Nonlinear interactions in magnetised piezoelectric semiconductor plasmas

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Based on hydrodynamic model of plasmas an analytical investigation of frequency modulational interaction between co-propagating high frequency pump and acoustic mode and consequent amplification (steady state and transient) of the modulated waves is carried out in a magnetised piezoelectric semiconductor medium. The phenomenon of modulational amplification is treated as four wave interaction process involving cubic nonlinearity of the medium. Gain constants, threshold-pump intensities and optimum-pulse duration for the onset of modulational instabilities are estimated. The analysis has been performed in non-dispersive regime of the acoustic mode, which is one of the preconditions for achieving an appreciable initial steady-state growth of the modulated signal wave. It is found that the transient gain diminishes very rapidly if one chooses the pump pulse duration beyond the maximum gain point. Moreover, the desired value of the gain can be obtained by adjusting intensity & pulse duration of the pump and doping concentration of the medium concerned.

1 Introduction

The nonlinear interactions of co-propagating beams have been a field of interest since the origin of physical optics. The scattering of light by sound or low frequency electromagnetic wave is essentially the same process as diffraction of light by grating. It affords a convenient means of controlling frequency, intensity and direction of an optical beam. The modulation processes make a large number of applications possible including impressing of information onto optical beams and their transmission, switching of lasers for generation of giant optical pulses, mode-locking, optical beam deflection and so on. The fabrication of some of the optical and microwave devices such as acousto-optic modulators, low noise amplifiers etc. is based on the interaction of the incident pump beam with an acoustic wave or a low-frequency electromagnetic wave.

Due to vast technological potentialities, a number of researchers have concentrated their experimental and theoretical studies on nonlinear interactions in solid state plasmas. An important field of study in nonlinear acoustics is amplification/attenuation and frequency mixing of waves in semiconductors (especially in III-V compounds), because of its immediate relevance to optical communication systems. Looking at the potential uses of the semiconductors in modern opto-electronic industries, analytical investigation of some of the basic nonlinear processes in such crystals is of considerable importance. If the medium concerned is piezoelectric, such studies become more significant from energy gain/loss point of view.

Among semiconductor media, nearly centro-symmetric one is more friendly to technologists. In these semiconductors, the dominant nonlinear optical processes may be described in terms of third-order optical susceptibility $\chi^{(3)}$, which is in general, a complex quantity and capable of describing the interference between various resonant and nonresonant processes. The third-order susceptibility tensor $\chi^{(3)}$ can be conveniently used to explain modulation processes in a Kerr active medium.

A large amount of work on nonlinear instability in solid-state plasmas is available elsewhere. Anderson et al. have found that the growth of instability in LiNbO$_3$ is large enough for experimental demonstration of amplitude modulation and envelope soliton. The recent activity in this area is the modes of wave propagation in optical fibres. The discussion has so far been restricted mostly to steady-state solutions. The deformation with time, of a laser pulse propagating in a medium with an intensity dependent refractive index, provides an example of transient nonlinear effects. Such transient effects are of practical importance particularly when the pump is a short travelling pulse. Hence the studies should not be restricted to steady-state solutions only.
Motivated by the above discussion, we seek to study in analytical terms, the interaction of co-propagating high frequency pump with an acoustic mode and amplification of resulted modulated wave in a magnetised piezoelectric semiconductor plasma medium. In this medium, the nonlinearity is induced by the intense pump. This nonlinearity in response amplifies the acoustic wave at the expense of the pump. We have also reported the nature of transient gain via steady-state gain, threshold pump intensities and optimum pulse duration for the onset of the modulational instability. As far as we know, no such attempt has been made to determine the transient gain via steady-state gain of the modulated wave and related parameters in magnetised piezoelectric semiconductor plasmas. Finally, we have made exhaustive numerical analysis with a set of data appropriate for a piezoelectric crystal duly irradiated by 10.6 µm CO₂ laser to establish the validity of the analysis.

