Studies on convergence point of the multi-strand yarn spinning

Xinjin Liu\textsuperscript{1,a} & Xuzhong Su\textsuperscript{2}

\textsuperscript{1}School of Textile and Clothing, Jiangnan University, Wuxi 214122, P R China
\textsuperscript{2}Key Laboratory of Eco-Textile, Ministry of Education, Jiangnan University, Wuxi 214122, P R China

Received 27 February 2013; revised received and accepted 5 July 2013

In this study, convergence point of multi-strand yarn spinning has been investigated. Firstly, the complete convergence point state is studied for two-strand yarn spinning. Then, one condition such that two strands are kept symmetric on the convergence point is given correspondingly. Secondly, a system of the equations governing the general multi-strand yarn spinning with \( n \) strands is presented by introducing a series of virtual intermediate variables. After eliminating the intermediate variable, the partial convergence point state \( F_i \) and \( \omega_i \) of multi-strand yarn spinning is given. It is observed that the complete convergence point for two-strand yarn spinning can be determined by the densities and radius of two strands and the spun yarn.

Keywords: Convergence point, Multi-strand yarn spinning, Self-contained system, Yarn

Multi-strand yarn spinning is a new kind of spinning idea, which is implemented by feeding multiple rovings into the apron zone at a predetermined separation simultaneously on a ring spinning frame. The convergence point of multi-strand yarn spinning plays an important role in controlling the stability of the spinning procedure and qualities of spun yarn.\textsuperscript{1} Therefore, the research on convergence point of multi-strand yarn spinning, especially two-strand yarn spinning, has attracted more and more attention and many interesting results have been established.\textsuperscript{1,5}

Two-strand yarn spinning (sirospun) is one of the most widely used new spinning methods invented by the Division of Textile Industry Laboratories of the CSIRO, Australia and IWS together at around mid seventies. This is conducted on a conventional ring frame by simultaneously feeding two rovings into the apron zone at a predetermined separation.\textsuperscript{6} As an effective new spinning technology, the research on two-strand yarn spinning has attracted more and more attentions at present, especially the convergence point analysis.\textsuperscript{4,6-8} For the convergence point of two-strand spinning, a theoretical model was established first by considering the force balance, and then two equations with three variables, namely \( f \) (tension), \( \alpha \) (angle with the twist point axis), and \( m \) (elastic torque) were obtained correspondingly.\textsuperscript{2}

However, the model could not be solved since the numbers of independent equations were less than that of the independent variables. Then, in order to make the model closed, an experimental procedure was adopted.\textsuperscript{9} Furthermore, for overcoming this difficulty, one improved system was given by He \textit{et al}.\textsuperscript{4}, and some equations were provided by considering the system obeying the basic laws in mechanics, including force balance, mass conservation and energy conservation, and partial convergence point of two-strand yarn spinning was determined by solving these equations.\textsuperscript{10} Finally, another three equations describing the torque equation in the horizontal direction and angular momentum conservation respectively were added to He’s model,\textsuperscript{5} and finally the system become self-contained in this case. Motivated by all these research works, attempts have been made in this study to give the complete convergence point state of two-strand yarn spinning based on the studies as mentioned above.

Three-strand yarn spinning can be designed for smart fabric, which have many advantages over two-strand spinning yarn. Three-strand spinning yarn can be prepared in a single processing step, and far-reaching implications have been found to emerge for its use in applications including intelligent textile and multi-functional materials.\textsuperscript{1} Therefore, in this study, according to two-strand yarn spinning analysis, the convergence point state of the general multi-strand yarn spinning has been investigated by setting a series of virtual intermediate variables. However, the solved convergence point state of multi-strand yarn spinning is found difficult to use directly since the density \( \rho_i' \) and radius \( R_i' \) of the virtual intermediate strands are difficult or even impossible to determine in practical applications. The study includes (i) investigation on
the theoretical model and convergence point for two-strand yarn spinning and to obtain the complete convergence point state; (ii) investigation on the convergence point of three-strand yarn spinning according to two-strand yarn spinning analysis, by introducing a series of virtual intermediate variables; and (iii) investigation on the general convergence point of multi-strand yarn spinning with \( n \) strands.

