MHD effects on heat and mass transfer in flow of a viscous fluid with induced magnetic field

N P Singh

Department of Mathematics, CL Jain (PG College, Firozabad-283 203)

and

Atul Kumar Singh

Department of Mathematics, VSSD College, Kanpur 208 002

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The present study is concerned with flow of an incompressible electrically conducting viscous fluid past a steady moving semi-infinite vertical porous plate under the action of uniform transverse magnetic field taking into account the induced magnetic field and constant heat flux. The analytical solutions for velocity field, temperature distribution, induced magnetic field, current density, concentration field and the expressions for shear stress and rate of heat transfer are obtained and discussed in detail.

1 Introduction

Many transport processes exist in nature and in industrial application in which the transfer of heat and mass occurs simultaneously as a result of combined buoyancy effects of thermal diffusion and diffusion of chemical species. In the last decades an intensive research effort has been devoted to problems on heat and mass transfer in view of their application to astrophysics, geophysics and engineering. The phenomenon of heat and mass transfer is also encountered in chemical process industries such as food processing and polymer products. In engineering applications, the concentration differences are created by either injecting the foreign gases or by coating the surface with evaporating material which evaporates due to the heat of the surface. In practice, hydrogen, helium, water-vapour, oxygen, ammonia etc. are the foreign gases which are injected in the air flowing past bodies. Thus, for flows past vertical surfaces, there are buoyancy forces which arise due to temperature differences and concentration differences.

Several authors have analysed the flow past an infinite vertical plate by the presence of free convective flow. Raptis and Soundelgekar studied MHD flow of an electrically conductive fluid past a steady moving vertical porous plate. Singh and Rathor extended this study to discuss MHD flow of a viscous fluid of equal kinematic and magnetic viscosity between two perfectly conducting infinite porous plates. Singh further modified the problem and presented a solution for MHD convective flow past a steadily moving infinite vertical porous plate with mass transfer under constant heat flux and induced magnetic field. The aim of the present study is to present a detailed analysis of the problem of Singh under slightly changed boundary conditions and to discuss the effects of mass transfer on the flow of an electrically conducting viscous fluid through a porous medium past a steadily moving infinite vertical porous plate under the action of a uniform transverse magnetic field with constant heat flux and induced magnetic field.

2 Formulation of Problem

Consider the flow of an incompressible, electrically conducting viscous fluid past a vertical porous plate which moves steadily in the vertically upward direction. Assuming cartesian coordinate system the X-axis is chosen along the plate and Y-axis normal to it. A constant magnetic field is applied normal to the plate. The level of concentration of foreign mass is assumed very low so that the Soret and Dufour effects are neglected. Under these assumptions the governing equations for the flow (under usual Boussinesq’s approximation Pr) are as follows

\[
\begin{align*}
\frac{d y}{d y} & = 0 \\
-\nu \frac{d u}{d y} & = g \beta (T - T_w) + g \beta' (C - C_w) + \nu \frac{d^2 u}{d y^2} 
\end{align*}
\]
where

\[ u = 0, \theta \to 0, C \to 0, H \to 0, \text{ as } y \to \infty \]  \hspace{1cm} \text{(11)}

where, \( S_i = \frac{\nu}{D} \) (Schmidt number),

\[ P_f = \frac{\mu C_p}{K} \] (Prandtl number)

\[ P_m = \sigma \mu_r (\text{Magnetic diffusivity}) \]

\[ G_r = \frac{g \beta Q u^2}{k v_0^2} \] (Grashof number)

\[ E_r = \frac{k v_{ii}}{Q \nu C_p} \] (Eckert number)

\[ G_m = \frac{g \beta \nu (C_m - C_s)}{v_0^2} \] (Modified Grashof number)

\[ H = \frac{H_s}{v_0} \sqrt{\frac{\mu_r}{\rho}} \] (Induced magnetic field)

\[ M = \frac{H_m}{v_0} \sqrt{\frac{\mu_r}{\rho}} \] (Magnetic parameter)

