Electric field dependence of microwave losses in strontium titanate

U C Nathani, N S Panwar & B S Semwal
Department of Physics, HNB Garhwal University Campus Pauri, Pauri (Garhwal), UP 246 001

Received 6 August 1998; revised 28 December 1999; accepted 7 February 2000

Electric field dependent microwave losses in pure single strontium titanate crystal have been theoretically calculated at microwave frequency 5 GHz. The loss tangent consists of a contribution, which is quadratic in applied biasing plus a field independent contribution. This variation of tan δ with applied field is noticeable in the vicinity of the Curie temperature. In the high temperature region, the field’s effect ceases and the increase in loss arises mainly due to higher order anharmonic terms.

1 Introduction

Ferroelectrics have been a subject of considerable interest because of their wide practical applications in various fields like ceramic industry, optical communication, memory display, holographic storage media, etc. Microwave losses in single crystalline strontium titanate are surprisingly low above the Curie temperature and thus SrTiO 3 is potentially a good substance to study the loss mechanism in dielectric materials. SrTiO 3 is commercially available in relatively high purity as single crystals.

In a previous paper 1 (hereafter referred to as I), studied the functional dependence of the microwave loss tangent have been theoretically on parameters such as temperature and frequency, using Green’s function method 3 in pure anharmonic ferroelectric SrTiO 3 crystal with the help of Pytte’s modified Hamiltonian 4, considering anharmonic effects up to fourth order and the results compared with that of others. The aim of the present paper is to discuss the variation of loss tangent (tan δ) at microwave frequency with the applied external electric field in this perovskite.

It is now well known that several interesting temperature dependent properties of ferroelectrics result from the temperature dependence of the low lying transverse optic mode of vibration 5. One of the very interesting property of these crystals is the electric field dependence of the low frequency transverse optic modes. A considerable review is available in the literature 6-14 and references therein. The electric field dependence of the ferroelectric soft mode is studied by augmenting the Hamiltonian with a fourth order term in the phonon coordinates and transforming this Hamiltonian with the help of a unitary transformation. The dynamics of these soft modes is studied in the presence of external electric field induced phonon scattering processes, which are not present in the absence of an electric field.

2 Hamiltonian and the Green’s Functions

The Hamiltonian which includes the anharmonicity up to fourth order in the potential energy due to interactions of the soft mode coordinates, resonant interactions, scattering and electric dipole moment terms is considered. Hamiltonian, using the approach of I (for zero field case) in the form of soft mode coordinates can be written as:

$$ H = \frac{1}{4} \sum_{k} \omega_{k}^{2} (A_{k}^{+} A_{k}^{+} A_{k}^{+} B_{k}^{+} B_{k}^{+}) + \frac{1}{4} \sum_{k} \omega_{k}^{2} (A_{k}^{+} A_{k}^{+} A_{k}^{+} B_{k}^{+} B_{k}^{+}) + \frac{1}{4} \sum_{k} \omega_{k}^{2} (A_{k}^{+} A_{k}^{+} B_{k}^{+} B_{k}^{+}) + (\Gamma_{m}^{2}/2) \sum_{k} A_{k}^{+} A_{k}^{+} A_{k}^{+} A_{k}^{+} + (\Gamma_{s}^{2}/4) \sum_{k} A_{k}^{+} A_{k}^{+} A_{k}^{+} A_{k}^{+} + \frac{1}{2} \sum_{k} V^{2} A_{k}^{+} A_{k}^{+} A_{k}^{+} A_{k}^{+} + (\Gamma_{v}^{2}/2) A_{k}^{+} A_{k}^{+} + (\Gamma_{v}^{2}/4) A_{k}^{+} A_{k}^{+} + \sum_{k} H^{2} A_{k}^{+} A_{k}^{+} A_{k}^{+} A_{k}^{+} + \sum_{k} H^{2} A_{k}^{+} A_{k}^{+} A_{k}^{+} A_{k}^{+} + \sum_{k} C(k, k, k) A_{k}^{+} A_{k}^{+} A_{k}^{+} A_{k}^{+} + \sum_{k, k, k} D(k, k, k) A_{k}^{+} A_{k}^{+} A_{k}^{+} A_{k}^{+} - D(\alpha, \beta, \gamma) A_{k}^{+} A_{k}^{+} A_{k}^{+} A_{k}^{+} \ldots (I)$$
or, one can write:

\[ H = H + H' . \]

