conditions. The results show that GA is a viable tool for optimum design of laminated composite plates with cutouts.

Composite structures find wide applications in aerospace, civil and mechanical industries where weight reduction and directional properties are the criteria. Cutouts are unavoidable in structures like aircraft, used for door openings, in the fuselage, flow passages, wiring connections, etc. Sometimes cutouts are used merely to reduce the structural weight like in the web members of aircraft wings. Here, an attempt is made to optimize the free vibration response of composite plate in the presence of an elliptical cutout.

Kam and Lal1 dealt with maximization of stiffness of the laminated composite plates through constrained multi-start global optimization approach. Finite element method is used to evaluate the objective function. The objective is to determine the optimal fibre angles and layer group thicknesses of the laminated plates for maximum stiffness. Vellaichamy et al.2 have given an optimum design with cutouts in laminated composite structures based on linear analysis. The design variables are aspect ratio and orientation of the ellipse and their values are so chosen that the value of the maximum failure criterion around the circumference of a hole is a minimum. Calhan and Weeks3 used the principles of genetic algorithm for optimum design of composite laminates. The design variables considered were the laminate orientations and stacking sequence required for maximum laminate strength or stiffness with minimum weight. Kogiso et al.4, applied genetic algorithm with memory for design of minimum thickness composite laminates subject to strength, buckling and ply contiguity conditions. Mahesh et al.5, have employed genetic algorithm for optimal design of a turbine blade. A simplified design method is developed to determine the optimum combination of layer materials, orientations and their thickness. The present optimization problem is concerned with the finding of the optimum cutout dimensions and its orientation, aspect ratio, ply orientations, ply thicknesses of rectangular plates with elliptical cutout keeping first and second natural frequencies as constraints. Minimizing weight of the plate with constraints on first and second natural frequencies with elliptical and circular cutouts have also been discussed.

Finite Element Formulation for Free Vibration

The problem is formulated for a plate of thickness \(h\) composed of orthotropic layers of thickness \(h_i\) with fibres oriented at angles \(\theta\) and \(-\theta\).
The displacements $u$, $v$ and $w$ based on Yang, et al. theory, considered for the analysis. These are

$$u = u_0(x,y,t) + z \psi_x(x,y,t),$$

$$v = v_0(x,y,t) + z \psi_y(x,y,t),$$

$$w = w(x,y,t).$$

In Eq. (1) $u$, $v$, and $w$ are the displacements along $x$, $y$, and $z$ directions respectively. $u_0$, $v_0$ are the midplane displacements while $\psi_x$, $\psi_y$ are rotations about $x$ and $y$ axis.

The plate is divided into a number of isoparametric nine-noded quadratic elements with 5 DOF per node ($u,v,w,\psi_x,\psi_y$). In the presence of cutout, trapezoidal elements are taken rather than rectangular elements of uniform size because the former gives better accuracy and also would be convenient for proper mapping. The rectangular plate with elliptical cutout is shown in Fig. 1.

Following the strain energy principle, the governing equation for free vibration analysis is given as

$$[M] \{ \ddot{\delta} \} + [K] \{ \delta \} = 0,$$

where $[M]$ and $[K]$ are the mass and stiffness matrix of the plate and $\delta$ is a vector of global degrees of freedom.

**Optimization Formulation**

Genetic algorithms (GAs) are chosen for finding the optimum solution. GAs mimic the principles of genetics and natural selection to constitute search and optimization procedure.

**Coding in genetic algorithm**—The variables namely ply materials, ply angles, group ply thickness, ratio of cutout length to plate length, ratio of major axis to minor axis of the ellipse, orientation of cutout are coded in a binary strings. A sixteen layer laminate is considered for the optimization study.

Each ply angle is coded in a 2-bit string. Only practical ply orientations 0°, 45°, -45° and 90° are considered for the design. String length for orientation of the elliptical cutout is kept as 6 so that a uniform 3° step can be given in a group of 0° to 189°. The ratio of cutout size is varied from 0.1 to 0.5 times the plate length. Since this ratio is very crucial as far as frequency and weight is concerned, close values are considered by keeping string length to be 6. For the shape of the cutout, i.e., ratio of major axis to minor axis of the ellipse, strings of 6 bits are considered. Overall thickness of the plate is kept constant. The thickness of all ±45° plies, all 90° and 0° plies are termed as group thickness belonging to a particular orientation and is obtained as follows—The group thickness corresponding to a ply orientation which occurs maximum number of times in a typical laminate is assigned thickness ($t$) and its value is obtained from genetic code. For other two groups the thickness corresponding to the ply angle which occurs minimum number of times is taken as 1 mm uniformly for each ply in its group if it occurs less than four times, otherwise its group ply thickness is assigned as 4 mm. The group thickness for the
Table 1—Material properties

