Evaluation of linewidth (damping) of zero-sound mode in liquid $^3$He

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Using the phonon part of the effective Hamiltonian of Mishra and Tripathy [J Low Temp Phys, 75 (1989) 79], we have evaluated the linewidth $\Gamma_k$ of the zero-sound mode as a function of the momentum transfer $k$ for liquid $^3$He by two methods is evaluated. Introducing a parameter for local field correction, our calculated values of $\Gamma_k$ are good agreement with the experimental data.

1 Introduction

The interparticle potential in liquid $^3$He mainly consists of a strong hardcore repulsion plus a weak Van der Waals's attraction. Landau\(^1\) predicted two types of collective mode exist in this system. They are ordinary (first) sound mode and the zero-sound mode and arise due to density fluctuations and distortions of the Fermi sphere. Ordinary sound is a propagating mode in the hydrodynamics (collision dominated) regime, whereas zero-sound is a well defined mode in the collisionless regime. The Fermi sphere for ordinary sound just dilates and contracts spherically symmetrically, whereas the zero-sound distortion is concentrated in the longitudinal direction and the transverse radius of the Fermi surface is unaffected. The velocities of both these sound waves have been expermentally determined\(^2\). The zero-sound velocities also vary with pressure\(^3\). Zero-sound velocities are damped because the mode coupled to a more complicated excitations through the scattering of quasiparticles\(^4\). Inelastic neutron scattering experiments were conducted on liquid $^3$He by Skold \textit{et al.}\(^5\) to study its elementary excitation spectrum for wave vectors in the range $8 \text{ nm}^{-1} < k < 22 \text{ nm}^{-1}$. They found that at a temperature of 15 mk there exists a well defined zero-sound mode of energy $13k$ for wave vectors in the range $8 \text{ nm}^{-1} < k < 1.41 \text{ nm}^{-1}$. Subsequently inelastic neutron scattering experiments with improved resolution were repeated by Hilton \textit{et al.}\(^6\). There results not only confirmed the existence of a well defined zero-sound mode with frequencies in agreement with those obtained by Skold \textit{et al.}\(^5\), but also provided measurements for the linewidths (damping) of these excitations. Sherm \textit{et al.}\(^7\) performed the inelastic neutron scattering measurements on the liquid at 120 mk and pressure of $(0, 5, 10$ and $20) \times 10^5$ Pa for wave vectors in the range $3 \text{ nm}^{-1} < k < 20 \text{ nm}^{-1}$. This represents the first experimental information on the density dependence of the zero-sound frequency and damping at finite wave vectors. Very recently Fak\(^8\)\(^,\)\(^9\) performed high resolution neutron inelastic scattering measurements on liquid $^3$He at a temperature of 120 mk and pressure between 0 and 20 bars. It was found that at SVP the linewidth increases nearly linearly with the wave vector. For wave vectors smaller $k$ than about $0.7 \text{A}^{-1}$ there is a clear increase in the linewidth with increasing pressure, in agreement with a previous analysis\(^7\). For wave vectors larger than $0.8 \text{A}^{-1}$ the linewidth decreases with increasing pressure. However, the amount of decrease is model-dependent. Different damping mechanism of the zero-sound mode has been discussed. At low temperature where the number of thermally excited phonon (zero-sound) is negligible, there different decay modes are available. In one decay mode, a phonon decays into two other phonons. This is a three phonon process which is kinematically allowed only if the dispersion is anomalous. At SVP the present data show clear anomalous dispersion: the zero-sound linewidth increases smoothly with the wave vector. There is no decrease in the width at the wave vector where the dispersion becomes normal. These observations show that three phonon processes can be ruled out as the dominating decay channel.
for the zero-sound mode. The second decay mechanism is Landau damping. A phonon decays by exciting a single quasiparticle-hole pair. The particle-hole excitation will have the same wave vector and energy as the decaying phonon. Hence, this mechanism requires a direct overlap between the zero-sound dispersion and the particle-hole band. If the quasi-particle mass is enhanced \((m^*/m - 3)\) this mechanism is separated only for wave vectors about 1.4\(\text{Å}^{-1}\), and cannot explain the large width of the zero-sound peak observed at small wave vectors. On the other hand, if the quasi-particle mass is of the order of the bare mass, Landau damping could occur for small wave vectors as well. However, since the separation in energy the zero-sound mode and particle-hole band increases with increasing pressure while the zero-sound width at low wave vectors increases or remains constant, it seems that Landau damping is not the dominating decay mechanism. The third decay mechanism is multipair damping. Here a phonon decays into multiple pairs. Since several particle-hole excitations are created, the kinematical constraints are relaxed and the mechanism could be active for all wave vectors. This is the most dominant damping mechanism for the zero sound mode. Although there exists a large number of theoretical calculations on the zero-sound aspect, but none of them is found to be very satisfactory in evaluating the damping of the zero-sound mode.