Here Hamiltonian \( H \) and the notation used are the same as used in 1. In brief the first two terms in Eq. (1) above represent the harmonic contributions to the energy from all modes other than those which become unstable in the harmonic approximation. The third term gives contribution to the energy from the modes which become unstable at the transition temperature, \( \Gamma'' \); above represent the third and fourth-order anharmonic force constants, \( V' \) describes the total interaction of the displacements, \( H' \) denotes the scattering terms and the terms within mother brackets are electric dipole moment terms. The rest are the cross terms of anharmonic force constants, resonant iteration and scattering terms with electric dipole moment terms. The rest are the cross terms of anharmonic force constants, resonant iteration and scattering terms with electric dipole moment terms. The primed summation in \( H \) includes \( K = 0 \). The term \( H_{1} \) is the Hamiltonian which contains the electric moment terms. In order to include the effect of electric field of the various dynamic properties, it is found that the most significant electric field dependent term (-\( \alpha \varepsilon E \cdot \vec{A}_{e} \)) in \( H \) is linear in \( \vec{A}_{e} \) and will give no contribution if the Hamiltonian is treated without transformation. Such transformations have been used by many workers, whenever the effect of electric field on some properties of ABO type perovskites is desired. The authors have previously used this technique in several studies. Such a transformation is a mathematical tool to study theoretically the electric field dependence of the soft mode frequency and hence various dynamical properties of these perovskites. The present study will help in understanding the facts and ideas like fundamental mechanism of the phase transition, the effect of electric field on the phase transition mechanism, critical behaviour near the phase transition etc. in ABO type ferroelectric perovskites.

On transforming \( H \) with the help of transformation operator \( S = i g E B^{\rho \omega} \), which generates unitary transformation according to general scheme:

\[ H_{1} = \exp (i \tau S) H \exp (i \tau S) \]

\[ = H + [ H, S ] - \frac{1}{2} [ [ H, S ] , S ] + ... \]

The transformed modified Hamiltonian becomes:

\[ H_{1} = \sum_{k} \omega_{k}^{\rho} \left( A_{k}^{\rho} A_{k}^{\omega} + B_{k}^{\rho} B_{k}^{\omega} \right) \]

\[ + \sum_{k} \frac{1}{4} \omega_{k}^{\rho} \left( A_{k}^{\rho} A_{k}^{\omega} + B_{k}^{\rho} B_{k}^{\omega} \right) \]

\[ + \frac{1}{2} \sum_{k} V' A_{k} \cdot A_{k} + \frac{1}{2} A_{k} \cdot F' A_{k} A_{k}^{\omega} \]

\[ + \frac{1}{2} A_{k} \cdot F' A_{k} A_{k}^{\omega} + \frac{1}{2} H A_{k} A_{k}^{\omega} \]

\[ + E' \sum_{k} ( k, k_{2}, k_{3}, k_{4}) A_{k} \cdot A_{k}^{\omega} + \sum_{k} \sum_{k'} ( k, k', k_{2}, k_{3}, k_{4}) A_{k} \cdot A_{k} \cdot A_{k} \cdot A_{k} \cdot A_{k}^{\omega} \]

\[ + E' \sum_{k} ( k, k_{2}, k_{3}, k_{4}) A_{k}^{\rho} A_{k}^{\omega} + \sum_{k} \sum_{k'} ( k, k', k_{2}, k_{3}, k_{4}) A_{k}^{\rho} A_{k}^{\omega} A_{k}^{\rho} A_{k}^{\omega} \]

\[ + \frac{1}{2} \sum_{k} V' A_{k}^{\rho} A_{k}^{\omega} + \frac{1}{2} A_{k}^{\rho} A_{k}^{\omega} + \frac{1}{2} H A_{k}^{\rho} A_{k}^{\omega} \]

\[ + E' \sum_{k} ( k, k_{2}, k_{3}, k_{4}) A_{k}^{\rho} A_{k}^{\omega} + \sum_{k} \sum_{k'} ( k, k', k_{2}, k_{3}, k_{4}) A_{k}^{\rho} A_{k}^{\omega} A_{k}^{\rho} A_{k}^{\omega} \]

\[ + \frac{1}{2} A_{k}^{\rho} A_{k}^{\omega} + \frac{1}{2} H A_{k}^{\rho} A_{k}^{\omega} \]