**Theoretical model and convergence point for two-strand yarn spinning**

Two-strand yarn spinning structure is shown in Fig.1. According to analysis reported earlier\(^4\),\(^5\), the system should obey the basic laws in mechanics, such as force balance, momentum equation, mass conservation, energy conservation and torque equation. Therefore, the governing equations for this system can be written as follows:

**Force balance**

\[
F_1 \cos \alpha_1 + F_2 \cos \alpha_2 = F \quad \ldots (1)
\]

\[
F_1 \sin \alpha_1 = F_2 \sin \alpha_2 \quad \ldots (2)
\]

**Momentum equation**

\[
\rho_1 u_1 R_1^2 u_1 \cos \alpha_1 + \rho_2 u_2 R_2^2 u_2 \cos \alpha_2 = \rho \dot{u} R^2 \quad \ldots (3)
\]

\[
\rho_1 u_1 R_1^2 u_1 \sin \alpha_1 = \rho_2 u_2 R_2^2 u_2 \sin \alpha_2 \quad \ldots (4)
\]

\[
\rho_1 u_1 R_1^2 \omega_1 R_1^2 \cos \alpha_1 + \rho_2 u_2 R_2^2 \omega_2 R_2^2 \cos \alpha_2 = \rho \dot{u} R^2 \omega^2 \quad \ldots (5)
\]

\[
\rho_1 u_1 R_1^2 \omega_1 R_1^2 \sin \alpha_1 = \rho_2 u_2 R_2^2 \omega_2 R_2^2 \sin \alpha_2 \quad \ldots (6)
\]

**Mass conservation**

\[
\rho_1 u_1 R_1^2 + \rho_2 u_2 R_2^2 = \rho \dot{u} R^2 \quad \ldots (7)
\]

**Energy conservation**

\[
\rho_1 u_1 R_1^2 u_1^2 + \rho_2 u_2 R_2^2 u_2^2 + \rho_1 u_1 R_1^2 \omega_1^2 R_1^2 + \rho_2 u_2 R_2^2 \omega_2^2 R_2^2 = p u^2 R^2 + p u^2 \omega^2 R^2 \quad \ldots (8)
\]

**Torque equation**

\[
M_1 \cos \alpha_1 + M_2 \cos \alpha_2 + R_1 F_1 \sin \alpha_1 + R_2 F_2 \sin \alpha_2 = M \quad \ldots (9)
\]

\[
M_1 \sin \alpha_1 - M_2 \sin \alpha_2 - R_1 F_1 \cos \alpha_1 + R_2 F_2 \cos \alpha_2 = 0 \quad \ldots (10)
\]

where \( F \), \( M \) and \( \omega \) are tension, elastic torque and angular velocity in the two-strand yarn below the convergence point respectively; \( \rho \), \( u \) and \( R \) are density, velocity and radius of the spun yarn respectively; \( F_i \), \( M_i \) and \( \rho_i \) are tension, elastic torque and density in the two strands above the convergence point; \( u_i \) and \( R_i \) are velocity and radii of two strands; \( \alpha_i \) are angles of two strands with the twist point axis for \( i = 1, 2 \); and \( \omega_i \) are angular velocities between two strands for \( i = 1, 2 \).

In all Eqs (1) - (10), Eqs (1), (2), (9) and (10) show static models and are valid only for the steady spinning, while Eqs (3) - (8) show fluid models. This self-contained static-fluid model can be able to describe this kind of complex dynamic process completely in theory. It is obvious that the convergence point is completely determined by the ten variables, namely \( F_1 \), \( F_2 \), \( \alpha_1 \), \( \alpha_2 \), \( u_1 \), \( u_2 \), \( R_1 \), \( R_2 \), \( M_1 \), \( M_2 \), \( \omega_1 \) and \( \omega_2 \), which can be theoretically obtained by Eqs (1) - (10), and no empirical or semi-empirical inputs are needed. In the following relationships, the complete convergence point is given according to the Eqs (1) - (10):

Firstly, by solving the above Eqs (3) and (4), we can get

\[
\cos \alpha_1 = \frac{m_1^2 + m^2 - m_2^2}{2mm_1} \quad \ldots (11)
\]

\[
\cos \alpha_2 = \frac{m_2^2 + m^2 - m_1^2}{2mm_2} \quad \ldots (12)
\]

where \( m_1 = \rho_1 u_1^2 R_1^2 \), \( m_2 = \rho_2 u_2^2 R_2^2 \), \( m = \rho \dot{u}^2 R^2 \).