2 Theoretical Formulation

2.1 Effective nonlinear polarisation

We have considered the well known hydrodynamic model of plasmas of infinite extent to study the modulational interaction between intense pump wave and acoustic signal and consequent amplification (steady-state and transient) of the modulated wave, in an obliquely magnetised n-type piezoelectric semiconductor. It is assumed that the origin of the said interaction lies in the effective third-order nonlinear optical susceptibility \( \chi^{(3)} \). The medium is immersed in static magnetic field \( B \), lying in \( x-z \) plane and inclined at an angle \( \theta \) with \( x \)-direction. The medium is irradiated by an intense hybrid pump wave expressed as:

\[
E_0 = (E_0, \mathbf{x} + E_0, y) \exp \left[ i(k_0x - \omega_0t) \right] \quad \text{(1)}
\]

This model enables us to replace the electron plasma or the streaming electrons by a charged fluid characterised by a few macroscopic parameters like mean carrier density, mean velocity etc. of the plasma fluid and thus makes the analysis of modulational interaction and other related phenomena, simple. The basic equations governing the modulational instability are:

\[
\frac{\partial n_e}{\partial t} + n_e \frac{\partial v_e}{\partial x} + n_1 \frac{\partial v_1}{\partial x} + v_0 \frac{\partial n_0}{\partial x} = 0 \quad \text{(4)}
\]

\[
\frac{\partial E}{\partial x} + \frac{\beta}{\varepsilon} \frac{\partial^2 u}{\partial x^2} \frac{\partial E}{\partial x} + \frac{\beta}{\varepsilon} \frac{\partial E}{\partial x} \frac{\partial u}{\partial x} = \frac{e}{\varepsilon} \frac{\partial^2 u}{\partial x^2} \quad \text{(5)}
\]

\[
\frac{\partial^2 u}{\partial t^2} + 2\eta_0 \frac{\partial u}{\partial t} + \frac{\beta}{\rho} \frac{\partial E}{\partial x} = \frac{e}{\rho} \frac{\partial^2 u}{\partial x^2} \quad \text{(6)}
\]

Eqs (2) and (3) are the linearised momentum transfer equations in which \( v_0 \) and \( v_e \) are the zeroth and first order oscillatory fluid velocities, and \( B_0 \) and \( B_1 \) are the applied static and pump magnetic fields, respectively. The parameter \( \nu \) is the phenomenological collision frequency of electrons. We have neglected the term \( v_1 (\partial v_1/\partial t) \) in Eq. (2) under the assumption \( \omega_0 \gg \nu \) in \( \omega_0 \). In Eq. (3) the Lorentz force terms \( v_0 \times B_1 \) and \( v_1 \times B_1 \) can be safely neglected if we assume that the transverse acoustic wave is propagating in such a direction of the crystal that it produces a longitudinal electric field. Eq. (4) is the continuity equation with \( n_e \) and \( n_1 \) being the unperturbed and perturbed electron densities, respectively. The Poisson equation (Eq. (5)) determines the space charge field \( E_0 \); \( \varepsilon \) and \( \beta \) are the total permittivity and piezoelectric constants of the medium, respectively. Eq. (6) describes lattice vibrations in the piezoelectric crystal of material density \( \rho \) in which \( \gamma_0 \) and \( \gamma_1 \) are the respective damping parameter and the crystal elastic constant. \( \mathbf{u}(x,t) = \mathbf{u} \exp \left[ i (k_0 x - \omega_0 t) \right] \) denotes displacement of lattice points from their mean position with \( \omega_0 \ll \omega_0 \).

