Hub Covering Location Problem under Gradual Decay Function

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This paper developed a mathematical model framework using the single allocation strategy for hub covering location problem. The model encompassed the transportation time covering under gradual decay function. Within a certain transportation time for the origin and destination nodes among two hubs, the route is fully covered, and beyond another specified transportation time the route is not covered. Between these two given service times, the coverage is linear for the routes. A tabu search has been used to solve some problems such as CAB and AP. The quality of solutions obtained using TS have been compared with the achieved one using CPLEX solver. Plenty of experimental study evaluated the performance of the gradual decay function.

Keywords: Hub location, Gradual covering, Tabu search, Networks

Introduction

Hub-and-spoke networks have been implemented within the supply chains, telecommunication, air transportation networks, tourism routing, postal delivery networks and so forth in recent years. There are three important cases of hub location problem: (i) hub median, (ii) hub center and (iii) hub covering problem. In this research, the hub-to-hub allotment of the transshipment is discounted by a factor $0 \leq \alpha \leq 1$. $\alpha$ accounts for economies of scale that result from mass transportation between hub nodes. In the literature, two different kinds of the covering models are discerned: The hub location set covering (HSCP) models minimize the number of facilities required to cover service time for each O-D pair, notwithstanding the maximal hub covering problem (MHCP) maximizes the flow of O-D in the network with a given fixed number of hubs.

One troublesome assumption in the MHCP is the sudden interruption of coverage (i.e. O-Ds within the coverage transportation time are covered completely, while O-Ds beyond that time are not covered at all). This sounds to be not practical modeling supposition in numerous conceivable applications. This problematic assumption also occurs in the maximal covering location problem. Berman et al. presented this assumption and extend the previous model named gradual coverage decay model. To handle the troublesome assumption of the MHCP, the gradual covering idea is used in our research. In this model, the coverage provided for O-D via two hubs is assumed gradually diminishes according to some decay function, rather than suddenly. In this, a generalized MHCP with two coverage radii $\omega$ and $\beta$ ($\omega < \beta$), If an O-D can be covered with two hubs in less than time $\omega$, it will be fully covered, if service time is between $\omega$ and $\beta$, the O-D will be partially covered. Finally, if the time is more than $\beta$, the O-D will never be covered. The model in this paper is called gradual hub covering problem (GHCP).

HSCP and MHCP with single and multiple allocations are provided by Campbell. Afterwards, more researches such as Kara and Tansel, Tan and Kara, Qu and Weng, Calik et al., Karimi and Bashiri, Zarandi and Davari and Eghbalizarch et al. work on the hub covering location problem. As mentioned formerly, Berman et al. presented the gradual covering problem. The other researchers such as Drezner et al., Drezner et al. and Berman and Wang work on the gradual covering location model. Interested readers can see Farahani et al. and Berman et al. and the references therein which provide an overview of the covering location problem models.

GHCP applications motivate us to handle a model formulation beside a solution approach. This model solves some real world problems such as: Tourism service, Postal delivery system and Airline...
transportation. For the large-scale GHCP, however, we still may have recourse to heuristic approach. If every O-D pair happened to a single point, then the problem became a general gradual covering problem. It means that the gradual covering problem is a special case of GHCP. Maximal covering location problem is a particular occasion of the gradual covering problem. Accordingly, GHCP is NP-hard since the maximal covering problem has been proved NP-hard\(^5\). Therefore, we appeal to meta-heuristic approach.

Solution for maximal hub covering problem, which is the special case of GHCP, does not research more. The meta-heuristic solution approaches to find an excellent solution for large-scale GHCP, does not research more. The researches which applied TS are for example Klincewicz\(^15\) and Skorin-Kapov and Skorin-Kapov\(^16\). GA is used in Carello et al.\(^17\), Topcuoglu et al.\(^18\) and Kratica et al.\(^19\). Cunha et al.\(^20\) and Ernst and Krishnamoorthy\(^21\) are run SA for p-hub median problems.

The rest of this paper is organized as follows: Section 2 describes a modeling framework. Section 3 presents a TS based meta-heuristic solution approach. The results of the experimental study are reported in Section 4. In the end, Section 5 concludes this paper.

**Modeling structure**

We assume that there is a pair of radii \((\omega, \beta)\) specified for the problem. O-D \((i, j)\) is considered to be fully covered if \(t_{ij} = t_{ik} + \alpha t_{km} + t_{mj} \leq \omega\), where \(t_{ij}\) is the transportation time between O-D \((i, j)\). In this case, the amount of flow that is covered \(w_{ij}\) (Flow between O-D \((i, j)\)) in this route. O-D \((i, j)\) is regarded to be not covered if \(t_{ij} \geq \beta\). Eventually, O-D \((i, j)\) is considered to be partially covered if \(\omega \leq t_{ij} \leq \beta\). Let \(f(t_{ij})\) be a non-increasing function for \(t_{ij} \leq \omega\) with \(f(\omega) = 1\) and \(f(\beta) = 0\). Then, the amount of flow that is covered is given by \(f(t_{ij})w_{ij}\). Hence, the gradual covering matrix \(A = a_{km}^{ij}\) which is \(n \times n \times n \times n\) can now be expressed as follows:

\[
a_{km}^{ij} = \begin{cases} 
1 & \text{if } t_{ik} + \alpha t_{km} + t_{mj} \leq \omega \\
 f(t_{ij}) & \text{if } \omega < t_{ik} + \alpha t_{km} + t_{mj} \leq \beta \\
 0 & \text{o.w}
\end{cases} \quad \ldots (1)
\]

In the literature on the gradual covering problem, three decay functions are described. These are called as stepwise, linear and not concave functions. The linear coverage function which is employed in this research is described in (2).

\[
f(t_{ij}) = \frac{\beta - (t_{ik} + \alpha t_{km} + t_{mj})}{\beta - \omega} \quad \ldots (2)
\]

This research employs the linear decay coverage function which is adjacent to the reality of the hub location problem (i.e. Satisfaction of customer is normally decreasing in a linear manner\(^10\)).