<table>
<thead>
<tr>
<th>Code</th>
<th>Material</th>
<th>$E_1/E_2$</th>
<th>$G_{12}/E_2$</th>
<th>$G_{45}/E_2$</th>
<th>$v_{12}$</th>
<th>$\rho \text{ kg/m}^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Graphite/Epoxy</td>
<td>40</td>
<td>0.6</td>
<td>0.6</td>
<td>0.25</td>
<td>1500</td>
</tr>
<tr>
<td>2</td>
<td>Glass/Epoxy</td>
<td>3.0</td>
<td>0.5</td>
<td>0.5</td>
<td>0.4</td>
<td>1800</td>
</tr>
<tr>
<td>3</td>
<td>Kevlar/Epoxy</td>
<td>14.82</td>
<td>0.375</td>
<td>0.375</td>
<td>0.34</td>
<td>1460</td>
</tr>
<tr>
<td>4</td>
<td>Boron/Epoxy</td>
<td>10</td>
<td>0.3</td>
<td>0.3</td>
<td>0.275</td>
<td>2000</td>
</tr>
</tbody>
</table>

Table 2—Variation of nondimensional frequency parameter $\lambda$ with length to thickness rate for angle ply laminate (graphite/epoxy, BC-II, [0°/45°/-45°/90°])

<table>
<thead>
<tr>
<th>$b/a$</th>
<th>$a/h$</th>
<th>present</th>
<th>Ref. 7</th>
<th>present</th>
<th>Ref. 7</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.5</td>
<td>4.13</td>
<td>4.24</td>
<td>3.892</td>
<td>4.03</td>
</tr>
<tr>
<td>2.0</td>
<td></td>
<td>6.08</td>
<td>6.13</td>
<td>5.805</td>
<td>5.87</td>
</tr>
<tr>
<td>4.0</td>
<td></td>
<td>7.727</td>
<td>7.72</td>
<td>7.4172</td>
<td>7.42</td>
</tr>
<tr>
<td>10.0</td>
<td></td>
<td>8.195</td>
<td>8.13</td>
<td>7.8545</td>
<td>7.80</td>
</tr>
</tbody>
</table>

remaining ply angle is obtained directly from the total thickness. If however, the number of times any two ply angles occur are equal but lower than the remaining ply angle then the group thickness of the maximum occurring ply angle is subtracted from total thickness and the remaining thickness is equally divided for the two group thicknesses. Elseif, the number of times any two ply angles occur are equal and higher than the third, then the group thickness of the maximum occurring ply angle is equally assigned to the group thickness of the ply angle which is equal in number and the remaining thickness is assigned to the third group thickness. The range of maximum group ply thickness is 0.6 mm to 0.8 mm and its string length is 5. Two bit strings are sufficient to decide the stacking sequence of the materials. Materials and their code numbers are given in Table 1. The total string length is 87 for the 16 layer laminate. In the following, one gets a typical string $010010\ldots010011101110110101111$ and the laminate is a antisymmetric angle ply. The length and width of the plate is taken as 0.32 m and 0.6 m and $a/h$ ratio is 20. The boundary conditions for the laminated plate considered are as follows:

BC-I: Simply supported on all edges:

$$u = w = \psi_0 = 0 \text{ at } x = 0, a,$$

$$v = w = \psi_y = 0 \text{ at } y = 0, b,$$
BC-II: Clamped at all corners and hinged at edges:

\[ w = \psi_x = 0 \text{ at } x = 0, a, \]

\[ w = \psi_y = 0 \text{ at } y = 0, b, \]

\[ u = v = w = \psi_x = \psi_y = 0 \text{ at all corners.} \]

