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Mathematical modelling of the washing zone of an industrial rotary vacuum washer

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The flow of miscible fluids through the beds of finite length to investigate the mechanism of displacement washing is presented through a diffusion-dispersion model. The model is based on phenomenon of diffusion-dispersion in porous cylindrical particles, *e.g.*, fibers. The data of 4^{th} stage industrial brown stock washer is used for simulation. Empirical relations have been found between key parameters such as cake thickness, fiber porosity, bed porosity and fiber consistency. Concentration of black liquor solids in discharge pulp and efficiency parameters have been correlated using least square principle.

Keywords: Washing process, Displacement ratio, Percent efficiency, Wash yield, Distribution ratio, Peclet number

The problem of heat and mass transfer during flow through the packed beds of solid and semi solid particles is of interest to the chemical engineers as well as the mathematicians. Vast literature related to longitudinal dispersion in porous media is available. However, the problem lies with predicting the behaviour of sharp interface between the two miscible fluids having identical dynamical and kinematical properties. The problem has substantial importance in the determination of the efficiency of solvent, filtrate recovery in the washing of filter cakes etc.

Pulp and paper industry is highly chemical, labour and water intensive industry. During the process of paper making certain operations such as chipping, pulping, washing, bleaching, chemical recovery and effluent treatment are involved. Washing of pulp fibers is one of the major operations in the manufacturing of paper. It is also a complicated operation as the cooked chemicals have to be recovered and the pollution must be kept to an acceptable minimum. Brown stock washing is one of such steps which significantly influences the economy as well as the environment.

The removal of soluble and insoluble impurities from the cake pores with the help of weak wash liquor or clean water is called washing process. During this process, the solute present in the irregular void channels of the bed diffuse out of fiber pores. Displacement of solute from these pores is associated with diffusion like dispersion of wash liquid in the direction of flow, commonly known as longitudinal or axial dispersion.

Black liquor solids mainly consist of dissolved lignin and inorganic ions like Na^+, Mg^{++}, Ca^{++} and K^+. Lignin, if not separated from the pulp stream before bleaching, consumes excess of bleaching chemicals and generates undesirable effluents such as AOX, colour, BOD, COD etc. More use of these chemicals in the bleaching section leads to the environment as well as water pollution. Ideally, there should neither be overflow of black liquor solids nor of the fibers with the filtrate leaving the washing plant. However, these ideal situations can never be met in the industry.

The pulp washing operations are classified into two categories *viz.* dilution thickening and displacement washing. In dilution thickening, the pulp slurry is diluted thoroughly, mixed with wash liquor and then thickened by filtering or pressing. In displacement washing wash liquor or clean water passes through the pulp bed in a piston like situation, pushing out the liquor associated with the pulp bed. In comparison to the dilution thickening, displacement washing is more effective, if the amount of wash liquor to be added is same.

The aim of present study is to mathematically model the washing behaviour of brown stock washer in pulp and paper industry, considering various factors which influence the washing behaviour. The purpose of pulp washing is not only to remove the insoluble or soluble impurities but to do it with minimum amount of water or weak wash liquor. It is because of the reason that large amount of water to be used will result in long time to evaporate the spent water and large quantity of weak wash liquor will immerse out chemicals hazardous to the environment. Hence, pulp washing is the balance between cleanliness of pulp and the amount of wash liquor to be used.

Mathematical model

A variety of mathematical models[1-7] have been developed to study the behaviour of washing process. Majority of the investigators have followed linear adsorption isotherm whereas Grahs[1] and Al-Jabari *et al.*[3] have followed the Langmuir adsorption isotherm.

In the proposed model, packed bed (Fig. 1) is assumed to be composed of porous, compressible and cylindrical particles such as fibers. The behaviour of average solute concentration flowing through the bed as well as the concentration of solute adsorbed on the fiber surface is studied. Langmuir adsorption isotherm has been followed to relate the concentration of solute adsorbed on the fiber surface and the intrapore solute concentration.

The packed bed, in the present study is divided into two zones namely zone of flowing liquor and the zone of fibrous material. Mass transfer takes place from fiber pores to fiber surface and from fiber surface to bulk fluid and vice versa. Mass transfer resistance through the thin stagnant film is controlled by fluid film mass transfer coefficient k_f. Two types of porosities, bed porosity ε and fiber porosity β have been taken into account. Diffusion through the fiber phase is represented by intrafiber diffusion coefficient D_F. Since the molecular diffusivity is very small as compared to the axial dispersion, therefore, the plug flow is controlled by axial dispersion coefficient D_L only.