Physically a signal wave creates acoustic perturbation \( \omega_0, k_0 \) and consequent electron density perturbation at the acoustic frequency. This perturbation which couples non-linearly with the incident pump wave to generate a modulated wave at the frequencies \( \omega_0 \pm \omega_p \), following Guha et al. and using above mentioned equations in the collision dominated regime \( \nu >> k_0 v_0, \omega_0 \omega_p \), we get

\[
\frac{\partial^2 n_e}{\partial t^2} + 2\eta_0 \frac{\partial n_e}{\partial t} + \omega_p^2 n_e = -n_e \frac{\beta}{\varepsilon} \frac{\partial^2 u}{\partial x^2} \left( \frac{\omega_0^2 + \nu^2}{\omega_0^2} \right) = \frac{E_0}{\mu} \frac{\partial u}{\partial x} \quad \text{(7)}
\]

where \( E_0 = \frac{e}{m} \left[ \mathbf{E}_0 + v_0 \times B_1 \right] = \frac{e}{m} \mathbf{E}_0 \),

\[
\omega_p^2 = \left( \omega_0^2 + \nu^2 \right) \left( \frac{\omega_0^2}{\omega_0^2 + \nu^2} \right) \quad \text{with} \quad \omega_{\text{cycl}} = \left( \frac{-e B_{\text{eff}}}{m} \right)
\]

are the \( x \) and \( z \) components of cyclotron frequency and \( \omega_{\text{cycl}} = \left( \frac{n_e e^2}{\mu m} \right) \) is the electron plasma frequency of the
medium. The Doppler shift has been neglected under the assumption \( \omega_o >> \nu \gg k_w \omega_o \). In the above analysis the effect of pump magnetic field (B) is neglected by considering that \( \omega_o \) and \( \omega_o \) are comparable to \( \omega_o \).

The modulation process in the medium considered above must fulfill the phase matching conditions \( k_w = k_0 \pm k_t \), i.e. \( \omega_o \) = \( \omega_o \) \pm \( \omega_o \), known as momentum and energy conservation relations, respectively. Here we have considered only the resonant sideband frequencies (\( \omega_o \pm \omega_o \)) by assuming a long interaction path (considering the crystal of infinite length) so that higher order scattering terms become off-resonant. Thus the only waves which couple with the sound waves are the incident (\( \omega_o \)) and first-order scattered waves (\( \omega_o \pm \omega_o \)).

On using Eqs (5) to (7), the perturbed electron density oscillating at the forced wave frequencies i.e. upper and lower sideband frequencies may be obtained as:

\[
n_1(\omega_o, k_w) = \frac{i k_{iw}^2 B_i E_0 \omega_o}{c \rho (\omega_o + 2 i \omega_o \gamma - k_w^2 \omega_o)} \left[ \frac{\omega_o^2 - \omega_o^2 - i \nu \omega_o + i k_w E_0}{\omega_o^2 - i \nu \omega_o + i k_w E_0} \right]^{-1} \quad (8)
\]

Here \( \nu = 4 \nu / \rho \) is the velocity of acoustic wave in the medium. The electron density perturbation at sideband frequencies is assumed to vary as exp \( i (k_w x - \omega_o t) \). The density perturbations thus produced affect the propagation characteristics of the generated wave, which can be examined by employing the general electromagnetic wave equations.

By neglecting the effects of transition dipole moment, the resonant components of the induced nonlinear current densities \( J_x \) may be expressed as:

\[
J_x = -en_x(\omega_o, k_w) \nu_0 \quad \ldots (9a)
\]

\[
J_x = -en_x(\omega_o, k_w) \nu_0\quad \ldots (9b)
\]

in which the asterisk denotes the complex conjugate of the respective term.

Now treating the induced polarisation \( P_x \) at the modulated frequencies as a time integral of \( J_x \), we get the effective nonlinear polarisation of the medium as:

\[
P_x = P_x + P_x = \frac{i k_{iw}^2 B_i E_0 \omega_o}{\omega_o \rho (\omega_o + 2 i \omega_o \gamma - k_w^2 \omega_o)} \left[ \frac{1}{\omega_o} (\delta_o^2 - i \nu \omega_o + i k_w E_0) \right]^{-1}
\]