Mishra and Tripathy\(^1\) have developed a theoretical model for liquid \(^3\)He, according to which liquid \(^3\)He is visualized as an interacting system of helium atoms and the quanta of its zero-sound mode. In this model, they have obtained an effective Hamiltonian for the system by applying a suitable canonical transformation and the Feynman diagrammatic technique. They have obtained a value of \(-2.55k\) for the average binding energy per particle against the experimental value\(^18\) of \((-2.47 \pm 0.01)\) \(k\). This theory also gives rise to a value of 14.1 \(\text{nm}^{-1}\) for the critical wave vector \(K_c\) which matches with experimental data\(^5\). In another work\(^19\), they have been able to reproduce an effective mass of \(^3\)He atom \((m^* = 2.99m)\) which exactly matches with the experimental data of Wheatley and co-workers\(^20\). They have also evaluated\(^21\) the velocity of zero-sound (204.2 m/s for liquid \(^3\)He at \(P = 0\) and \(T = 0\)) using a self-consistent method. An exact agreement with the experimental value\(^22\) was obtained by introducing a term into theory to account for the local field correction to the bare particle-hole propagator beyond the random phase approximation (RPA) in a parametric way. Taking the phonon part of the effective Hamiltonian, Singh and Mishra\(^23\) have determined the sound velocities as a function of pressure to obtain values in good agreement with the experimental data\(^24\). Singh et al.\(^25\) have also evaluated the temperature variation of the pressure dependent zero-sound velocities in liquid \(^3\)He. Very recently Singh and Mishra\(^26\) have determined the pressure-dependent effective mass of the \(^3\)He atom in normal liquid \(^3\)He and the results are in good agreement with the recent experimental data of Greywall\(^27\). Tripathy and Mishra\(^28\) have calculated the binding energy per nucleon in the ground state of nuclear matter with the same approach using the collective description of the inter-particle interactions. Mishra and Mishra\(^29\) have evaluated the energy of the zero-sound mode as a function of wave vector \(k\).

In this paper, we present a method of evaluation of the damping of zero-sound mode as a function of wave vector \(k\). By confining ourselves to the phonon part of the effective Hamiltonian of Mishra and Tripathy\(^17\), an equation for the phonon frequency has been obtained and solved. Writing down the renormalized phonon frequency in terms of its real and imaginary parts, damping has been introduced in the expression for the energy of the zero-sound mode. Taking the present calculated values\(^29\) for the phonon frequency for each value of \(k\), the energy equation has been solved self-consistently. The calculation is repeated by taking the experimental value of the phonon frequency\(^6\). In both cases, our calculated values for damping are in good agreement with that of the experimental data\(^6\).

### 2 Calculation of the Damping of the Zero-Sound Mode at \(P = 0\) and \(T = 0\)

The energy of the zero-sound mode as a function of the wave vector \(k\) can be obtained by solving the following equation\(^17\)

\[
\Omega(k)^2 = \Omega_{\text{ph}}(k) + 2 \Omega_{\text{ph}}(k) \text{Re} \pi_0 [\mathbf{k}, \Omega(k)]
\]

where \(\Omega(k)\) is the re-normalized zero-sound frequency, \(\Omega_{\text{ph}}(k)\) is the bare phonon in the \(k\)-th mode defined as:

\[
\Omega_{\text{ph}}(k) = \sqrt{\frac{n \nu(o) h}{m}}\]