Model is developed on certain important lines and for model development the following assumptions are made:

(i) System is isothermal.
(ii) Bed is macroscopically uniform and fibers are randomly oriented in the horizontal plane.

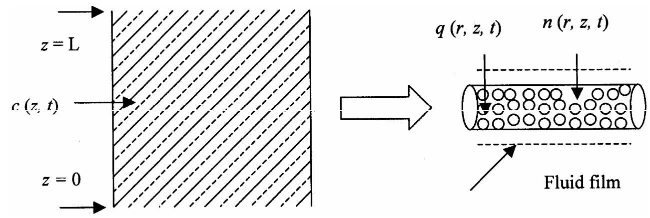

Fig. 1—Schematic representation of the packed bed of pulp fibers

(iii) Fibers are porous and of uniform cylindrical size. Fiber diameter as well as the fiber length is very small as compared to the axial distance.
(iv) The intrafiber diffusion coefficient and axial dispersion coefficient are independent of the cake thickness, fiber radius and solute concentration.
(v) The movement of the solute within fiber pores is described mathematically by Fick's law.
(vi) Average solute concentration is defined over the bed cross section.

Model for the particle phase

The diffusion equation describing the movement of solute within fiber pores is defined as:

$$D_F\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}\right)=\frac{\partial w}{\partial t} \qquad \ldots(1)$$

where w is the local intrafiber solute concentration. By definition w does not distinguish between the solute adsorbed on the particle surface and the solute within fiber pores. To distinguish between the solute adsorbed on the fiber surface and the solute within fiber pores, it is assumed that the surface diffusion effects are negligible. The transport of solute within fibers is effectively described by diffusion equation involving both the concentration of solute adsorbed on the fiber surface and the intrapore solute concentration. The driving force for diffusion is taken to be the intrapore concentration gradient. The intrafiber diffusion equation can be written as:

$$D_F\left(\frac{\partial^2 q}{\partial r^2}+\frac{1}{r}\frac{\partial q}{\partial r}\right)=\frac{\partial q}{\partial t}+C_F\,\frac{(1-\beta)}{\beta}\frac{\partial n}{\partial t} \qquad \ldots(2)$$

By definition of local intrafiber solute concentration at any radial distance '*r*' following relation is applied $w=\beta\, q+C_F(1-\beta)n$. It is assumed that local equilibrium prevails in the individual intrafiber pores.

Adsorption isotherms

To relate the intrapore solute concentration and the concentration of solute adsorbed on the fiber surface Langmuir adsorption isotherm is followed. Vast literature is available for linear adsorption isotherm[2,4-9]. It is followed due to the reason that linear adsorption isotherm linearise the differential equation describing the behaviour of fluid flow and reduces the mathematical complexities. However, paucity is found in case of Langmuir adsorption isotherm. Grahs[1], Al-Jabari *et al.*[3], Crotogino *et al.*[10] and Trinh *et al.*[11] have studied the Langmuir adsorption isotherm for pulp fibers. The mass balance equation for adsorption isotherm is given by:

$$\frac{\partial n}{\partial t} = k_1 \frac{q}{C_F}(N_0 - N_S - n) - k_2 n \qquad \ldots(3)$$

At equilibrium, Eq. (3) simplifies to Langmuir kinetics as:

$$n = \frac{q(N_0 - N_S)}{(C_F k^{-1} + q)} \qquad \ldots(4)$$

Model for bulk fluid

The transport phenomenon in the porous media having void fraction ε is described by one dimensional axial dispersion model involving axial dispersion coefficient. The mass balance equation for bulk fluid is:

$$D_L \varepsilon \frac{\partial^2 c}{\partial z^2} = u\varepsilon \frac{\partial c}{\partial z} + \varepsilon \frac{\partial c}{\partial t} + \frac{2(1-\varepsilon)D_F}{R} \left.\frac{\partial q}{\partial r}\right|_{r=R} \qquad \ldots(5)$$

Initial and boundary conditions

The concentration gradient is assumed to be zero at the centre of the particle, *i.e.*,

$$\frac{\partial q}{\partial r} = 0 \text{ at } r = 0 \text{ and } t > 0 \qquad \ldots(6)$$