On simplifying Eq. (10) and rearranging the various terms, we obtain

\[
P_x = \frac{2 \varepsilon_0 \alpha^2}{m^2 \omega_o^4} \frac{k_{iw}^2 (\delta_o^2 - \nu^2)}{\omega_o^2 - k_w^2 \omega_o^2 + 2 i \omega_o \gamma} \times \frac{1}{\omega_o} (\delta_o^2 - i \nu \omega_o + i k_w E_0)^{-1}
\]

\[
\left[ (\delta_o^2 + \nu^2) - 2 \frac{2 \nu k_w E_0}{\omega_o (\omega_o^2 - k_w^2 \omega_o^2 + 2 i \omega_o \gamma)} \right]^{-1}
\]

\[
\left[ (\delta_o^2 + \nu^2)^2 + \frac{4 \nu k_w E_0}{\omega_o (\omega_o^2 - k_w^2 \omega_o^2 + 2 i \omega_o \gamma)} \right]^{-1}
\]

\[
\left[ \frac{2}{\omega_o} (5 \nu^2 k_w^2 - 3 \delta_o^2 k_w^2 + 2 \delta_o^2 k_w^2) (\delta_o^2 + \nu^2)^{\frac{3}{2}} E_0 \right]^{-1}
\]

\[
\ldots (11)
\]

in which \( A = \kappa^2 k_w^2 \nu^2, \kappa^2 = \frac{B^2}{2 \varepsilon_0} \) and \( \delta = \omega_o - \omega_o \).

Using Maclaurin’s power series to expand Eq. (11), one obtains after some algebraic simplification as:

\[
P_x = \frac{2 \varepsilon_0 \alpha^2}{m^2 \omega_o^4} \frac{k_{iw}^2 (\delta_o^2 - \nu^2)}{\omega_o^2 - k_w^2 \omega_o^2 + 2 i \omega_o \gamma} \times \frac{1}{\omega_o} (\delta_o^2 - i \nu \omega_o + i k_w E_0)^{-1}
\]

\[
\left[ \frac{2}{\omega_o} (5 \nu^2 k_w^2 - 3 \delta_o^2 k_w^2 + 2 \delta_o^2 k_w^2) (\delta_o^2 + \nu^2)^{\frac{3}{2}} E_0 \right]^{-1}
\]

\[
\ldots (12)
\]

2.2 Effective third-order susceptibility and steady-state gain

The third-order induced polarisation due to cubic nonlinearity at modulated frequencies \( \omega_o \) is defined as:

\[
P_x^{(3)} = \varepsilon_0 \chi_x^{(3)} \quad \ldots (13)
\]

where \( \varepsilon_0 \) is the free space permittivity.

From Eq. (12) one may obtain the third-order polarisation as:

\[
P_x^{(3)} = \frac{2 \varepsilon_0 \alpha^2}{m^2 \omega_o^4} \frac{k_{iw}^2 (\delta_o^2 - \nu^2)}{\omega_o^2 - k_w^2 \omega_o^2 + 2 i \omega_o \gamma} \times \frac{1}{\omega_o} (\delta_o^2 - i \nu \omega_o + i k_w E_0)^{-1}
\]

Using Eqs (13) and (14), the third-order susceptibility may be obtained as:

\[
\chi_x^{(3)} = \frac{2 \varepsilon_0 \alpha^2}{m^2 \omega_o^4} \frac{k_{iw}^2 (\delta_o^2 - \nu^2)}{\omega_o^2 - k_w^2 \omega_o^2 + 2 i \omega_o \gamma} \times \frac{1}{\omega_o} (\delta_o^2 - i \nu \omega_o + i k_w E_0)^{-1}
\]

\[
\ldots (14)
\]

Here \( \varepsilon_1 \) is the high frequency dielectric constant of the medium.

This equation reveals that the third-order susceptibility \( \chi_x^{(3)} \) (via \( P_x^{(3)} \)) couples the perturbed density wave at sideband frequencies \( \omega_o \) and the acoustic wave at \( \omega_o \) to produce modulational interaction. It is evident from Eq. (15) that \( \chi_x^{(3)} \) is a complex quantity and hence may be expressed as:

\[
\chi_x^{(3)} = \chi_x^{(3)} + i \chi_x^{(3)}
\]