The interparticle potential \(\nu(k)\) at \(k = 0\) is

\[

\nu(o) = 4\pi b^2 h^2/m
\]
where $b$ denotes the free particle scattering for a pair of helium atoms and $\Omega_0 [k, \Omega(k)]$ denotes the bare-particle-hole propagator evaluated as $\omega = \Omega(k)$. With the set of dimensionless unit as discussed in Ref. 17, we write
\[
\Omega_{ph}^{(o)}(k) = \Omega_0^{(o)}(k) = a_0 k \tag{3}
\]
where $\Omega_0^{(o)}(k)$ is the velocity of zero-sound in the lowest order.
\[
\Omega_0^{(o)}(k) = (4b/3\pi)^{1/2} = 1.846 \tag{4}
\]
Now $\beta \pi_0 [k, \Omega(k)]$ in the dimensionless unit assume the form:
\[
\beta \pi_0 [k, \Omega(k)] = [1 - \Omega_0^{(o)}(k)] \left[ \frac{3/8k^2}{(3/16k^3)} \right] \left[ \frac{(\omega(k) - k^2 - 2k)}{\omega(k) - k^2 + 2k} \right] - \left[ \frac{(\omega(k) + k^2 - 2k)}{\omega(k) + k^2 + 2k} \right] \tag{5}
\]
The expression given in Fig. 5 has been obtained for $T = 0$. We now write renormalized zero-sound frequency $\Omega(k)$ as of difference, real and imaginary part.
\[
\Omega(k) = \beta(k) - i r_k \tag{6}
\]
where $\beta(k)$ is the real part of $\Omega(k)$ and $r_k$ is the imaginary part known as zero-sound damping. Using Eq. (6), Eq. (1) assumes the form after separating out the real and imaginary part and taking only the real part as:
\[
w^2(k) = r_k^2 + \Omega_0^{(o)}(k) \left[ (\omega(k) - w_{ph}(k)) \sum_{\omega} \rho \right] \left[(\omega(k) - w_{ph}(k))^2 + \frac{1}{k^2} \right]
\]
Eq. (7) has been solved for each $k$ and for two sets of values of $\omega(k)$. The first set is our calculated values for $\omega(k)$ and the other set is the experimental value for $\omega(k)$. The calculated values of the linewidth $\Gamma_k$ are in poor agreement to that of the experimental data. In order to improve our result, we include the local field correction in a parametric way by writing Eq. (1) in the following way:
\[
\Omega_0^{(o)}(k) = \Omega_{ph}^{(o)}(k) + 2 \omega_k \Omega_{ph}^{(o)}(k) R_{\pi_0} [k, \Omega(k)] + \left( \delta/2 \right) a_0 k \tag{8}
\]
where $\delta$ is an adjustable parameter which accounts for local field correction.

Now using Eq. (6) and making dimensionless Eq. (8) assumes the form:
\[
w^2(k) = r_k^2 + \Omega_0^{(o)}(k) \left[ \log \frac{[w(k) - k^2 - 2p k]^2 + r_k^2}{[w(k) + k^2 - 2p k]^2 + r_k^2} \right] - \frac{w(k) r_k}{k^3} \left[ \tan^{-1} \frac{w(k) - k^2 + 2p k}{r_k} \right] + \tan^{-1} \frac{w(k) + k^2 - 2p k}{r_k} - \tan^{-1} \frac{w(k) + k^2 + 2p k}{r_k} \tag{9}
\]
For small $k$ the Eq. (9) reduces to:
\[
w^2(k) = r_k^2 + \Omega_0^{(o)}(k) \left[ \frac{2w^2(k)}{w^2(k) + r_k^2} \right] - \frac{12}{5} \frac{k^2}{(w^2(k) + r_k^2)^2} + \frac{4r_k^2}{w^2(k)} + \frac{8k^2 r_k^6}{w^6(k)} \left( \frac{136}{5} \frac{k^2 r_k^6}{w^8(k)} \right) \tag{10}
\]
Finally we solve Eqs. (9) and (10) self-consistently for different values of $\delta$ and for our calculated values of $\omega(k)$ and experimental values of $\omega(k)$ for each $k$. The results have been given in Table 1 and 2 for both these methods. We need $\delta = 5.44$ and $\delta = 6.76$ to match our calculated values of zero-sound linewidth $\Gamma_k$ with the experimental value in two different methods. Our calculated values of $\Gamma_k$ (meV) have been compared with other model calculations and with experiment in Table 3.
3 Discussion of the Results

In the present work, using the phonon part of the effective Hamiltonian, we have evaluated the linewidth

\[ \Gamma_k (\text{meV}) = \frac{\hbar}{2} \delta_n (\text{meV}) \]

Table I — Calculation of linewidth \( \Gamma_k \) taking our calculated values of \( w(k) \) (Ref. 29)

<table>
<thead>
<tr>
<th>( K (\text{Å}^{-1}) )</th>
<th>( \frac{W(k)}{\text{cal}} (\text{meV}) )</th>
<th>( \frac{\Gamma_k}{\text{cal}} (\text{meV}) )</th>
<th>( \frac{\Gamma_k}{\text{exp}} (\text{meV}) )</th>
<th>( \frac{\Gamma_k}{\text{ref}} (\text{meV}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.2948</td>
<td>0.1237</td>
<td>0.1024</td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>0.4465</td>
<td>0.1633</td>
<td>0.1327</td>
<td></td>
</tr>
<tr>
<td>0.3</td>
<td>0.5431</td>
<td>0.2311</td>
<td>0.2046</td>
<td></td>
</tr>
<tr>
<td>0.4</td>
<td>0.7068</td>
<td>0.3214</td>
<td>0.2789</td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>0.7344</td>
<td>0.3869</td>
<td>0.3054</td>
<td></td>
</tr>
<tr>
<td>0.6</td>
<td>0.7930</td>
<td>0.4217</td>
<td>0.3718</td>
<td>0.27 ± 0.04</td>
</tr>
<tr>
<td>0.7</td>
<td>0.8060</td>
<td>0.5086</td>
<td>0.4132</td>
<td>0.32 ± 0.05</td>
</tr>
<tr>
<td>0.8</td>
<td>0.8706</td>
<td>0.5213</td>
<td>0.4372</td>
<td>0.36 ± 0.05</td>
</tr>
<tr>
<td>0.9</td>
<td>0.9482</td>
<td>0.5847</td>
<td>0.5416</td>
<td>0.55 ± 0.05</td>
</tr>
<tr>
<td>1.0</td>
<td>1.0430</td>
<td>0.6289</td>
<td>0.5916</td>
<td>0.78 ± 0.07</td>
</tr>
</tbody>
</table>