Secondly, according to the Eqs (1), (2), (5), (6), (11) and (12), we have

\[
F_1 = \frac{m_1}{m} F ; \quad F_2 = \frac{m_2}{m} F
\]

\[
\omega_1 = \frac{u_1 R_1^2}{u R_1^2} \omega ; \quad \omega_2 = \frac{u_2 R_2^2}{u R_2^2} \omega
\]

![Fig. 1—Two-strand yarn spinning structure](image-url)
Thirdly, according to the Eqs (9) and (10), we have
\[
M_1 = \frac{M \sin \alpha_2 + R_F \cos(\alpha_1 + \alpha_2) - R_2 \omega_1}{\sin(\alpha_1 + \alpha_2)}
\]
\[
M_2 = \frac{M \sin \alpha_1 + R_F \cos(\alpha_1 + \alpha_2) - R_1 \omega_1}{\sin(\alpha_1 + \alpha_2)}
\]

Finally, by solving Eqs (7) and (8), we found that the variables \( \frac{u_1}{u} \) satisfy the following cubic equation:
\[
a_i \left( \frac{u}{u_i} \right)^3 + b_i \left( \frac{u}{u_i} \right)^2 + c_i \frac{u}{u_i} + d_i = 0 \quad \ldots \text{(13)}
\]
where
\[
a_i = \left( \rho_i \left( R_i^2 + R_i^2 \omega_i^2 \right) - \rho_i \left( R_i^2 + R_i^2 \omega_i^2 \right) \left( \frac{R_i \rho_i}{R_i \rho_i} \right)^3 \right)
\]
\[
b_i = 3 \rho_i \left( R_i^2 + R_i^2 \omega_i^2 \right) \left( \frac{R_i \rho_i}{R_i \rho_i} \right)^2
\]
\[
c_i = -3 \rho_i \left( R_i^2 + R_i^2 \omega_i^2 \right) \left( \frac{R_i \rho_i}{R_i \rho_i} \right)^2 - R_i \rho_i \left( \frac{R_i \rho_i}{R_i \rho_i} \right)
\]
\[
d_i = \rho_i \left( R_i^2 + R_i^2 \omega_i^2 \right) \left( \frac{R_i \rho_i}{R_i \rho_i} \right)^3 - R_i \rho_i \left( \frac{R_i \rho_i}{R_i \rho_i} \right) \quad \text{for} \quad i = 1, 2 \quad \text{and if} \quad i = 1, \text{then} \quad j = 2; \quad \text{if} \quad i = 2, \text{then} \quad j = 1.
\]
According to Eq. (13), it is easy to get the solutions of \( \frac{u_1}{u} \) and \( \frac{u_2}{u} \). However, the result is very complex.

Therefore, in the following, we give one condition such that \( u_1 = u_2 \).

**Theorem 1**—If \( R_1 = R_2 = R_0 \) and \( \rho_1 = \rho_2 = \rho_0 \), then \( u_1 = u_2 \) (i.e. \( F_1 = F_2 \), \( \alpha_1 = \alpha_2 \), \( M_1 = M_2 \) and \( \omega_1 = \omega_2 \)) if and only if
\[
\left( \frac{R_0 \rho_0}{R_i \rho_i} \right)^3 = \frac{\left( R_i \rho_i \right)^2}{4 \left( 1 + R_i \rho_i \omega_i^2 \right)} \quad \ldots \text{(14)}
\]