The assumptions to construct our model are: 1) single allocation strategy, 2) interconnected hub nodes, 3) the triangle inequality for \(t_{ij}\). We define the decision variables of the model as follows: \(x_{ikmj} = 1\) if the node via hub \(k\) and \(m\) to node \(j\) under coverage constraint; otherwise \(x_{ikmj} = 0\). The model imposes these variables to be binary, but actually they are restricted as a real variable. \(z_{ik} = 1\) if node \(i\) allocated to hub at node \(k\); otherwise \(z_{ik} = 0\). \(z_{ik} = 1\) means node \(k\) is a hub. \(0 < y_{ij} \leq 1\) show the flow ratio of the O-D \((i, j)\) pair, which is gradually covered by located hubs.

The inputs of the model are \(p, \omega, \beta, w_{ij}\) and \(t_{ij}\) which defined heretofore. According to the mentioned variables and inputs, the GHCP is formulated and available on http://wp.kntu.ac.ir/hkarimi/files/ghcp_model.pdf.

In the GHCP, the objective function accounts for the maximization of the total flows between nodes, which covered (serviced) by located hubs. A constraint specifies that exactly \(p\) hubs would be chosen. Some constraints ensure that each O-D can be routed just via two hubs, which do not have to be different. Also, some constraints ensure that an assignment cannot be made unless there is a hub at node \(k\). Moreover, some constraints require that the flow of the O-D pair be serviced if only there were at least one or two hubs, which could fully or partially cover the flow. As before-mentioned, GHCP is NP-hard. Moreover, the number of variable and constraint accentuate that solving this model for a large problem is challenging, and for a small problem, the commercial solver may achieve the exact solution with reasonable computational time. Hence, we implement TS as a heuristic solution approach.

**Tabu search**

TS manages the solution method to escape from a local optimum and to shift to before unexamined areas of the solution space\(^16\). In our TS approach, a
heuristic procedure to allocate non-hub nodes to hub nodes in each move is proposed. We define total flow \( TF_{ik} \) for each non-hub and its potential hub as

\[
TF_{ik} = \sum_{j \in N} \sum_{m \in H} w_{ij} a_{ij} \quad \forall i \in N, k \in H \quad \ldots(3)
\]

Where \( N \) is set of nodes and \( H \) is set of selected hub. Then, for each non-hub, a hub node can be found by maximizing the total flow as follows:

\[
IS = \left\{ (i \in N, k \in H) \mid \max_k TF_{ik} = TF_{ik} \right\} \quad \ldots(4)
\]

where IS is an index set. Hence, the single allocation strategy reaches using allocate \( (I \in IS) = hub \ (k \in IS) \).

Using heuristics to solve the large-scale problems concern with the quality of the solutions obtained. The exact solutions in this research are gained by CPLEX solver. The pseudo-code of TS developed for this study is available on http://wp.kntu.ac.ir/hkarimi/files/pseudo_codets.pdf

**Computational Study**

Computational examination, in this section, wants to evaluate the performance of meta-heuristic solution approach by comparing it with optimal or best-known solution. The qualification examination of heuristic approaches needs to some practical test benchmarks. These instances affirm a range of problem attributes. In this research, CAB and AP is applied. All the tests for TS heuristic were implemented in MATLAB 7.5 and run in an experimental computer which is equipped with 3GB of RAM and a Pentium microprocessor running at 2.53 GHz.

The TS results for CAB display that the more available hubs impel much CPU time consumption. \( \beta \) and \( \alpha \) don’t impose the model to use much computational time in a specific number of hub. Obviously, the results indicate that there is a significant difference between TS and CPLEX for CAB dataset. ANOVA test is employed to compare these solution approaches. TS clearly leads to the higher objective function value than the CPLEX solver. The results tables and the useful figures can be downloaded from http://wp.kntu.ac.ir/hkarimi/files/paperresults.pdf

**Concluding remarks**

The impact of using a gradual covering decay function within the context of a hub-and-spoke network is researched in this paper. This study develops a modelling formulation which embodies the service time requirements in a single allocation hub-and-spoke network under gradual covering decay function. The linear decay function is considered in this paper, since it is more practical. Our model formulation can be implemented in tourism, postal delivery and airline services. Moreover, TS is presented as solution approach. The computational study shows that the TS achieves high-quality solutions in a reasonable CPU time (average optimality gap of 0.036%). The TS solutions are compared with the corresponding optimal objective function values which are obtained by CPLEX solver. The results reveal that for these problems, the solution approach yielded better solutions than the CPLEX in the same CPU time. This research identifies the impact of the number of hubs \( (P) \), discount factor \( (\alpha) \), first time service \( (\omega) \) and second time service \( (\beta) \) on the objective function value. When the value of the number of hubs, first time service and second time service are increased, the flow in the network can be more. In addition, by increasing discount factor, the objective function value will be reduced.

**References**