It is assumed that external mass transfer resistance exists at r = R and the mass transfer to the fiber surface is controlled by film resistance mass transfer coefficient:

$$-D_F\left(\frac{\partial q}{\partial r}\right) = \frac{k_f \beta}{K}\left(q|_{r=R} - c\right) \text{ at } r = R \text{ and } t > 0 \qquad \ldots(7)$$

At the inlet of the bed, it is assumed that there will be no loss of solute from the bed through the plane at which the displacing fluid is introduced:

$$uc - D_L \frac{\partial c}{\partial z} = uC_S \text{ at } z = 0 \qquad \ldots(8)$$

To avoid the unacceptable conclusions that fluid will pass through the maximum or minimum in the interior of the bed, the concentration gradient is taken to be zero at the bed outlet:

$$\frac{\partial c}{\partial z} = 0 \text{ at } z = L \text{ and } t > 0 \qquad \ldots(9)$$

Initially, $c = q = C_0$ and $n = N_0$

To solve the model numerically, the model is first converted into dimensionless form and then the technique of orthogonal collocation on finite elements is applied. The details of the numerical technique and the model can be obtained from Arora *et al.*[12,13].

Experimental Procedure

As the drum rotates inside the vat various zones such as cake formation, dewatering, washing, drying, blow, discharge and dead zone are formed. In the present study, the washing zone is modeled mathematically because of its complex nature. The data of non-wood pulp made up of wheat straw has been considered in the present study.

The data of an industrial brown stock washer was collected from a paper mill. Pulp samples were collected from the 4[th] stage of a rotary vacuum washer. The diameter of the washer was 5.7912 m and width was 4.8768 m. The Kappa number of the pulp was 15, *p*H was 12 and temperature during washing was 323 K. The degree of beating was 24-25°SR. The fractional submergence of the drum was 40%. Pressure drop was 15,000-20,000 Pa.

On the basis of these samples experiments of consistency, density and viscosity of the liquid, concentration of black liquor solids and consistency and density of fibers have been performed in the laboratory. The inlet and outlet consistencies were in the range of 1-2 and 10-12% respectively. Viscosity and density of black liquor were 7.2×10^{-4} kg/ms and 1003.5 kg/m^3 respectively. Cake thickness, interstitial velocity, axial dispersion coefficient, hydraulic mean radius of pores, bed porosity has been calculated using the relations given by Kukreja *et al.*[5], Potucek[14,15] and Shiraishi[16]. The experimental data for sodium and lignin is presented in Tables 1 and 2.

Analysis of sodium and lignin

The sodium concentration was calculated using flame photometer with the help of the method given by Trinh & Crotogino[17]. The concentration of sodium adsorbed on the fiber surface is plotted against the concentration of sodium in the bulk fluid (Fig. 2). The lignin concentration is determined using an ultraviolet spectrophotometer operating at wavelength of 280 nm with the help of the method given by Potucek[6]. The concentration of lignin adsorbed on the fiber surface is plotted against the concentration of lignin in the bulk fluid (Fig. 3).

Calculation of industrial parameters

The performance of pulp washing system is based on the quantum of the black liquor solids removed with the amount of wash liquor added. The performance of brown stock washing can be categorized as wash liquor usage, solute removal and efficiency parameters. These parameters have been measured experimentally by using the relations given by Kukreja *et al.*[5] and Potucek[14,15].

Wash liquor usage parameters

During pulp washing operation, the amount of wash water added, is sent to the evaporator for further treatment. If more chemicals are used in this section, it will have perilous effect on environment whereas excess use of water will increase the load on evaporator. Therefore, there needs to be a balance in the amount of wash water added and the impurities to be removed.

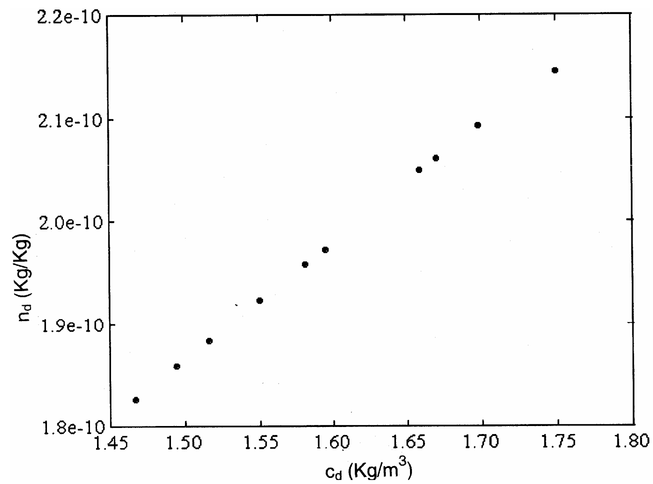

Fig. 2—Behaviour of sodium adsorbed on fiber surface versus sodium in the discharged pulp

