Distribution of the Sun glitter sizes on the sea surface, derived from theoretically and in situ experiments

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The distribution of sun glitter areas on the sea surface is discussed. Theoretical distribution, which is obtained for Gaussian sea surface is verified using experimental data. Sun glitter images, taken with a high time and spatial resolution digital camera are developed by using specially designed FORTRAN program which calculates the statistical characteristics of glints. The experimentally defined distribution of glint areas (or reciprocal of Gaussian curvature at the specular reflection points) is compared with theoretical distribution. It is pointed, that the main causes of small divergence must be the following effects: 1) the real waved surface is not completely Gaussian 2) the solid angle of the sun is finite but not zeros. Using glint characteristics the possibility of oil films detection on the sea surface is discussed, also.

[Keywords: Gaussian Sea surface, Sun glintter, Glint images, Image process, Remote sensing]

Introduction

Despite the number of investigations have been provided on sun glitters, the important task – the validation of theoretical distribution density of sun glints sizes by “in situ” experiments have not been fulfilled yet. Aim of present work is to make this validation. Theoretical study with numerical simulations of the statistical characteristics SP (average and distribution of number SP, distribution of radii of curvature at SP) for a Gaussian sea surface has been carry out in Gardashova and Gardashov1. Experimentally (laboratory and in situ) investigation of glint characteristics were realized by Nosov et al.2. Where the asymptotic behavior of distribution,

\[ W(\hat{\rho}) \propto \frac{1}{\hat{\rho}^3} \]

was especially analyzed, also. But in the comparison with the theoretical formula were not performed2.

If suppose, that the distance \( L \) from the surface to the point of observation is greater than the radii of curvatures \( r_1 \) and \( r_2 \) at the specular point (SP), then the viewed glint area \( A \) will not depend on the angle of incidence and will be determined by the expression3:

\[ A = \frac{\varepsilon}{4} \rho \]

....(1)

Where, the quantity \( \rho = \left| r_1 r_2 \right| = \frac{1}{|\omega|} \) is the reciprocal to total curvature \( \omega \) at the SP; \( \varepsilon \) is the solid angle of the Sun (or of the other small light source). For a Gaussian uniform surface, \( \varepsilon = \zeta(x, y) \) the statistical distribution of the total curvature,

\[ \omega = \frac{\zeta_{xx} \zeta_{yy} - \zeta_{xy}^2}{(1 + \zeta_x^2 + \zeta_y^2)^2} \]

at the SP with the gradients,

\[ \gamma_x = \zeta_x(x, y) = 0 \quad \gamma_y = \zeta_y(x, y) = 0 \]

has been derived by Languet-Higgins3. Here, the distribution density \( W(\omega) \), of

\[ \omega = \Omega \]

\[ \frac{\zeta_{xx} \zeta_{yy} - \zeta_{xy}^2}{(1 + \zeta_x^2 + \zeta_y^2)^2} \]
has been expressed in terms of contour integrals. A comparatively simple expression for \( W(\omega) \) is convenient for use in practical computations, has been derived in Gardachov\(^4\).

**Theoretical distribution of Gaussian curvature**

The expression of \( W(\omega) \) from (Gardachov, 2000), be written in terms of the dimensionless curvature \( \varpi = \omega \sqrt{3H} \) (the parameter \( H \) is related to the average of \( \omega^2 \) by \( 3H = \langle \omega^2 \rangle \)), where the average is taken on all points of the surface \( z = \zeta(x, y) \), but not only on SP, at \( \gamma_x = 0, \gamma_y = 0 \) has the following form:

\[
W(\varpi) = \frac{1}{\Phi(t)} \left[ \frac{(t^2 - t + 1)^{\frac{3}{2}}}{\sqrt{t(1-t)}} \right] \exp\left(-\varpi \sqrt{t^2 - t + 1}\right) \frac{n}{\sqrt{m(\alpha)}} \, d\alpha
\]

for \( \varpi < 0 \) and

\[
W(\varpi) = \frac{1}{\Phi(t)} \left[ \frac{(t^2 - t + 1)^{\frac{3}{2}}}{\sqrt{t(1-t)}} \right] \exp\left(-\varpi \sqrt{t^2 - t + 1}\right) \frac{n}{\sqrt{m(\alpha)}} \left[-F\left(\sqrt{m(\alpha)}\right)\right] d\alpha
\]

for \( \varpi \geq 0 \).

where, \( F(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-t^2) dt \) is the error function,

\[ m(\alpha) = \frac{(t + 1)\sqrt{t^2 - t + 1}}{t} (1 - k \sin^2 \alpha) \]

The function \( \Phi(t) \) is expressed in terms of elliptical integrals:

\[ K(k) = \int_0^\frac{\pi}{2} \frac{1}{\sqrt{1 - k^2 \sin^2 \alpha}} \, d\alpha \]

and

\[ E(k) = \int_0^\frac{\pi}{2} \sqrt{1 - k^2 \sin^2 \alpha} \, d\alpha \]

as

\[ \Phi(t) = \sqrt{1 - t^2} E(k) - \frac{1 - t}{1 + t} tK(k) \text{ at } k = \frac{\sqrt{1 - 2t}}{\sqrt{1 - t^2}} \]

In fact, \( \Phi(t) \) is a very slow decreasing monotonic function with maximum and minimum values at and \( \Phi(0) = 1 \) and \( \Phi\left(\frac{1}{2}\right) = \frac{\pi}{2\sqrt{3}} \approx 0.907 \) respectively.