4 Solution

The solution of Eq. (10) under boundary conditions Eq. (11) is

\[ C = c^{-e^{(y)}} \] \hspace{1cm} \text{(12)}

In order to obtain a solution of the coupled non-linear system of Eqs. (7) to (9), we expand \( u, H \) and \( T \) in powers of the Eckert (\( E_r \)) number which is very small and is justified (Raptis') for incompressible fluids. Hence we assume:

\[ u(y) = u_i(y) + E_i u_i(y) + O(E_r^2) + ... \]

\[ H(y) = H_i(y) + E_i H_i(y) + O(E_r^2) + ... \]

\[ \theta(y) = \theta_i(y) + E_i \theta_i(y) + O(E_r^2) + ... \]

Substituting Eq. (13) into Eqs. (7) to (9) and equating the coefficients of the like powers of \( E_r \) and neglecting the terms in \( E_r^2 \) and higher order, we obtain six equations. Solving these equations under the corresponding boundary conditions, we get following solutions,

\[ \theta_1 = \frac{1}{P_r} e^{-P_r \theta_1} \] \hspace{1cm} \text{(14)}

\[ H_1 = (h + A_5 + A_6) e^{-P_r \theta_1} - A_5 e^{-P_r \theta_1} - A_6 e^{-P_r \theta_1} \] \hspace{1cm} \text{(15)}

\[ u_i = (U + A_3 - A_4) e^{-P_r \theta_1} - A_3 e^{-P_r \theta_1} - A_4 e^{-P_r \theta_1} \] \hspace{1cm} \text{(16)}

3 Non-Dimensional Quantities and Transformed Equations

We introduce the following non-dimensional quantities:

\[ u = \frac{u}{U_0}, \theta = \frac{v v_0}{U_0}, k = \frac{k v_0}{U_0}, C = \frac{C - C_m}{C_m - C_s}, \]

\[ U = \frac{U_0}{v_0}, K = \frac{K v_0}{U_0}, \text{ and } \theta = \frac{(T - T_i) k v_0}{Q v_0} \]

Introducing these in Eqs. (2) to (6) and neglecting the stars over them, following non-dimensional equations are obtained:

\[ \frac{d^2 u}{dy^2} + \frac{M}{P_m} \frac{dH}{dy} + \frac{d u}{dy} = -G T - G_m C \] \hspace{1cm} \text{(7)}

\[ \frac{1}{P_m} \frac{d^2 H}{dy^2} + \frac{d H}{dy} + \frac{M}{P_m} \frac{du}{dy} = 0 \] \hspace{1cm} \text{(8)}

\[ \frac{d^2 \theta}{dy^2} + \frac{P_r}{P_m} \frac{d \theta}{dy} = -P_e E \left[ \left( \frac{du}{dy} \right)^2 + \frac{1}{P_m} \left( \frac{dH}{dy} \right)^2 \right] \] \hspace{1cm} \text{(9)}

\[ \frac{d C}{dy} + \frac{d^2 \omega}{dy^2} + \frac{1}{S_i} \frac{d^2 C}{dy^2} = 0 \] \hspace{1cm} \text{(10)}

\[ u = U_0, \frac{d \theta}{dy} = -1, C = 1, H = h \text{ (say) at } y = 0 \]
\[ t_2 = K_1 e^{-P_s y} - P_i, A_1 e^{-P, y} + A_2 e^{-P, y} \]
\[ A_{13} e^{-P, y} - A_{14} e^{-P, y} + A_{15} e^{-P, y} + A_{16} e^{-P, y} + A_{17} e^{-P, y} + A_{18} e^{-P, y} + A_{19} e^{-P, y} \]
\[ A_{20} e^{-P, y} + A_{21} e^{-P, y} + A_{22} e^{-P, y} + A_{23} e^{-P, y} + A_{24} e^{-P, y} + A_{25} e^{-P, y} + A_{26} e^{-P, y} + A_{27} e^{-P, y} + A_{28} e^{-P, y} + A_{29} e^{-P, y} + A_{30} e^{-P, y} \]
\[ A_{31} e^{-P, y} + A_{32} e^{-P, y} + A_{33} e^{-P, y} + A_{34} e^{-P, y} + A_{35} e^{-P, y} + A_{36} e^{-P, y} + A_{37} e^{-P, y} + A_{38} e^{-P, y} + A_{39} e^{-P, y} + A_{40} e^{-P, y} \]
\[ A_{41} e^{-P, y} + A_{42} e^{-P, y} + A_{43} e^{-P, y} + A_{44} e^{-P, y} + A_{45} e^{-P, y} + A_{46} e^{-P, y} + A_{47} e^{-P, y} + A_{48} e^{-P, y} + A_{49} e^{-P, y} + A_{50} e^{-P, y} \]