Table 1—Experimental data for sodium

β	ε	k_1/k_2	C_0	C_F	u/L×10^2	D_F/R^2×10^2	k_f×10^6
0.912	0.968	0.003146	8.3341	47.059	2.4475	2.73	1.8471
0.908	0.966	0.003388	8.3341	50.000	2.4595	2.98	1.8398
0.904	0.966	0.003539	8.3341	51.470	2.4716	3.24	1.8328
0.902	0.964	0.003639	8.3341	52.941	2.4797	3.42	1.8284
0.898	0.962	0.003771	8.3541	55.882	2.4850	3.67	1.8206
0.894	0.961	0.003914	8.3541	57.353	2.4964	3.94	1.8147
0.889	0.959	0.004114	8.3541	60.294	2.5125	4.34	1.8068
0.885	0.957	0.004262	8.3782	63.235	2.5143	4.66	1.7987
0.882	0.956	0.004362	8.3782	64.759	2.5221	4.87	1.7950
0.880	0.955	0.004461	8.3782	66.177	2.5300	5.09	1.7915

Table 2—Experimental data for lignin

β	ε	k_1/k_2	C_0	C_F	u/L×10^2	D_F/R^2×10^2	k_f×10^6
0.918	0.968	0.002939	8.3341	47.059	1.8348	2.15	1.8408
0.912	0.966	0.003123	8.3341	50.000	2.0209	2.27	1.8411
0.910	0.966	0.003214	8.3341	51.470	2.1135	2.85	1.8413
0.907	0.964	0.003306	8.3341	52.941	2.2071	3.17	1.8415
0.902	0.962	0.003481	8.3541	55.882	2.3117	3.35	1.8402
0.900	0.961	0.003573	8.3541	57.353	2.4056	5.22	1.8404
0.894	0.959	0.003756	8.3541	60.294	2.5968	5.48	1.8407
0.889	0.957	0.003928	8.3782	63.235	2.6232	5.72	1.8391
0.887	0.956	0.004019	8.3782	64.759	2.7171	6.46	1.8393
0.884	0.955	0.004110	8.3782	66.177	2.8137	6.94	1.8394

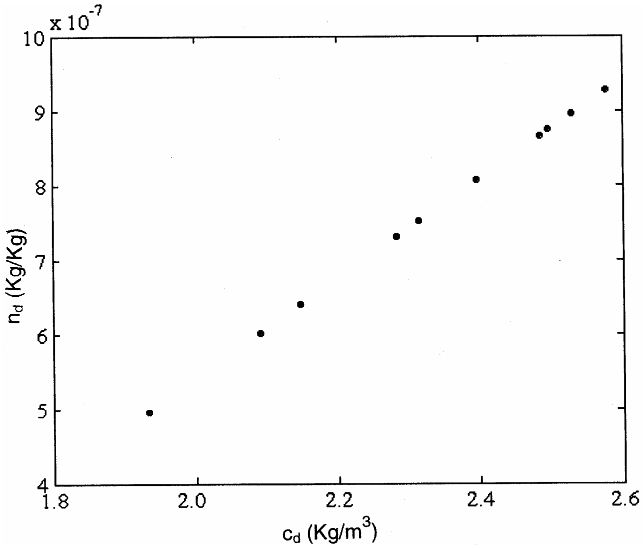

Fig. 3—Behaviour of lignin adsorbed on fiber surface versus lignin in the discharged pulp

Dilution factor

It is also characterized as an excess of wash water used. It is the difference of wash liquor entering and the wash liquor leaving with the washed pulp and calculated by the formula:

$$DF = L_S - L_d \quad \ldots(10)$$

Wash liquor ratio

It is the ratio of the wash liquor entering to the liquor leaving with the washed pulp. By definition, wash liquor ratio is:

$$WR = L_S / L_d \quad \ldots(11)$$

WR = 1 signifies the minimum wash ratio. It is the ideal case in which the concentration of black liquor solids in the wash liquor is same as that of the filtrate in the unwashed cake.

Solute removal parameters

Solute removal parameters depend upon the amount of dissolved solids removed during the washing stage or operation. These parameters act as significant tool to predict the amount of bleach chemical consumption.