The mean values of \( \rho \) and \( N \), the number of SP with gradients \( (\gamma_x, \gamma_y) \) per unite area can be defined by

\[
\langle \rho \rangle = \frac{\pi (1 + \gamma_x^2 + \gamma_y^2)}{\sqrt{3H}} \int \frac{\sqrt{t^2 - t + 1}}{2\Phi(t)} dt \]

\[
\langle N \rangle = \frac{2\sqrt{3H}}{\pi \sqrt{t^2 - t + 1}} \Phi(t) W_2(\gamma_x, \gamma_y)
\]

respectively. Here, \( W_2(\gamma_x, \gamma_y) \) is the distribution density of surface gradients\(^5\).

As we can see, the distribution \( \rho(\varpi) \) depends on only one dimensionless parameter, \( t \) which is the indicator of sea wave structure. The parameters \( t \) and \( H \) are evaluated by using surface wave energy spectrum moments. The parameter \( t \) may lie only between 0 and \( \frac{1}{2} \). The value \( t = 0 \) corresponds to the surface, which consists of two distinct systems of long-crested waves to intersect the each other at a small angle. The value \( t = \frac{1}{2} \) might occur in a variety of circumstances – for instance, when the surface is isotropic or when angular spread of energy is small and has a certain ‘peakedness’\(^3\).

To regard the light reflection problem, the distribution of the, \( \bar{\rho} = \frac{1}{|\rho|} \) is very important. This distribution can be directly obtained from the:

\[
W_\rho(\bar{\rho}) = \frac{1}{\bar{\rho}^2} \left[ W_2\left(\frac{1}{\bar{\rho}}\right) + W_2\left(-\frac{1}{\bar{\rho}}\right)\right]
\]

Note that from the general formula (4) for mean value of \( \bar{\rho} \), in the considered case of zero gradients of SP (i.e. \( \gamma_x = 0, \gamma_y = 0 \)), we find:

\[
\langle \bar{\rho} \rangle = \frac{\pi}{2\Phi(0)} = \frac{\pi}{2} \approx 1.57
\]

For \( t = 0, \ t = \frac{1}{4} \) and, \( t = \frac{1}{2} \) correspondingly, we find:

\[
\langle \bar{\rho} \rangle = \frac{\pi}{2\Phi(0)} = \frac{\pi}{2} \approx 1.57 \quad \langle \bar{\rho} \rangle = \frac{\pi}{2\Phi\left(\frac{1}{4}\right)} \approx 1.5277
\]
and
\[ \langle \bar{p} \rangle = \frac{\pi}{2 \Phi \left( \frac{1}{2} \right)} \sqrt{3} = \frac{3}{2} = 1.5 \]

As we see, for all possible values of \( t \), which lay in the interval \( \left[ 0, \frac{1}{2} \right] \), the mean value of \( \bar{p} \) changes insignificantly and approximately can be taken as, equal to 1.53.

**In situ experiments, its processing and discussion**

To validate the theoretical formula (6) (in other words (2), (3)), a series of sun glint images on waved basin have been taken with digital camera of high temporal and spatial resolution. (One of those images is shown in Fig.1.) Further, those images have been processed by FORTRAN program, particularly designed by us for calculation the glints parameters, calibration information (the image of the bright sphere with the radius 2.43 cm, located in the center of occupied by glints zone) was used.

Using the relationship, \( \frac{A}{\langle A \rangle} = \frac{\bar{p}}{\langle \bar{p} \rangle} \), which follows from (1), the distribution \( W(\bar{p}) \) can be simply derived from the distribution \( W(A) \). The theoretical curve \( W(\bar{p}) \) (at \( t = \frac{1}{2} \)) and its histograms are shown in fig.2. As we see, there is quite good closeness between theoretical and empirical distributions. We think that the main causes of deviation are: 1) the otherness of actual waved water surface from Gaussian one; 2) the distinction of the Sun from a point light source. Indeed, when closely inspecting the Fig.2, we see that some large glints are formed by confluence of small glints. It tells us, that if we will produce experiment with light source which has a solid angle essentially smaller than the Sun, then, some large glints will be resolved. Thereat, the number of lager glints will decrease and the number of small glints will increase. As a result, the histogram \( W(\bar{p}) \) in Fig.2 (color curves) must be much closer to theoretical curve.

**About the remote sensing of oil firms on the glint statistical characteristics**

The statistical characteristics of SP of sea surface are very sensitive to changes in the reflecting surface and they can change in wide range, depending on
geometrical and physical characteristics of surface, namely, the degree and structure of waves, presence of flow, internal waves and oil films on the surface. Therefore, the sun glint statistical characteristics may be used for remote sensing these effects. For this, it is necessary to investigate the interrelation between the geometrical and physical characteristics of surface and the statistical characteristics of SP – Sun glitters.