(Satisfies the boundary condition) \[ \ldots(22) \]

### 6 Discussion

In order to get the physical insight into the problem
\[ \left( \frac{d u}{d y} \right)_{y=0}, \left( \frac{d \theta}{d y} \right)_{y=0}, \right. \]
\[ u, \theta, H, J \] are observed taking numerical values of \( M, G_n, Sc \) and \( U \) and represented graphically. To be realistic, following Raptis and Soundalgekar \(^9\), Agrawal \(^1\) and Jha \(^1\), the value of \( P_s \) is taken as 0.71 (for air) and the Eckert number is taken very small i.e., 0.01 which is justified for incompressible fluids. The values of Schmidt number are taken for the gases hydrogen (\( S_c = 0.22 \)), helium (\( S_c = 0.30 \)), water-vapour (\( S_c = 0.60 \)), oxygen (\( S_c = 0.66 \)) and ammonia (\( S_c = 0.78 \)) at temperature 25°C and 1 atmosphere pressure. The velocity and temperature profiles, induced magnetic field, current density versus \( y \) are shown in Figs 1-4 but shear stress and rate of heat transfer versus \( G_n \) are shown in Figs 5-6. The species concentration and rate of diffusion are shown in Table 1.

### 7 Conclusions

The conclusions of the present analysis are as follows.

(i) An increase in \( G_n \) or \( M \) leads to an increase in the velocity but velocity decreases with increase in \( S_c \).

(ii) Due to downward motion of the plate, there is a fall in the velocity.

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**Fig. 1 — Velocity profiles versus \( y \)**
Table 1—Effect of $S_c$ on species concentration and rate of mass diffusion

<table>
<thead>
<tr>
<th>S no.</th>
<th>$S_c$</th>
<th>$y$</th>
<th>$C$</th>
<th>$\left( \frac{\partial C}{\partial y} \right)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.22</td>
<td>0.5</td>
<td>0.805834</td>
<td>-0.49708</td>
</tr>
<tr>
<td>2</td>
<td>0.22</td>
<td>1.0</td>
<td>0.802529</td>
<td>-0.17655</td>
</tr>
<tr>
<td>3</td>
<td>0.30</td>
<td>0.5</td>
<td>0.860708</td>
<td>-0.21286</td>
</tr>
<tr>
<td>4</td>
<td>0.30</td>
<td>1.0</td>
<td>0.740818</td>
<td>-0.18879</td>
</tr>
<tr>
<td>5</td>
<td>0.60</td>
<td>0.5</td>
<td>0.740818</td>
<td>-0.22830</td>
</tr>
<tr>
<td>6</td>
<td>0.60</td>
<td>1.0</td>
<td>0.548812</td>
<td>-0.20047</td>
</tr>
<tr>
<td>7</td>
<td>0.60</td>
<td>0.5</td>
<td>0.718924</td>
<td>-0.474491</td>
</tr>
<tr>
<td>8</td>
<td>0.60</td>
<td>1.0</td>
<td>0.516854</td>
<td>-0.34122</td>
</tr>
<tr>
<td>9</td>
<td>0.60</td>
<td>0.5</td>
<td>0.677057</td>
<td>-0.528105</td>
</tr>
<tr>
<td>10</td>
<td>0.78</td>
<td>1.0</td>
<td>0.458416</td>
<td>-0.357857</td>
</tr>
</tbody>
</table>

(iii) An increase in $G_m$ or $M$ leads to an increase in the temperature but temperature decreases with increase in $S_c$.