Wash yield

Wash yield is an important solute removal parameter used in the industry. It is the ratio of dissolved solids removed to the dissolved solids entering with the unwashed pulp. If one assumes that the solute is split in the same proportion as the liquor, it is easy to calculate how much solute is left in the thickened pulp. For dilution/extraction washing operations wash yield can be calculated using the inlet and outlet solute consistencies as:

$$WY = [(C_{yd} - C_{yi})/\{C_{yd}(100 - C_{yi})\}]100 \quad \ldots(12)$$

Displacement ratio

Displacement ratio at any stage is the ratio of the actual possible reduction in the dissolved solids to the maximum possible reduction. This factor is commonly used in the industry to check the functioning of the equipment and the reduction in the black liquor solids. The effect of sorption is ignored in the displacement ratio. Kukreja *et al.*[5] and Potucek[6] have calculated the displacement ratio using the following formula:

$$DR = \frac{C_0 - c_d}{C_0 - C_S} \quad \ldots(13)$$

Efficiency parameters

Efficiency parameters constitute an important part of the study of pulp washing operation. These parameters help to check the efficiency of the equipment and the removal of black liquor solids from the unclean pulp.

Percent efficiency

Percent efficiency is the key factor to check the efficiency of the equipment. It signifies percent of black liquor solids removed during washing operation. Higher the percent efficiency, higher will be the percent of solutes removed and therefore, higher will be the efficiency of the equipment. Kukreja *et al.*[5] has calculated percent efficiency using displacement ratio, inlet and outlet consistencies of pulp respectively:

$$\%efficiency = [1 - \{(1 - DR)(100 - C_{yd})/(100 - C_{yi})\}]100 \quad \ldots(14)$$

Equivalent displacement ratio

In equivalent displacement ratio, the washer of given consistencies is compared to a hypothetical washer of standard inlet consistency of 1% and outlet consistency of 12%. The hypothetical washer has the same dilution factor and soda loss as the actual washer. Its formula is:

$$1 - EDR = (1 - DR)\,(DCF)\,(ICF) \quad \ldots(15)$$

where $DCF = L_d/7.333$ and $ICF = 99(L_i + DF)/[L_i(99 + DF) - L_d(99 - L_i)(1 - DR)]$

DCF and *ICF* are discharge correction factor and inlet correction factor respectively. The values of these industrial parameters are given in Table 3.

Analysis of data

In the present study, experiments were carried out for removal of lignin and sodium ions adsorbed on fiber surface. A comparative study of the input data is made. The distribution ratio D_F/R^2 is found to be higher in case of lignin as compared to sodium. It signifies the fact that more time will be consumed to remove the lignin adsorbed on the fiber surface as compared to that of sodium ions where the distribution ratio is comparatively small. It may be due to the fact that pulp is washed for long time in case of sodium as compared to that of lignin.

The film resistance mass transfer coefficient is found to be more or less constant in case of lignin however small variations can be found in case of sodium ions. Fiber porosity is found to be higher in case of lignin as compared to that of sodium. Due to this reason variation in u/L is found in case of lignin whereas not much variation in u/L ratio is found in case of sodium.

In Fig. 4 the behaviour of two distribution ratios D_F/R^2 and u/L is presented. It is observed that in case of sodium, D_F/R^2 is dependent on u/L whereas for lignin, latter is found to be less effected by u/L. The mass transfer rate in case of sodium ions is comparatively higher as compared to that of lignin. It is due to the small molecular size of the sodium ions as compared to that of lignin and the difference in the residence time of two. It results in more swelling of pore size, which contributes to more diffusion and less mass transfer resistances.

In Fig. 5 the ratio of rate kinetics is plotted against the distribution ratio. It is observed from the figure that the ratio of rate kinetics increases with the increase in distribution ratio. In case of sodium ions the rate kinetics is higher and very steep as compared to that of lignin.

Results and Discussion

On the basis of the experimental results obtained, the relationships among different parameters have been established. These relations are discussed here.