(damping) of zero-sound mode as a function of momentum transfer \( k \) for liquid \(^3\)He. In one method, we use our calculated values of zero-sound frequency \( w(k) \text{cal} \) and in another method we take the experimental \( w(k) \text{Exp} \) value for each \( k \). To get a good agreement with the experimental data, we use an adjustable parameter which accounts for the local field correction. Our calculated values for \( \delta = 5.44 \) in the second method [using \( w(k) \text{Exp} \)] are in better agreement with the experimental data. Also calculated values \( \Gamma_k \) (in both these methods) increases with the wave vector \( k \). We have not found any decrease in the linewidth at any \( k \). Our evaluated values of \( \Gamma_k \) appears to be in agreement with the analysis of Paramaynow\(^9\),\(^31\) model \( (m^*/m = 1) \) and a calculation in which an effective mass has \( k \) dependence \(^9\),\(^32\) \[ m^*(k) \text{ for } p = 0 \]. The model potential chosen by us in a contact interaction potential which is totally repulsive in nature. However, the interaction potential for liquid \(^3\)He contains both repulsive and attractive part. It is our firm believe that taking more realistic potential for liquid \(^3\)He which contains both attractive and repulsive part, we may be able to notice decrease of linewidth \( \Gamma_k \) for some \( k \) up to \( k_c \) as per experimental observation\(^8\),\(^9\). The work is under investigation.

Table 2 — Calculation of linewidth \( \Gamma_k \) taking experimental value of \( W(k) \text{Exp} \) (Ref. 6)

Fermi energy \( E_F = 4.96 \text{ K} \), Fermi Wave vector \( k_F = 0.786(\text{Å}^{-1}) \)

<table>
<thead>
<tr>
<th>( K/k_F )</th>
<th>( K(\text{Å}^{-1}) )</th>
<th>( W(k) \text{Exp} (\text{meV})(\text{Ref.6}) )</th>
<th>( \Gamma_k (\text{meV}) )</th>
<th>( \Gamma_k (\text{meV}) )</th>
<th>( \Gamma_k (\text{meV}) )</th>
<th>( W(k) \text{Exp} (\text{meV}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>7.86 \times 10^{-3}</td>
<td>0.0116</td>
<td>0.064</td>
<td>0.0630</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.10</td>
<td>0.076</td>
<td>0.1082</td>
<td>0.0812</td>
<td>0.0805</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.20</td>
<td>0.1572</td>
<td>0.2078</td>
<td>0.0855</td>
<td>0.0834</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.30</td>
<td>0.2358</td>
<td>0.3412</td>
<td>0.1288</td>
<td>0.1124</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.40</td>
<td>0.3144</td>
<td>0.4712</td>
<td>0.1894</td>
<td>0.1632</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.50</td>
<td>0.3930</td>
<td>0.6512</td>
<td>0.3078</td>
<td>0.2715</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.61</td>
<td>0.4800</td>
<td>0.72 ± 0.03</td>
<td>0.3206</td>
<td>0.2910</td>
<td>0.27 ± 0.04</td>
<td></td>
</tr>
<tr>
<td>0.64</td>
<td>0.5300</td>
<td>0.82 ± 0.03</td>
<td>0.4319</td>
<td>0.4004</td>
<td>0.32 ± 0.05</td>
<td></td>
</tr>
<tr>
<td>0.74</td>
<td>0.5800</td>
<td>0.875 ± 0.03</td>
<td>0.4737</td>
<td>0.4116</td>
<td>0.27 ± 0.06</td>
<td></td>
</tr>
<tr>
<td>0.80</td>
<td>0.6300</td>
<td>0.93 ± 0.03</td>
<td>0.4960</td>
<td>0.4252</td>
<td>0.36 ± 0.05</td>
<td></td>
</tr>
<tr>
<td>0.90</td>
<td>0.7100</td>
<td>1.03 ± 0.03</td>
<td>0.5857</td>
<td>0.5020</td>
<td>0.55 ± 0.05</td>
<td></td>
</tr>
<tr>
<td>0.99</td>
<td>0.7800</td>
<td>1.10 ± 0.03</td>
<td>0.6200</td>
<td>0.5462</td>
<td>0.48 ± 0.02</td>
<td></td>
</tr>
</tbody>
</table>
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References