(iv) Due to downward motion of the plate, there is an increase in the temperature.

(v) An increase in $G_m$ or $M$ leads to a decrease in induced magnetic field but increases with increase in $S_c$.

(vi) Due to downward motion of the plate, there is an increase in induced magnetic field.

(vii) An increase in $G_m$ or $M$ leads to an increase in the value of $J$ but it decreases with increase in $S_c$.

(viii) The current density decreases due to downward motion of the plate.

Fig. 2. Temperature profiles versus $y$

Fig. 3—Induced magnetic field versus $\gamma$
Fig. 4 — Current density versus \( j \)

Fig. 5 — Shear stress versus \( \gamma \)
(ix) An increase in $S_c$ or $M$ leads to an increase in the value of skin-friction but it increases with increase in $G_m$.

(x) The skin-friction decreases due to downward motion of the plate.

(xi) An increase in $S_c$ or $M$ leads to an increase in the value of $N_a$ but it decreases with increase in $G_m$.

(xii) Due to downward motion of the plate, there is a fall in the rate of heat transfer.

(xiii) An increase in $S_c$ decreases species concentration and mass diffusion.

(xiv) The species concentration is an exponentially decreasing function of $y$. Therefore species concentration and mass diffusion tend to zero as $y$ tends to infinity.

Acknowledgement
The authors are thankful to the learned referee for suggestions to improve the paper.

References
Appendix

\[
A_1 = (1 + Pw), \quad A_2 = P_w(1 - M^2), \quad A_3 = \frac{MP_wG_r}{P_r}
\]

\[
A_4 = \frac{MP_wG_{in}}{S}, \quad A_5 = \frac{A_4}{(P_r - A_3 P_r + A_3)},
\]

\[
A_6 = \frac{A_4}{(S_n^2 - A_4 S_n + A_2)}, \quad A_7 = \frac{M(\alpha - S_0)},
\]

\[
A_8 = \frac{1}{1 - P_r} \left[ MA_5 + \frac{G_r}{P_r} \right], \quad A_9 = \frac{1}{1 - P_r} \left[ MA_5 + \frac{G_r}{P_r} \right],
\]

\[
A_{10} = \frac{(M + A_7 - A_8 - A_9)^2}{2(P_r - 2)}, \quad A_{11} = \frac{(M + A_7 - A_8 - A_9)^2 + A_7}{2(P_r - 2)},
\]

\[
A_{12} = \frac{A_3 + A_5}{2}, \quad A_{13} = \frac{(A_3 + A_5) S_n}{2(P_r - 2 S_n)}, \quad A_{14} = \frac{2A_1 A_7}{P_r + L_2},
\]

\[
A_{15} = A_3 A_0, \quad A_{16} = \frac{2A_1 A_0}{P_r + S_n},
\]

\[
A_{17} = \frac{2P_r(A_3 + A_5 A_0 + A_7 A_8)}{(P_r + S_n)},
\]

\[
A_{18} = \frac{2P_r S_n(A_3 A_0 + A_5 A_0)}{L_2(L_2 + P_r)},
\]

\[
A_{19} = \frac{2L_2 S_n(A_3 A_0 + A_5 A_0 + A_7 A_8)}{(L_2 + S_n)},
\]

\[
A_{20} = \frac{A_9}{P_r}, \quad A_{21} = \frac{A_{10}}{P_r}, \quad A_{22} = \frac{A_{11}}{(4 - 2A_1 + A_2)},
\]

\[
A_{23} = \frac{A_{12}}{2L_2 (4L_2^2 - 2L_2 A_3 + A_2)},
\]

\[
A_{24} = \frac{A_{12}}{2P_r (4P_r^2 - 2P_r A_3 + A_2)},
\]

\[
A_{25} = \frac{A_{13}}{2S_n (4S_n^2 - 2S_n A_3 + A_2)},
\]

\[
A_{26} = \frac{A_{14}}{(1 + L_2) \left[ (1 + L_2)^2 - (1 + L_2) A_3 + A_3 \right]},
\]