**Proof**—It is easy to see that if \( R_1 = R_2 = R_0 \) and \( \rho_1 = \rho_2 = \rho_0 \), Eq. (13) is identical for \( i = 1 \) and \( i = 2 \), and can be rewritten as
\[
b_i \left( \frac{u}{u_i} \right)^2 + c_i \frac{u}{u_i} + d_i = 0 \quad i = 1, 2 \quad \ldots \text{(15)}
\]
where
\[
b_i = 3 \rho_i \left( R_i^2 + R_i^2 \omega_i^2 \right) \left( \frac{R_i \rho_i}{R_i \rho_i} \right)^2
\]
\[
c_i = -3 \rho_i \left( R_i^2 + R_i^2 \omega_i^2 \right) \left( \frac{R_i \rho_i}{R_i \rho_i} \right)^2 - R_i \rho_i \left( \frac{R_i \rho_i}{R_i \rho_i} \right)
\]
\[
d_i = \rho_i \left( R_i^2 + R_i^2 \omega_i^2 \right) \left( \frac{R_i \rho_i}{R_i \rho_i} \right)^3 - R_i \rho_i \left( \frac{R_i \rho_i}{R_i \rho_i} \right) \quad \text{for} \quad i = 1, 2 \quad \text{and if} \quad i = 1, \text{then} \quad j = 2; \quad \text{if} \quad i = 2, \text{then} \quad j = 1.
\]

In Theorem 1, one condition such that two strands are symmetrical is given, which is an equality about the radii and density of two strands \( (R_0, \rho_0) \) and radii, density, velocity and angular velocity of the spun yarn \( (R, \rho, u, \omega) \) [Eq. (14)]. Actually, the input variables \( R_0, \rho_0, u \) and \( \omega \) are related with the cops diameter and spindle rate rather than the variables \( R \) and \( \rho \). Therefore, the result of Theorem 1 is far from the practical application at present.

According to the analysis in Theorem 1, we can get the complete convergence point state for two-strand yarn spinning when \( u_1 = u_2 \) as follows:
\[
u_1 = u_2 = \frac{R_i \rho_i}{2 R_0 \rho_0} u
\]
\[
\cos \alpha_1 = \cos \alpha_2 = \frac{2 R_0^2 \rho_0}{R_i \rho_i}
\]
\[
\omega_1 = \omega_2 = \frac{R_i \rho_i \omega}{2 R_0 \rho_0}
\]
\[
F_1 = F_2 = \frac{R_i \rho_i}{4 R_0^2 \rho_0} F
\]
\[
M_1 = M_2 = \frac{R_i \rho_i}{4 R_0^2 \rho_0} M - \frac{FR_i \rho_i \left( R_i \rho_i \omega \right)^2}{8 R_0^3 \rho_0^2}
\]

**Convergence point for three-strand yarn spinning**

In this section, on the basis of two-strand yarn spinning analysis, the convergence point of three-strand yarn spinning is discussed by introducing a series of virtual intermediate variables. Three-strand yarn spinning structure is shown in Fig.2. The system should also obey the basic laws in mechanics.
such as force balance, momentum equation, mass conservation, energy conservation and torque equation just as two-strand case. However, if we use the analysis method of two-strand spinning directly, ten equations can be obtained with fifteen variables. Therefore, the system cannot be self-contained. To overcome this difficulty, virtual intermediate process is set and a series of virtual intermediate variables are introduced. Corresponding structure is shown in Fig. 3, in which the spinning process is decomposed into two steps, namely firstly, any two strands are twisted into a new virtual strand, and then the virtual strand and the rest one strand are twisted into a final spun yarn. According to the analysis above, the governing equations for the system shown in Fig. 3 can be written as follows:

\[ F_1 \cos \alpha_1 + F_2 \cos \alpha_2 = F' \cos \alpha' \]  \hspace{1cm} (16)
\[ F_1 \sin \alpha_1 = F_2 \sin \alpha_2 + F' \sin \alpha' \]  \hspace{1cm} (17)
\[ F' \cos \alpha' + F \cos \alpha_3 = F \]  \hspace{1cm} (18)
\[ F_3 \sin \alpha_3 = F' \sin \alpha' \]  \hspace{1cm} (19)