\[
\ldots (16)
\]
where

\[
\chi^{(3)}_0 = \frac{-2 e^2 A e^{2 \omega_0^2} k_1^2 (\delta^2 - v^2)}{m^2 \omega_0^4 (\delta^2 + v^2)^2 [\omega_0^2 - k_1^2 v^2]^2 + 4 \omega_0^2 Y_a^2}
\]

and

\[
\chi^{(3)}_0 = \frac{-4 e^2 A e^{2 \omega_0^2} k_1^2 (\delta^2 - v^2) \omega_a Y_a}{m^2 \omega_0^4 (\delta^2 + v^2)^2 [\omega_0^2 - k_1^2 v^2]^2 + 4 \omega_0^2 Y_a^2}
\]

here the subscripts \(r\) and \(i\) to the susceptibility represent its real and imaginary parts, respectively. The above Eqs (17) describe the steady-state optical response of the medium in the present configuration and govern the nonlinear wave propagation through the medium. It is evident that the nonlinear susceptibility is influenced by the unperturbed carrier concentration through \(\omega_0\) and the external magnetic field \(B\), through \(\omega_a\). Now the imaginary part (Eq. (17b)) of the third-order susceptibility can be employed to obtain steady-state growth and the real part (Eq. (17a)) is useful in exploring the dispersion characteristics of the medium for the modulated wave.

From Eq. (17a), it may be inferred that \(\chi^{(3)}_0\) can have positive and negative values depending upon the relative values of \(\omega_0\) and \(v\). Positive dispersion of the modulated wave occurs \(\omega_0 >> v\) at and under such circumstances the nonlinear refractive index falls off with distance from the beam axis. Snell's law allows us to conclude that beam velocity of the modulated wave increases with the distance from the beam axis. This leads to the occurrence of self-focusing of the signal. As \(\chi^{(3)}_0\) becomes more positive, one may expect more effective self-focusing of the modulated wave. However, for non-dispersive acoustic mode at \(\omega_0 = k_1 v_a\), we observe anomalous dispersion of the modulated wave in the medium as \(\chi^{(3)}_0 = 0\). It can further be concluded that for non-dispersive acoustic mode, there is no change in the nonlinear refractive index of the medium due to induced current density.

In order to explore the possibility of modulational amplification in the semiconductor plasma, we employ the relation

\[
\alpha_c = \frac{k_1}{2 E_0} \chi^{(3)}_0 |E_0| I
\]

here \(\alpha_c\) is the effective nonlinear absorption coefficient and \(E_0\) is the lattice dielectric constant of the medium.

The nonlinear growth of the modulated signal is possible only if \(\alpha_c\) obtained from Eq. (18) is negative. Thus it can be inferred that the growth of the modulated wave can be achieved only when \(\chi^{(3)}_0\) is negative i.e. \(\delta^2 > v^2\). Hence the growth is possible only in highly doped semiconductors. This condition can easily be satisfied by adjusting the doping level of the medium. The growth of the modulated wave obtained from above equations is as below:

\[
g_c = \frac{-2 e^2 A e^{2 \omega_0^2} k_1^2 (\delta^2 - v^2) \omega_a Y_a}{e_1^2 m^2 \omega_0^4 (\delta^2 + v^2)^2 [\omega_0^2 - k_1^2 v^2]^2 + 4 \omega_0^2 Y_a^2} |E_0| I
\]

or the steady-state gain of the modulated wave in \(n\)-InSb crystal (data are given in the Section 3) may be obtained as:

\[
g_c = 1.385 \times 10^{-5} I_n
\]

where we have defined \(I_n = \frac{1}{2} \eta e_0 c_0 |E_0| I^2\) in which \(c_0\) is the speed of light in vacuum and \(\eta\) being the background refractive index of the crystal.