In Table 2, the results for nondimensional frequency parameter \( \lambda \), are given for rectangular plates with various \( a/h \) ratios. The boundary conditions employed for numerical computation are taken from Akhras et al.\(^7\), which is “four edges are hinged and free to move in both tangential and normal inplane directions but fixed at all corners” (BC-II). The present results are in conformity with Akhras et al.\(^7\). It is observed that the frequency parameters decreases when the aspect ratio of the plate increases. This is due to the decrease in effective stiffness of the plate with increase in the aspect ratio. Simply supported boundary condition (BC-I) is used for further analysis. Fig. 2 shows the variation of fundamental frequency parameter with ply angle and number of layers. The maximum and minimum values of fundamental frequency parameter occur at 0° and 90° respectively. Due to the effect of coupling in 2-layer laminate it gives lesser frequency parameter compared to other layers. Fig. 3 shows the variation of second frequency parameter with ply angle and number of layers. Here the 45° ply orientation gives maximum frequency parameter except for 2-layer. In both figures, when the number of ply increases, the variation in fundamental frequency parameter approaches the orthotropic plate values. Fig. 4 shows the variation of the fundamental frequency with ply angles for various \( ea/a \) ratios. Here it is observed that the frequency parameter is maximum at 0° and minimum at 90° in all the cases. Variation is very steep between angles 0° and 15° when the ply angle is below 15° and above 75° the variation in frequency parameter with size of the cutout is not so predominant but in between it shows noticeable increase in the frequency parameter. Variation of the second natural frequency is given in Fig. 5, shows the variation is steep upto 15° and then it stabilises for smaller \( ea/a \) ratios but it keep decreasing with larger slope at higher \( ea/a \) ratios. When the cutout size is small it is observed that the second frequency parameter approaches a local peak value at 45°. It shows that the plate stiffness has increased at that orientation locally. But this behaviour is not observed for large size cutouts. It may be due to the fact that the increase in stiffness due to orientation is overcome by reduction in stiffness due to the material that has been removed. There is considerable change in trend after 75° between \( ea/a \) 0.4 to 0.6. The frequency parameter starts increasing. It can be observed from Fig. 6 that 0° ply orientation gives higher frequency parameter for all \( ea/a \) ratios for four layer antisymmetric angle ply laminate. When the \( ea/a \) ratio is small the frequency parameter is larger than other ratios at 0° and 90°. However, between 30° to 75° the trend is reversed. Fig. 7

![Fig. 4—Variation of fundamental frequency parameter with ply angle and \( ea/a \) ratio for 2-layer laminate (\( b/a=2.0, ea/eb=2.0, a/h=20.0, \text{Material: Graphite/Epoxy} \)](image)

![Fig. 5—Variation of second frequency parameter with ply angle and \( ea/a \) ratio for 2-layer laminate (\( b/a=2.0, ea/eb=2.0, a/h=20.0, \text{Material: Graphite/Epoxy} \)](image)
Fig. 6—Variation of fundamental frequency parameter with ply angle and $ea/a$ ratio for 4-layer laminate ($b/a=2.0$, $ea/eb=2.0$, $a/h=20.0$, Material: Graphite/Epoxy)

Fig. 8—Variation of fundamental frequency parameter with orientation angle of major axis of ellipse with $x$-axis ($b/a=2.0$, $ea/a=0.2$, $ea/eb=2.0$, $a/h=20.0$, Material: Graphite/Epoxy)

shows the variation of second frequency parameter with ply angle for various $ea/a$ ratios. It is seen that the frequency parameter is maximum at $45^\circ$ as compared to other orientation. But for $ea/a$ ratio beyond 0.6, there seems to be a shift to the left, in the location of maximum parameter value. Though the difference in frequency parameters for various $ea/a$ ratios at $0^\circ$ is not considerable, it keeps increasing with the ply angle, which is also true for the two layer case shown in Fig. 5. Variation of frequency parameter with the rotation angle of ellipse about $x$-axis is given in Fig. 8. This shows that there is a considerable variation in frequency.

**Optimization Problem**

*Design of plate for specified first and second natural frequencies—Initially antisymmetric angle*
Table 3—Optimum design values for hybrid rectangular plate with elliptical cutout and with constraints on first and second natural frequencies

<table>
<thead>
<tr>
<th>$e_a/a$</th>
<th>$e_a/e_b$</th>
<th>$\theta_{rot}$</th>
<th>Material code</th>
<th>$\theta$</th>
<th>Ply thickness $(m)$</th>
<th>$\lambda_1$</th>
<th>$\lambda_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.171</td>
<td>1.928</td>
<td>165.0</td>
<td>[4/4/1/1/4/2/1/3/1/2/1/1/4/1]</td>
<td>$[0/-45/-45/45/45/-45/0/-45/90/45/0/0]$</td>
<td>$h_0=0.001724$</td>
<td>13.35</td>
<td>22.238</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$h_{45}=0.000638$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$h_{90}=0.001$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4—Optimum value for minimum weight design of a rectangular plate with an elliptical and a circular cutout with constraints on first and second natural frequencies ($e_a/a=0.2$, $e_a/e_b=2.0$, $\theta_{rot}=0.0$)