\[
A_{27} = \frac{A_{15}}{(1 + P_r) \left[ (1 + P_r)^2 - (1 + P_r) A_3 + A_3 \right]},
\]

\[
A_{28} = \frac{A_{17}}{(L_2 + P_r) \left[ (L_2 + P_r)^2 - (L_2 + P_r) A_1 + A_2 \right]},
\]

\[
A_{29} = \frac{A_{18}}{(P_r + S_n) \left[ (P_r + S_n)^2 - (P_r + S_n) A_1 + A_2 \right]},
\]

\[
A_{30} = \frac{A_{19}}{(L_2 + S_n) \left[ (L_2 + S_n)^2 - (L_2 + S_n) A_1 + A_2 \right]},
\]

\[
A_{31} = \frac{A_{20}}{P_r (1 + P_r)}, \quad A_{32} = \frac{A_{21}}{P_r (1 + P_r)},
\]

\[
A_{33} = \frac{A_{22}}{2L_2 (1 - 2 L_2)}, \quad A_{34} = \frac{A_{23}}{2P_r (1 - 2 L_2)},
\]

\[
A_{35} = \frac{A_{24}}{2S_n (1 - 2 S_n)}, \quad A_{36} = \frac{A_{25}}{L_2 (1 - 2 L_2)},
\]

\[
A_{37} = \frac{A_{26}}{P_r (1 + P_r)}, \quad A_{38} = \frac{A_{27}}{S_n (1 + S_n)},
\]

\[
A_{39} = \frac{A_{28}}{(L_2 + P_r) (1 - L_2 - P_r)},
\]

\[
A_{40} = \frac{A_{29}}{(P_r + S_n) (1 - L_2 - P_r)},
\]

\[
A_{41} = \frac{A_{30}}{(L_2 + S_n) (1 - L_2 - S_n)},
\]

\[
A_{42} = \frac{A_{31}}{(L_2 + S_n) (1 - L_2 - S_n)},
\]

\[
A_{43} = \frac{A_{32}}{(P_r + S_n) (1 - L_2 - P_r)},
\]

\[
A_{44} = \frac{A_{33}}{(L_2 + P_r) (1 - L_2 - P_r)},
\]

\[
K_1 = 2A_{40} - 2A_1 A_{11} + 2P_r A_{12} - 3S_n A_1 - (1 + L_2) A_{14} + (1 + P_r) A_{15} + (1 + S_n) A_{16} - (L_2 + P_r) A_{17} + (P_r + S_n) A_{18} - (L_2 + S_n) A_{19},
\]

\[
K_2 = -A_{30} + A_{21} - A_{22} + A_{23} - A_{24} - A_{25} + A_{26} + A_{27} - A_{28} + A_{29} - A_{30}.
\]
\[ K_4 = A_1 + A_{12} - A_{31} + A_{41} - A_{51} \]
\[ - A_{60} + A_{17} + A_{18} - A_{29} + A_{49} - A_{59} \]

\[ K_4 = A_{12} - A_{31} + A_{40} - A_{50} \]
\[ - A_{42} - A_{32} + A_{41} + A_{51} - A_{52} - A_{51} \]

\[ K_5 = P_A + 2 A_{12} - 2 L_2 A_{11} - 2 P_r A_{51} - 2 S A_{31} \]

\[ -(1 + L_2) A_{3} - (1 + P_r) A_{37} - (1 + S_r) A_{38} \]
\[ -(L_2 + P_r) A_{40} + (P_r + S_r) A_{41} - (L_2 + S_r) A_{41} \]

\[ K_6 = L_2 A_3 + A_{30} - 2 A_{31} + 2 L_2 A_{22} - 2 P_r A_{23} \]
\[ + 2 S A_{24} + (1 + L_2) A_{25} - (1 + P_r) A_{26} - (1 + S_r) A_{27} \]
\[ + (L_2 + P_r) A_{28} - (P_r + S_r) A_{29} + (L_2 + S_r) A_{29} \]
\[ 2 L_2 = A_1 + \sqrt{A_1^2 - 4 A_2} \]