### 2.3 Transient gain

It is evident from Eq. (20) that a high power pulsed laser can only yield a significant growth of the modulated signal. But at this intensity of pump, the study of transient effects becomes important. The additional advantage of incorporating transient effects in our analysis is that we can also predict threshold pump intensity \(I_{th}\) for the onset of modulation process. We may also predict the optimum pulse duration necessary for the instability to occur. In general, the transient gain is related to steady-state gain \(g_t\) through the relation

\[
g_t = (2g_c \times \Gamma \tau_p)^{0.5} - \Gamma \tau_p
\]

where \(\Gamma\) being the optical photon life time, \(x\) is the interaction length and \(\tau_p\) is the pulse duration. For very short pulse \((\tau_p \leq 10^{-10}\) sec), the interaction length should be replaced by \(\frac{e_1 c_0 \tau_p}{2}\), where \(c_0\) is the speed of light in the crystal and is given by \(c_0 = \sqrt{\epsilon_0 \epsilon_r}\). By making \(g_t = 0\), in Eq. (21) we can obtain the threshold pump intensity as:

\[
I_{th} = \frac{\Gamma}{2 \times G_c e_1 I^2}
\]

with \(G_c = g_c/I_{th}\), the steady-state gain per unit pump intensity.

By assuming \(\Gamma = 4 \times 10^5\) sec \(^{-1}\) for \(n\)-InSb crystal, the estimate of \(I_{th}\) is obtained as 2.253 \times 10^5 \) WM \(^{-1}\). More-
over, for $\tau_p \geq 10^{-4}$ sec, the cell length can be taken equal to $x$, thus the transient gain becomes

$$g_i = \left( \Gamma \tau_p \right)^{1/2} \left[ \frac{x}{\tau_p} \right]^{1/2} \left( g_{\text{pr}} \right)^{1/2} \left( g_{\text{ex}} \right) \left( g_{\text{ad}} \right) \left( g_{\text{in}} \right)$$

(23)

By setting $g_i = 0$ in above relation, one can get an expression for the optimum pulse duration ($\tau_{p,\text{opt}}$) above which transient gain would not be possible as:

$$\tau_{p,\text{opt}} = \frac{g_{\text{pr}}^2}{\Gamma} \left( \frac{x}{\tau_p} \right)^{1/2} \left( g_{\text{ex}} \right) \left( g_{\text{ad}} \right) \left( g_{\text{in}} \right)$$

(24)

A calculation for n-InSb using the values of $g_e$ obtained earlier and $x = 10^{-3}$ gives

$$\tau_{p,\text{opt}} = 4.064 \times 10^{-11} \text{ sec}$$

(25)

This value of ($\tau_{p,\text{opt}}$) not only explains the washing out of the gain of modulated wave at large pulse duration but also suggests that optimum pulse duration can be increased by increasing intensity of the pump.

3 Results and Discussion

A detailed investigation about the nature of the steady-state gain reveals that an appreciable amplification ($g_e = 10^3 - 10^6 \text{ m}^{-1}$) of modulated wave is obtained, when $\omega_p = \omega_{\text{pr}}$, (condition of anomalous dispersion). The growth rate of the modulated wave is found independent of its frequency but depends on the frequency of the pump and acoustic wave; a fact in agreement with experimental observations. The expression for the steady-state growth rate as obtained from Eq. (19) has the usual variation $g_e \propto \left[ a k_p^2 \left( b \right) \left( 1 + k_p^2 \right) \right]^{1/2}$ as predicted by Drake et al. Moreover it can also be inferred from Eq. (22) that in the region of smaller magnitudes of $k_p$ (i.e. $\omega_p \gg k_p \omega_s$), $g_e$ increases with $k_p$ and becomes maximum at $\omega_p = \omega_s$. On increasing $k_p$ further, $g_e$ shows a steep decline, this is in agreement with the usual dependence quoted above.