<table>
<thead>
<tr>
<th>Material code</th>
<th>$\theta$</th>
<th>Ply thickness $(m)$</th>
<th>$\lambda_1$</th>
<th>$\lambda_2$</th>
<th>Weight obtained (Kg)</th>
<th>Weight actual (Kg)</th>
<th>Weight reduced (Kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I 1/1/1/3/3/3/3/3</td>
<td>$[-45/45/0/0/-45]$</td>
<td>$h_0=0.001666$</td>
<td>13.123</td>
<td>23.001</td>
<td>4.784</td>
<td>4.877</td>
<td>0.093</td>
</tr>
<tr>
<td>3/3/3/3/3/1/1/1/1</td>
<td>$45/-45/0/0/0/0/0$</td>
<td>$h_{45}=0.006$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>45/45/-45/-45/45/45</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>II 1/1/1/3/3/3/3/3</td>
<td>$[45/-45/-45/45/45]$</td>
<td>$h_0=0.001473$</td>
<td>13.164</td>
<td>23.063</td>
<td>4.75</td>
<td>4.838</td>
<td>0.088</td>
</tr>
<tr>
<td>3/3/3/3/3/1/1/1/1/1</td>
<td>$45/0/45/-45/0/0/0/0$</td>
<td>$h_{45}=0.0007161$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>45/0/45/-45/45</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

I—elliptical cutout, II—circular cutout.

In this design problem, 16 plies are used. The population size and number of generations are varied according to the number of variables used in the design. In the above optimization problem the weight of the plate is not taken into account for design. The bounds of the frequency parameter in constraints are taken as $\lambda_{1_{\min}}=13.0$, $\lambda_{1_{\max}}=13.25$ and $\lambda_{2_{\min}}=22.0$, $\lambda_{2_{\max}}=22.5$.

Table 3 gives the results of the optimization problem for a rectangular plate with a central elliptical cutout having constrained fundamental and second frequencies. The constraint violations are $g_1=0.01$, $g_2=0.0$, $g_3=0.154$, $g_4=0.346$. Since one of the constraint violation is zero, it can be argued that the obtained solution is close to the optimum solution.

Minimum weight problem of the plate with cut-out—Minimizing weight of the plate with circular and elliptical cutout is attempted for given first and second natural frequencies as constraints. In the problem formulated the weight of the plate is governed by the cutout area, materials and thicknesses of the individual ply. Area of the cut-out is fixed. The formulation of the problem is given as

Minimize $W$
subject to
$g_1 = \lambda_1 - \lambda_1 \geq 0$, $g_2 = \lambda_2 - \lambda_2 \geq 0$.

Limits on the design variables are $-90^\circ \leq \theta \leq 90^\circ$.

$\sum_{i=1}^{n_l} h_i = h$
$0.6 \leq t \leq 0.8$.

for $l=1, 2, \ldots, n_l$

Other governing parameters specified for the plate and elliptical cutout are $a/h=20.0$, $e_a/a=0.2$, $e_a/e_b=2.0$, $\theta_{rot}=0.0$. For circular cutout $e_a/e_b=1.0$ while other parameters are the same. In both the problems, the specified first and second natural frequencies are $\lambda_1=13.0$, $\lambda_2=23.0$. The GA parameters used are

Number of variables =33
Total string length =69
Population size =75
Number of generations =225
Probability of crossover =0.93
Probability of mutation =0.001
Sixteen layer laminate is used for the design problem. Size of cutout is fixed in both the cases. Table 4 gives the optimum design values for the rectangular plate with elliptical and circular cutouts for the minimum weight problem. From the test runs it is felt that unless there is constraint on area of cutout or area of wide variety of shape is available as choice it is better to specify the shape of the cutout as constant because it reduces the number of variables. The optimal sequences of materials in both cutouts are the same. The constraint violations are $g_1=0.123$, $g_2=0.001$, $g_3=0.164$, $g_2=0.036$ for elliptical and circular cutouts respectively. Since one of the constraint violation is close to zero, the above argument on optimum is also valid here.

**Conclusion**

On the basis of the investigation carried out, it is observed that the natural frequency of the plate is affected considerably in the presence of cutouts. Genetic algorithm (GA), can be successfully employed for design and tailoring of the composite plates for different type of objective function and design constraints with many number of design variables. With the flexibility in representing mixed variables (such as discrete and/or continues) and ability to include variables such as materials, GA allows an efficient way of solving engineering optimization problems, a matter which is not possible with traditional methods.

**Nomenclature**

- $a, b, h$: length, width and thickness of the plate in meters respectively.
- $a_e, b_e$: major and minor axis of ellipse respectively.
- $h_0, h_{90}, h_{45}$: ply thickness of 0°, 90° and 45° respectively.
- $n_l$: number of layers.
- $x, y$: cartesian coordinates.
- $\lambda_i$: $i$th non dimensional frequency parameter.
- $\rho$: density of the material.
- $\theta$: ply angle.
- $\theta_{mn}$: orientation angle of major axis of ellipse with x-axis.
- $W$: weight of the plate.

**References**