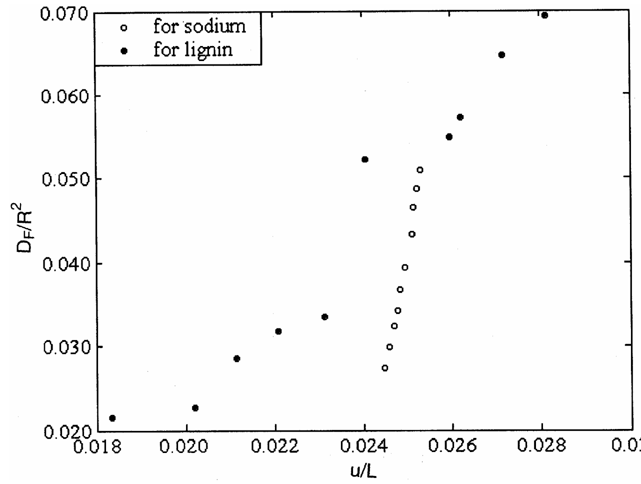

Fig. 4—Behaviour of D_F/R^2 versus u/L for sodium and lignin ions adsorbed on the fiber surface

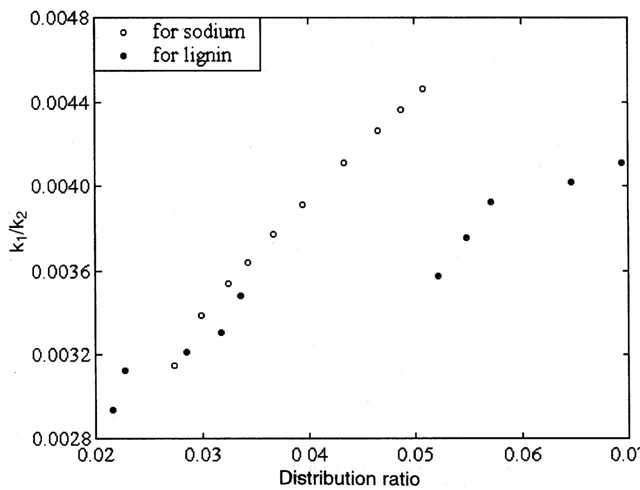

Fig. 5—Behaviour of rate kinetics (k_1/k_2) versus distribution ratio (D_F/R^2)

Table 3—Values of efficiency parameters

DR	%efficiency	WR	EDR	DF	WY
0.7901	80.3221	3.76417	0.4087	3.1488	0.9514
0.7962	80.9894	3.90725	0.3730	3.1215	0.9578
0.7996	81.3976	3.99027	0.3449	3.0346	0.9611
0.8010	81.5848	4.02437	0.3290	2.9601	0.9625
0.8092	82.3859	4.23972	0.3114	3.0542	0.9696
0.8108	82.6147	4.28402	0.2927	2.9523	0.9710
0.8144	83.0655	4.38909	0.2677	2.8584	0.9739
0.8190	83.5095	4.52578	0.2623	2.8821	0.9772
0.8216	83.7930	4.60412	0.2506	2.8604	0.9789
0.8249	84.1464	4.71037	0.2388	2.8610	0.9810

Effect of cake thickness on bed porosity

Cake thickness is found to influence the bed porosity significantly. It is observed that bed porosity increases linearly with the increase in cake thickness. On the basis of the experimental data following equation, using least square method is obtained:

$$\varepsilon = 0.9291 + 0.3859\, L \quad \ldots(16)$$

The values are found to be within the error of ±1%. The correlation coefficient is found to be 0.9726 at the confidence interval of 95% and the standard error estimate is 0.0011.

Effect of cake thickness on fiber porosity

Fiber porosity is also found to be effected by cake thickness and increases linearly with the cake thickness. On the basis of the experimental data of sodium, following equation, using least square method is obtained:

$$\beta = 0.8182 + 0.9815\, L \quad \ldots(17)$$

The values are within the error of ± 1%. The correlation coefficient is found to be 0.9793 at confidence interval of 95% and the standard error estimate is 0.0023

Effect of cake thickness on consistency of fibers

Fiber consistency is found to decrease linearly with the cake thickness. On the basis of the experimental data following equation is obtained using least square method.

$$C_F = 103.6259 - 558.1176\, L \quad \ldots(18)$$

The values are within the error of ± 4.7%. The correlation coefficient is 0.9793 and the standard error estimate is 1.3940. The values are found to be within the confidence interval of 95%.

Effect of concentration of black liquor solids in discharge pulp on wash yield

Wash yield is found to be dependent on concentration of black liquor solids in discharge pulp. The relation between two has been found using least square method. The values are within the range of ± 1%. The correlation coefficient is –0.9969. The values are found to be within the confidence interval of 99%. The two are related by the following equation.

$$WY = 1.1302 - 0.8458\, C_d \quad \ldots(19)$$

Effect of concentration of black liquor solids on % efficiency

Percent efficiency is also found to be affected by concentration of black liquor solids in the discharge pulp. Percent efficiency is related with concentration of black liquor solids in discharge pulp by following equation using least square method.

$$\%\,efficiency = 1.0341\times10^2 - 1.0989\times10^2\, C_d \quad \ldots(20)$$

The values are within the error of ± 1%. The correlation coefficient is –0.9997 at the confidence interval of 95%.