Here the notations used are exactly similar as used in our previous calculation\(^{10}\). For the study of dielectric constant and loss tangent, the following Green’s function is introduced:

\[ G_{\omega} ( t-t') = \langle <A_{\omega} ( t) A_{\omega} ( t') > \rangle \]

Writing equation of motion for the Green’s function [Eq. (3)] with the help of modified transformed Hamiltonian [Eq. (2)], Fourier transforming it and writing it in the form of Dyson’s equation, one obtains:

\[ G_{\omega} ( \omega, E ) = \]

\[ \frac{\omega / \pi}{\omega - \Omega} + \frac{\omega / \pi}{\omega - \Omega} - 2 \omega / \pi \Gamma ( \omega, E ) \]

Here \( \Omega = \omega_{0}^{\rho} + 2 \omega_{0}^{\omega} \Delta ( \omega, E ) \)

where

\[ \omega_{0}^{\omega} ( \omega ) = \omega_{0}^{\omega} + \frac{1}{2} F ( t ) : A_{\omega}^{\rho} ( t - t' ) \]

with

\[ \omega_{0}^{\rho} = \omega_{0}^{\rho} \left[ \omega_{0}^{\omega} + 2 V + 12 \Gamma_{1} g E + 24 \Gamma_{2} g E \right] \]

\[ + 4 \Gamma_{2} \left[ \left( 1 + N \right)^{2} \right] \right] \Omega / \omega_{0}^{\omega} - \left( 2 \Omega / \omega_{0}^{\omega} \right)^{2} \]

\[ \omega_{0}^{\omega} - \left( 3 \Omega / \omega_{0}^{\omega} \right)^{2} \]

and \( \Delta ( \omega, E ) = 18 \Gamma_{1} g E \left[ 2 \Omega / \omega_{0}^{\omega} - \left( 2 \Omega / \omega_{0}^{\omega} \right)^{2} \right] \]

\[ + 4 \Gamma_{2} \left[ \left( 1 + N \right)^{2} \right] \right] \Omega / \omega_{0}^{\omega} - \left( 3 \Omega / \omega_{0}^{\omega} \right)^{2} \]

\[ - \left( 1 - N^{2} \right) \Omega / \omega_{0}^{\omega} - \left( 3 \Omega / \omega_{0}^{\omega} \right)^{2} \]
\[ +16 \sum H^2_i \left[ (N_i^{+}N_i^{-}) (\bar{\Omega}_{i}^{+} + \bar{\Omega}_{i}^{-}) \right] / \]

\[ + \left[ \omega^2 - (\bar{\Omega}_{i}^{+} + \bar{\Omega}_{i}^{-}) \right] + \left[ 288 \Gamma_i \Omega_i^2 k^2 E^2 \right] \]

\[ + 4 E \sum B^2_i (k)^2 2 \bar{N} \left( 2 \bar{\Omega}_{i}^{+} / \omega^2 - (2 \bar{\Omega}_{i}^{-}) \right) \]

\[ + \left[ (-4 A \Lambda_i^2 \Sigma \ndot F_i \right]^2 + \]

\[ 64 g^2 E^2 \left( \sum H_i^2 / \Omega_i^2 \right) N \left( \Omega_i + (\omega^2 - \Omega_i) \right) \] ... (5a)

or

\[ \Delta(T,E) = AT + BT^2 + CTE^2 \] ... (5b)

and

\[ \Gamma(\omega, E) = 9 \pi \Gamma_i N \left[ \delta (\omega - 2 \bar{\Omega}_{i}^{+}) - \delta (\omega + 2 \bar{\Omega}_{i}^{-}) \right] \]

\[ 2 \pi \Gamma_i^2 \left[ (1 + 3 N^2) \left[ \delta (\omega - 3 \bar{\Omega}_{i}^{+}) - \delta (\omega + 3 \bar{\Omega}_{i}^{-}) \right] \right] \]

\[ + (2 \not\omega - 2 \bar{\Omega}_{i}^{+}) - \delta (\omega + 2 \bar{\Omega}_{i}^{-}) \right) \]

\[ + \pi \left[ (-2 A \Lambda_i^2 \Sigma \ndot F_i \right]^2 + \]

\[ 64 g^2 E^2 \left( \sum H_i^2 / \Omega_i^2 \right) N \left[ \delta (\omega - 2 \bar{\Omega}_{i}^{+}) - \delta (\omega + 2 \bar{\Omega}_{i}^{-}) \right] \] ... (6a)

or

\[ \Gamma(T,E) = AT + BT^2 + CTE^2 \] ... (6b)

Eqs (5b) and (6b), above show the temperature and electric field dependence of shift and width respectively, of optic phonons in frequency response.