But, Sun glitters can be clear-cut derived from the sea surface images captured only at low altitudes. As it is known, the angular resolution, $\beta$, of an optical system with the diameter of lens $d$, is defined as

$$\beta = 1.22 \frac{\lambda}{d}, \text{where } \lambda \text{ is the light wavelength.}$$

Then, the corresponding spatial resolution, $l$, is determined by the expression:

$$l = L \cdot \beta = 1.22 \frac{\lambda}{d} \cdot L, \text{where } L \text{ is the altitude of optical sensor (camera). For the typical values of parameters: } l = 0.1 \text{ m (distance between glints), } \lambda = 0.5 \cdot 10^{-6} \text{ m }, \text{ and } d = 0.3 \text{ m is obtained: } L \approx 57 \text{ km. It means that, for only the altitudes less than, } 57 \text{ km the extraction of glints from the image may be realized. Actually, due to influence of atmosphere (scattering and absorption) this altitude is substantially less. Now, as to occur how oil films on the sea surface can be identified by using SP statistical characteristics. According to the measurements of Cox and Munk}, \text{ the dispersions of surface gradients for clear sea surface depend on wind speed } v \text{ and these are:}$$

$$\sigma_x^2 = 3.16 \cdot 10^{-3} v \quad \sigma_y^2 = 0.003 + 1.92 \cdot 10^{-3} v$$

When the oil films cover the surface, these dispersions become 2-3 times less than that of clear surface. If the oil covered surface dispersions are taken as: $\sigma_x^2 = \sigma_y^2 / 3$, then to get the similar dispersion values of $\sigma_x^2$, $\sigma_y^2$, and $\sigma_x^2$, $\sigma_y^2$ for clear and oil-film covered sea surface, the wave spectrum must include the harmonics with the wavelengths of $\Lambda \geq \Lambda_{\text{min}} = 4.5 \text{ cm}$ and $\Lambda \geq \Lambda_{\text{min}} = 60.0 \text{ cm}$, respectively. For the parameters, $t$ and $H$, which were evaluated from the energy spectrum at the wind speed of $v = 10 \text{ m }/\text{s}$, were found: $t = 0.49$, $H = 3.97 \cdot 10^4 \text{ (m}^4\text{)}$ and $\overline{t} = 0.48$, $\overline{H} = 8.10 \cdot 10^{-3} \text{ (m}^{-4})$ for clear and oil-covered surfaces, respectively. The graph of $W(\rho)$ distribution calculated by the theoretical formula (2), (3) and (6) for clear and oil-covered surfaces, are shown in figure. The mean value of $\rho$ evaluated by equation (4) was $\langle \rho \rangle_{\text{clear}} = 0.014 m^2$ for a clear surface and $\langle \rho \rangle_{\text{oil}} = 9.6 m^2$ for oil covered surface. The average number of SP calculated by equation (5) was $\langle N \rangle_{\text{clear}} = 424 m^{-2}$ and $\langle N \rangle_{\text{oil}} = 1.8 m^{-2}$ for clear and oil films covered surface, respectively. As it’s seen, when oil film exists on the sea surface, the average number of SP is decreased about 200 times, the mean value $\langle \rho \rangle$ (in other words, the mean value of the sun glints sizes) increased about 700 times, and the graph of $W(\rho)$ shifts essentially to the right. By designing an appropriate optical system to sense the changes occurring in these parameters, the problem of remote sensing of oil films existing on the sea surface can be solved. For not very high remote sensing altitudes (when optical system resolves glints), an appropriate data may be a digital image of the Sun glints or other small point alike light source. For large altitudes, as a suitable data it can be used a backscattering light intensity distribution according to the method offered in (by taking into account influence of atmosphere, additionally).\(^{5}\)

![Image](image.png)

**Fig. 3** The theoretical distribution $W(\overline{\rho})$ at the wind speed $v = 10 \text{ m }/\text{s}$. (a) clear surface (b) oil covered surface

**Results and Discussion**

The statistical characteristics of the SP can be used for remote sensing of geometrical and physical characteristics of sea surface. For this, it is necessary
to investigate the interrelation between the geometrical and physical characteristics of surface and the statistical characteristics of SP – Sun glitters. These studies, both experimentally and theoretically, must be synergetic. As a result of these studies, the data bank of the statistical characteristics of the sun glitters can be created.

For example, for the beginning, as for this data it can be taken the experimentally determined the average number of the sun glitters, \( \langle N_S \rangle \ (m^{-2}) \), and the average radius of curvature of the sun glitters \( \langle \rho \rangle \ (m^2) \), with the precise registration: wind speed \( (m/c) \); wind direction; state of development and the stability of waves; the time-spatial spectrum of waves; flows; internal waves; oil films. Subsequently, this data bank can be enriched by the distribution densities \( W_N(N_S) \) and \( W_\rho(\rho) \). Having this data bank, it is possible to carry out the quantitative analysis of the sensitivity of the statistical characteristics of glints to the nature of waves, the presence of flows, the internal waves and the oil films, and further estimate the possibility of their remote sensing.

References