A mathematical model is of significance if it can be implemented in the industry. The performance of an industrial brown stock washer is characterized in terms of different efficiency parameters. The model proposed in the present study is also correlated with these efficiency parameters. Here, these efficiency parameters are measured from the model. The effect of Peclet number and distribution ratio (D_F/R^2) is checked on these efficiency parameters.

Displacement ratio

Potucek[6] and Kukreja[18] have mentioned the dependence of average solute concentration on time and therefore, the displacement ratio is also dependent on time. The breakthrough curves are plotted for displacement ratio with respect to time. It also depends upon the nature of the substance to be removed. In the present case sodium based solutes are used.

The behaviour of displacement ratio for different values of Peclet number and distribution ratio is shown in Fig. 6. It is observed that for large Peclet number and small distribution ratio, the actual reduction in black liquor solids converges to maximum possible reduction more rapidly as compare to the small Peclet number and large distribution ratio.

Percent efficiency

Percent efficiency depicts the percentage of black liquor solids removed during the washing operation. Higher %efficiency signifies better performance of the equipment. Percent efficiency is calculated using the following formula.

$$\%efficiency = \left(1 - \frac{(c_d - C_S)(100 - C_{yd})}{(C_0 - C_s)(100 - C_{yi})}\right)100 \quad \ldots(21)$$

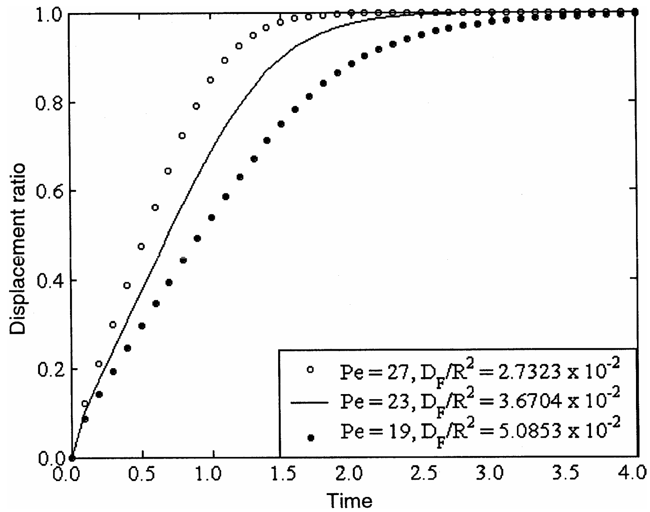

Fig. 6—Theoretical breakthrough curves of displacement ratio for different values of Peclet number and distribution ratio

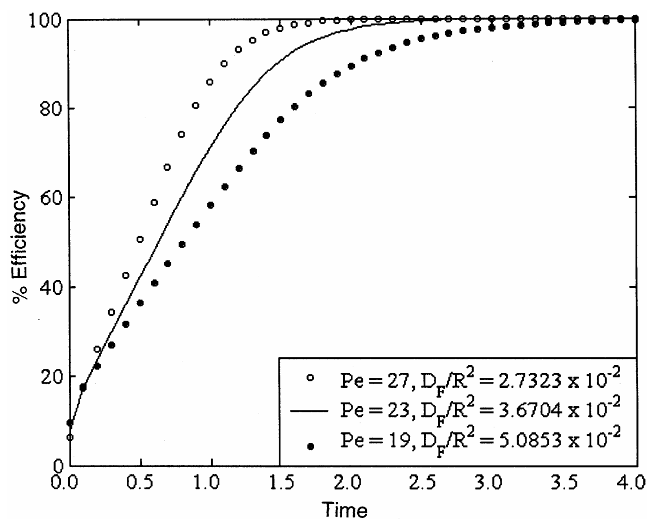

Fig. 7—Theoretical breakthrough curves of %efficiency for different values Peclet number and distribution ratio

The behaviour of solution profiles for %efficiency is shown for different values of Peclet number and distribution ratio in Fig. 7. It is observed that %efficiency approaches to the maximum limit for large Peclet number and small distribution ratio. Therefore, higher efficiency of the equipment can be achieved for large Peclet number and small distribution ratio.