3 Soft Mode Frequency

The soft mode frequency for optic mode [Eqs (4a) to (4c)] may be written as:

\[ \bar{\Omega}_{i}^{+} (\omega) = \omega_i^{+} + 2 \omega_i^{+} + 2 V + 12 \Gamma_i R E + 24 \Gamma_i \Omega_i^2 k^2 E^2 \]

\[ + \left[ F_i (r) : \Lambda_i^{+} (r') \right] > \]

\[ + \frac{\omega_i^{+}}{2} \left( 3 \bar{\Omega}_{i}^{+} / \omega^2 - (2 \bar{\Omega}_{i}^{+}) \right) \]

\[ + 8 \Gamma_i^2 \left[ (1 + 3 N^2) \Omega_i^2 / \omega^2 - (3 \bar{\Omega}_{i}^{+}) \right] \]

\[ -(1 + N^2) \Omega_i^2 / \omega^2 - (3 \bar{\Omega}_{i}^{+}) \] \]

\[ + 32 \sum H_i^2 \left[ (N_i^{+}N_i^{-}) (\bar{\Omega}_{i}^{+} + \bar{\Omega}_{i}^{-}) \right] \]

\[ + \left( \omega^2 - (\bar{\Omega}_{i}^{+} + \bar{\Omega}_{i}^{-}) \right] + \left[ 576 \Gamma_i \Omega_i^2 k^2 E^2 \right] \]

\[ + 8 E \sum B^2_i (k)^2 2 \bar{N} \left( 2 \bar{\Omega}_{i}^{+} / \omega^2 - (2 \bar{\Omega}_{i}^{-}) \right) \]

\[ + \left[ (-8 A \Lambda_i^2 \Sigma \ndot F_i \right]^2 + \]

\[ 128 g^2 E^2 \left( \sum H_i^2 / \Omega_i^2 \right) N \left[ \omega^2 - (\bar{\Omega}_{i}^{-}) \right] \]

or

\[ \bar{\Omega}_{i}^{+} (\omega) = (\omega_i^{+})^2 + 2 \omega_i^{+} V + \gamma_i T + \gamma_i E \]

\[ + \gamma_i T^2 + \gamma_i E + \gamma_i E^2 \] ... (7)

where \(\gamma_i, \gamma_i\) are coefficients of \(T\) and \(E\) is of \(T^2\) and \(\gamma_i, \gamma_i\) are the coefficients of \(E\) and \(E^2\) respectively, in \(\Delta(\omega_i E_i)\) [Eq. 5a]. The coefficients \(\gamma_i\) and \(\gamma_i\) depend upon third order anharmonic force constants and electric moment terms respectively. The coefficient \(\gamma_i\) depends upon fourth order anharmonic force constants, while \(\gamma_i\) and \(\gamma_i\) are the electric moment dependent terms.

The square of the soft mode frequency varies as the square of the applied electric field in agreement with the experimental results). Eq. (7) gives the temperature and electric field dependence of the soft mode frequency in a ferroelectric crystal in presence of an external electric field, besides the field and temperature dependences through re-normalized frequencies. The soft mode frequency which is imaginary in harmonic approximation, is stabilized in the presence of higher order anharmonic and electric moment terms. The \(T\) and \(E\) dependences of \(\bar{\Omega}_{i}^{+} (\omega)\) are due to third- and fourth-order anharmonicities respectively. The influence of electric field on this mode also affects the interaction of soft mode with other modes, thus giving rise to electric field dependence of various dynamical properties. Eq. (7) above can be rewritten as

\[ \bar{\Omega}_{i}^{+} (\omega) = \alpha + \beta T + \gamma E^2 \] ... (8)

where \(\alpha = (\omega_i^{+})^2 + 2 \omega_i^{+} V + \gamma_i E + \gamma_i E^2\)
\[ \beta' = \gamma + r' \]
and \[ r' = \gamma \]
Eq. (8) can be rewritten as:
\[ \Omega^{-2} = \beta'[T + \alpha' r' + \gamma'/\beta'T'] \]
or
\[ \Omega^{-2} = K(T - T'c + \epsilon'T') \]  ...(9)
where
\[ \beta' = K, \alpha' r' = -T'c \text{ and } \gamma'/\beta' = \epsilon' \text{ (the nonlinearity constant).} \]