In the present study, one can also estimate the order of the third-order nonlinear susceptibility $\chi^{(3)}$ of the crystal. After algebraic simplifications and adopting appropriate numerical approximations, Eq. (15) reduces to:

$$\chi^{(3)} = \frac{2 \pi \nu \nu_0}{\omega_0 \Gamma} \left( \frac{x}{\tau_p} \right) \left( \frac{g_{\text{pr}}}{g_{\text{ex}}} \right) \left( \frac{g_{\text{ad}}}{g_{\text{in}}} \right)$$

(26)

From above equation, for the crystals with $n_i = 10^{17}$ m$^{-3}$ and $10^{18}$ m$^{-3}$, the orders of $\chi^{(3)}$ are found 10$^{-7}$ esu and 10$^{-8}$ esu, respectively. This is fairly in agreement with the experimentally observed value ($\chi^{(3)} = 10^{-8}$ for $n_i = 10^{17}$ m$^{-3}$) for III-V compounds. Thus one may infer that heavily doped semiconductors are the most appropriate host for the modulation processes. Obviously, one may expect larger amplification in heavily doped medium. However, the doping level should not increase to such an extent that the plasma frequency exceeds the input pump frequency $\omega_p$. Otherwise, the pump will be reflected back and no amplification will be achieved. The static magnetic field $B$ (via $\omega_p$) also effectively increases $\chi^{(3)}$ of the medium. By raising the magnitude of $\chi^{(3)}$, it is possible to excite third-order nonlinear phenomena at much lower pump amplitudes. Thus in III-V crystals, modulational interaction in the infrared regime appears quite promising under the typical resonance condition viz. $\omega_p = \omega_{\text{pr}}$.

To appreciate the estimation of different parameters, we have done numerical analysis for n-InSb semiconductor at 77 K irradiated by 10.6 $\mu$m CO$_2$ laser. The analysis is based on the following set of parameters: $\mu = 0.014 m_e$, $\beta = 0.054 \text{ cm}$, $\rho = 508 \times 10^{-1} \text{ kg m}^{-1}$, $\varepsilon_p = 18.0$, $\nu = 4 \times 10^5 \text{ s}^{-1}$, $n_i = 10^{24} \text{ m}^{-3}$, where $m_e$ being the free electron mass.

The variation of steady-state gain $g_e$ with carrier density $n_i$ (via $\omega_p$) is shown in Fig. 1. The estimations are plotted for a non-dispersive acoustic mode i.e. for $\omega_p = \omega_s$ (with $k_p = 2.08 \times 10^3 \text{ m}^{-1}$, $\nu_0 = 4 \times 10^5 \text{ m s}^{-1}$ and $\omega_p = 8.32 \times 10^7 \text{ sec}^{-1}$). The steady-state gain increases somewhat parabolically with doping concentration.

The behaviour of transient gain $g_1$ as a function of pump pulse duration ($\tau_p$) with pump intensity $I_p$ as a parameter is plotted in Fig. 2. We have chosen $\tau_p$ in the range $10^{-3} \leq \tau_p \leq 10^{-8}$ sec and the cell length is taken as $10^{-1}$ m. For the given intensity of pump, $g_1$ first increases rapidly with $\tau_p$ and attains maximum value. On further increasing $\tau_p$, $g_1$ diminishes very rapidly and finally becomes zero. The plot also reveals that with rise in $I_p$, $g_1$ increases and the maximum gain point shifts towards larger value of $\tau_p$.

From above discussion, it may be concluded that the doping level and magnitude & orientation of the applied external magnetic field are the important parameters governing the steady-state gain. An appropriate choice of these parameters can lead to onset of modulational interaction using even low pump intensity and thus can replace the conventional idea of high laser pump. Applicable transient gain can also be obtained by suitably choosing pump pulse duration. The highlight of our study is that significant and reliable inferences regarding
Fig. 1 — Variation of steady-state gain ($g_s$) with plasma frequency ($\omega_p$) when $\omega_L = 0.9 \omega_0$, $\theta = 45^\circ$, $E_c = 10^7$ V m$^{-1}$.

Fig. 2 — Variation of transient gain ($g_t$) with the pump pulse duration ($\tau_p$) with the pump intensity ($I_m$) as a parameter, when $\omega_L = 0.9 \omega_0$, $\theta = 90^\circ$. (Curves I, II and III correspond to $I_m = 7.586 \times 10^{10}$, $1.165 \times 10^{11}$ and $1.60 \times 10^{11}$ W m$^{-2}$, respectively.)
nonlinear interactions in piezoelectric semiconductors can be drawn using a simple hydrodynamic treatment.

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