Wash yield

Wash yield of any stage is the ratio of dissolved solids removed to the dissolved solids entering with the unwashed pulp. Kukreja *et al.*[5] have calculated the

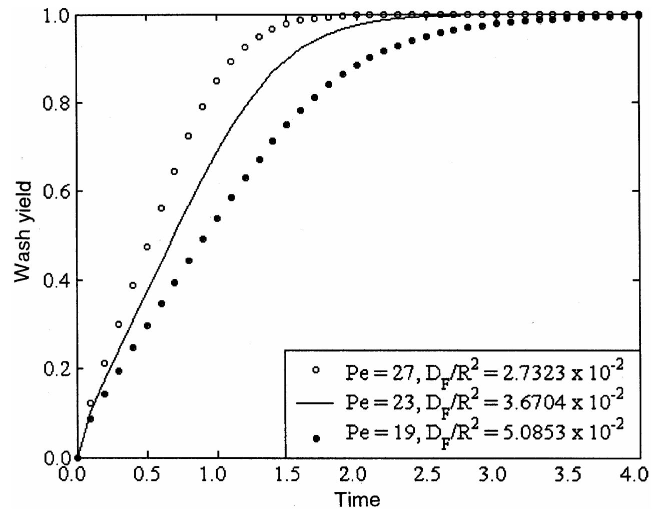

Fig. 8—Theoretical breakthrough curves of wash yield for different values of Peclet number and distribution ratio

wash yield by correlating it with %efficiency and wash ratio as:

$$WY = 1 - \exp(-\%efficiency \times WR) \quad \ldots(22)$$

For wash ratio equal to unity, wash yield is equivalent to the displacement ratio. Figure 8 gives the breakthrough curves of wash yield for different values of Peclet number and distribution ratio. It is observed that higher wash yield can be obtained for large Peclet number and small distribution ratio.

Conclusion

In this paper a mathematical model for washing zone of brown stock washer is proposed. The model includes the intrafiber diffusion as well as mass transfer resistances along with axial dispersion. On the basis of experimental results empirical relations have been found between %efficiency and concentration of black liquor solids in discharge pulp and between wash yield and concentration of black liquor solids in discharge pulp. The results are with in the range of ± 1%. On the basis of experimental results it has been found that bed porosity, fiber porosity and fiber consistency are dependent on cake thickness and changes linearly as the cake thickness changes. The efficiency parameters are also correlated with the model and it is observed that the efficient washing operations can be achieved for large Peclet number and small distribution ratio (D_F/R^2).

Acknowledgement

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Nomenclature

c	=	Concentration of solute in the liquor, kg/m^3
c_d	=	Concentration of solute in the discharged pulp, kg/m^3
C_d	=	Concentration of solute in the discharged pulp, dimensionless
C_F	=	Fiber consistency, kg/m^3
C_S	=	Shower liquor concentration, kg/m^3
C_{yd}	=	Outlet vat consistency, kg/kg
C_{yi}	=	Inlet vat consistency, kg/kg
C_0	=	Solute concentration in the vat, kg/m^3
D_F	=	Intrafiber diffusion coefficient, m^2/s
D_F/R^2	=	Distribution ratio, 1/s
D_L	=	Longitudinal dispersion coefficient, m^2/s
k	=	Mass transfer coefficient, dimensionless (k_1/k_2)
K	=	Volume equilibrium constant, dimensionless
k_f	=	Film mass transfer coefficient, m/s
k_1,k_2	=	Mass transfer coefficients, 1/s
L	=	Thickness of the bed, m
L_d	=	Amount of liquor in discharged pulp, kg of liquor/kg of pulp
L_i	=	Amount of liquor inside the vat, kg of liquor/kg of pulp
L_S	=	Amount of wash water, kg of water/kg of pulp
n	=	Concentration of solute adsorbed on the fibers, kg /kg
N_S	=	Shower concentration of solute adsorbed on the fibers, kg/kg
N_0	=	Initial concentration of solute adsorbed on the fibers, kg /kg
Pe	=	Peclet number, dimensionless (uL/D_L)
q	=	Pore liquid concentration, kg/m^3
r	=	Radial position in particle, m
R	=	Fiber radius, m
t	=	Time, s
u	=	Interstitial velocity through bed, m/s
z	=	Distance from point of introduction of solvent, m
β	=	Particle porosity, dimensionless
ε	=	Porosity of cake, dimensionless

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