4 Microwave Absorption

For a crystal model considered here, using Green's function [Eq. (3)] for microwave photons \( \omega << \omega + \omega_{0} \), one can write the expression for loss tangent as:
\[ \tan \delta = \frac{G^* (\omega)}{G (\omega)} = \frac{T (\omega)}{\omega} \]  ...(10)
Neglecting the change in Curie temperature due to applied electric field, the expression for loss tangent takes the form:
\[ (T - T_{c}) \tan \delta = \alpha + \beta T + \gamma T^{2} \]  ...(11)
The parameters \( \alpha, \beta, \gamma \) are given by:
\[ \alpha = 0 \]  ...(11a)
\[ \beta = -\frac{\epsilon_{0}}{A + CE} \beta' \]  ...(11b)
and \[ \gamma = -B \beta' \]  ...(11c)

Here it is seen that \( \alpha = 0 \) for a pure single crystal, \( \beta \) and \( \gamma \) are third- and fourth-order anharmonic interaction terms, respectively and are temperature dependent and they vary linearly with frequency \( \omega \). Expressions (11a) to (11c) show that \( \beta \) and \( \gamma \) are modified in the presence of an external electric field. For a pure crystal at a constant temperature, in paraelectric phase the loss tangent [Eq. (10)] can be expressed as \( \tan \delta = \tan \delta_{0} + \tan \delta' \), where \( \tan \delta_{0} \) is the field independent loss tangent and has been well discussed in I. In deriving the expression for loss tangent, different authors have used different methods, approximations and symbols. In the treatment adopted here, the authors have used Green's function technique and the Dyson's equation treatment has been found convenient to derive the shift and width of the frequency response function and hence to describe the properties of ABO3 type ferroelectrics. Inserting the values of \( \alpha, \beta \) and \( \gamma \) \( (0, 2.9556 \times 10^{-5} \text{ (GHz)}^{-1}, 1.55 \times 10^{-1} \text{ (GHz)}^{-1} \text{ K}^{-1} \text{ respectively}), T_{c} = 37 \text{ K and } T_{m} = 160 \text{ K} \) for SrTiO3 pure single crystal, one obtains, \( \tan \delta = 0.626 \times 10^{-3} \omega \), where \( \omega \) is in GHz. A linear variation of the loss tangent with the frequency obtained by the present authors is in agreement with all previous theoretical as well as experimental results. In weak fields the value of loss tangent of SrTiO3 increases and passes through a maximum when the temperature approaches the ferroelectric phase transition point. Even in weak fields the losses near the Curie temperature increase due to high domain mobility. Lowering of temperature decreases the loss, because this reduces the domain mobility. The nonlinear dependence of the polarization on the field, observed when strong fields are applied to ferroelectrics, gives rise to dielectric hysteresis loops. In general within a wide range of frequencies (right up to \( 10^{6} \text{ cps} \)) one can select a field which produces complete alignments of the domains along the field.

The term \( \tan \delta_{0} \) is the field dependent loss and increases with an increasing electric field [Eq. (11)]. The \( E' \) dependence of dielectric loss is evident from Eqs. (7) and (11). Such type of \( E' \) dependence has been reported by Rupprecht et al. They have measured the loss tangent of single SrTiO3 from 3 to 37 GHz, over a temperature range from 80 to 475 K. The field dependent part of \( \tan (\uparrow \text{ to } 7 \text{ GHz}) \) is proportional to the square of the electric field and to the frequency of the microwave field.