**Momentum equation**
\[ \rho u_1 R_1^2 u_1 \cos \alpha_1 + \rho u_2 R_2^2 u_2 \cos \alpha_2 = \rho' u' R'^2 u' \cos \alpha' \]  \hspace{1cm} (20)
\[ \rho_1 u_1 R_1^2 u_1 \sin \alpha_1 = \rho_2 u_2 R_2^2 u_2 \sin \alpha_2 + \rho' u' R'^2 u' \sin \alpha' \]  \hspace{1cm} (21)

**Mass conservation**
\[ \rho u_1 R_1^2 + \rho u_2 R_2^2 = \rho' u' R'^2 + \rho u R^2 \]  \hspace{1cm} (22)

**Energy conservation**
\[ \rho_1 u_1 R_1^2 u_1^2 + \rho_2 u_2 R_2^2 u_2^2 + \rho u_1 R_1^2 \omega_1^2 R_1^2 + \rho_2 u_2 R_2^2 \omega_2^2 R_2^2 = \rho' u' R'^2 u'^2 + \rho u R^2 \omega^2 R^2 \]  \hspace{1cm} (23)

**Torque equation**
\[ M_1 \cos \alpha_1 + M_2 \cos \alpha_2 = M' \cos \alpha' + R F \sin \alpha \]  \hspace{1cm} (24)
\[ M_1 \sin \alpha_1 - M_2 \sin \alpha_2 = R' F \sin \alpha' \]  \hspace{1cm} (25)

where \( F', \ M', \ \omega', \ \alpha' \) and \( u' \) are virtual intermediate variables.
In the following, the convergence point of three-strand spinning is discussed based on Eqs (16) - (27). By using a similar calculation process as two-strand spinning process, we can get the partial convergence point of three-strand spinning as follows after eliminating the intermediate variables:

\[
F_m = \frac{m}{m} F; \quad F_1 = \frac{m_1}{m} F; \quad F_2 = \frac{m_2}{m} F; \quad \ldots (28)
\]

\[
\omega_1 = \frac{u_1 R_1^2}{u R_1^2} \omega; \quad \omega_2 = \frac{u_2 R_2^2}{u R_2^2} \omega; \quad \omega_3 = \frac{u_3 R_3^2}{u R_3^2} \omega \quad \ldots (29)
\]

where \(m_i = \rho_i u_i^2 R_i^2\) for \(i = 1, 2, \ldots, n\).

We can also get \(\cos \alpha_i\), \(M_i\), and \(u_i\) by eliminating the intermediate variable \(\alpha_i\), \(M_i\) and \(u_i\). However, the density \(\rho_i\) and radius \(R_i\) are difficult or even impossible to determine in practical applications since the virtual intermediate strands do not actually exist, which also needs further study.

In this study, the convergence point of general multi-strand yarn spinning with \(n\) strands has been discussed by introducing a series of virtual intermediate variables. After eliminating the intermediate variable, the convergence point state of multi-strand yarn spinning has been given. However, the density \(\rho_i\) and radius \(R_i\) are difficult or even impossible to determine in practical applications since the virtual intermediate strands do not actually exist. Therefore, a more practical complete convergence point state of multi-strand yarn spinning should be further studied in the future.

Convergence point for multi-strand yarn spinning

In this section, according to three-strand yarn spinning analysis, the convergence point of the general multi-strand yarn spinning is discussed simply. Similar to the three-strand yarn spinning analysis, after eliminating a series of virtual intermediate variables, we can get the partial convergence point of the multi-strand yarn spinning with \(n\) strands as follows:

\[
F_1 = \frac{m_1}{m} F; \quad F_2 = \frac{m_2}{m} F; \quad \ldots (30)
\]

\[
\omega_1 = \frac{u_1 R_1^2}{u R_1^2} \omega; \quad \omega_2 = \frac{u_2 R_2^2}{u R_2^2} \omega; \quad \omega_3 = \frac{u_3 R_3^2}{u R_3^2} \omega \quad \ldots (31)
\]

Acknowledgement

Authors acknowledge with thanks the funding support by the National Natural Science Foundation of P. R. China under the Grant 11102072, the Natural Science Foundation of Jiangsu Province under the Grant BK2012254, Prospective industry-university-research project of Jiangsu Province under the Grant BY2012065 and Fundamental Research Funds for the Central Universities under the Grant JUSRP51301A.

References