Here the electric field dependence of \( T_{c} \) is neglected in order to show the variation. The variation of \( \tan \delta_{0} \) with \( E \) will be effected by change in \( T_{c} \), due to the electric field. The temperature dependence of \( \tan \delta_{0} \) is shown qualitatively, taking electric field as a parameter in the paraelectric phase (Fig. 1). The anisotropic constant which is a measure of anharmonicity is used here, as has been reported by Rupprecht et al., at a frequency 5 GHz. It is evident from the figure that in a ferroelectric crystal, the loss tangent increases with the applied electric field. The field's effect is noticeable near the Curie temperature and at high temperature the \( E' \) effect of the field ceases. This \( E' \) dependence is valid for a frequency range 2.3-6.5 GHz and a temperature of 90 K. Thus one may conclude that the field's effect is appreciable in the vicinity of the Curie temperature \( (T_{c}) \) and in the high temperature limit the anharmonicities are mainly responsible for the increase in loss tangent. In the vicinity of the Curie temperature the microwave loss increases anomalously. The soft mode frequency \( (2 \alpha(T-T_{c})^{1/2}) \), if the temperature is not too high is held responsible for this.
Fig. 1 — Temperature dependence of field dependent loss tangent (tan$\delta$) in single crystal strontium titanate, in the para-electric phase.

5 Conclusions

The expression for microwave absorption in anharmonic displacive ferroelectric crystals has been obtained, in presence of an external electric field. The transformed Hamiltonian [Eq. (2)] has been used in the calculations. This transformation is necessary because the electric field dependence of the soft mode frequency and hence the dielectric loss is governed by $g(=\alpha/\omega_0)$. If the term $(-\alpha E\delta_0)$ in the Hamiltonian [Eq. (1)] is treated without transformation, it will give zero Green’s function. Such transformation have been used by many workers, whenever the effect of electric field on some properties of displacive ferroelectrics is desired. The soft modes due to their large occupation number, should cause an appreciable scattering of other modes. It may be concluded that higher order anharmonic terms in the Hamiltonian stabilize the soft mode frequency to the lower temperature region in presence of electric field [Eq. (7)]. The soft ferroelectric mode, which has an imaginary frequency in the harmonic approximation, is re-normalized and the lowest order anharmonic interaction that can stabilize the mode is of fourth order. The square of the effective soft mode frequency varies as the square of the electric field, in agreement with experimental results. The influence of electric field on this mode also affects the interaction of soft modes with other modes in the presence of higher order anharmonic terms, thus giving electric field dependence of various dynamic properties, such as thermal conductivity, specific heat, attenuation constant, dielectric constant and loss tangent, etc. The soft mode contribution to these properties is particularly important in the vicinity of $T_c$ and is expected to give anomalous behaviour.

It follows from Sec 4, that microwave loss tangent strictly depends on the anharmonicity and electric field through $\beta (=\gamma_1+\gamma_2)$ and the field affects tan only in presence of the anharmonicity. Neglecting the change in Curie temperature due to the applied fields, above the phase-transition temperature, the results of loss tangent can be well represented by the temperature dependence of the dielectric loss tangent [Eq. (11)]. The parameter has zero value in pure crystal, which is in agreement with the previous experimental results. The extrinsic parameter $\beta$ and $\gamma$ arise due to three and four phonon anharmonic interaction terms of the lattice and are modified by the electric field. The to dependence of tan $\delta$ and hence of $\beta$ and parameters is obtained and is in agreement with the previous findings. In the vicinity of the Curie temperature $T_c$, soft mode frequency $Q$ becomes anomalously large, thus making $\Gamma(\omega)$ anomalously large. Thus the microwave loss increases anomalously. Soft mode may be held responsible for this anomalously increasing behaviour. The qualitative variation of tan $\delta$, taking electric field as a parameter is shown by making use of anisotropic constants, which is a measure of anharmonicity. The field’s effect is noticeable near the Curie temperature. At high temperatures the contribution due to higher order anharmonicities dominates over the contribution due to electric moment terms. Recently the authors have applied the Green’s function technique to KDP type ferroelectrics also.

References
<table>
<thead>
<tr>
<th>Reference</th>
<th>Authors</th>
<th>Journal/Book/Conference</th>
<th>Volume</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Bull Am Phys Soc.</td>
<td>6 (1961)</td>
<td>12</td>
</tr>
<tr>
<td>13</td>
<td>Rupprecht G &amp; Bell R O.</td>
<td>Phys Rev.</td>
<td>125 (1962)</td>
<td>1915</td>
</tr>
<tr>
<td>18</td>
<td>Nettleton R E.</td>
<td>Anu Phys.</td>
<td>20 (1967)</td>
<td>136</td>
</tr>
<tr>
<td>19</td>
<td>Silverman B D &amp; Joseph R I.</td>
<td>Phys Rev.</td>
<td>129 (1963)</td>
<td>2061</td>
</tr>
</tbody>
</table>