Performance analysis of a shallow solar pond water-heater

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A simple algebraic equation for water temperature is expressed in terms of time and space coordinate. This is useful for determining the water temperature at any instant of time at any position, along the length of the collector. The effects of collector-length, water-depth and the flow-velocity have been analysed. The performance of the system is also addressed under both the evaporation and without-evaporation conditions. For appreciation of analytical results, numerical calculations were made employing meteorological parameters for a typical winter day in Delhi.

1 Introduction

A shallow solar pond water-heater serves the dual purpose of collection and storage of solar energy in the same configuration. The depth of water in this solar water-heater is very small, typically only a few centimetres and owing to the shallow level of water, this system is called a shallow solar pond water-heater (SSP). The concept of a shallow solar pond water-heater emerged from the plastic bag water-heater which was conceived by the Japanese as early as 1930. Its use for the conversion of solar energy into low grade thermal energy has been a subject of intensive investigation, entailing both theoretical and experimental work, by several scientists for a number of years.\textsuperscript{1,2} Kudish and Wolf tested the performance of a compact shallow solar pond water-heater designed for camping and military use. Their prototype SSP water-heater consisted of a pillow type water bag encased in a wooden box with the bottom painted black and the top transparent, while the sides and the bottom were insulated.

At the top, the system had a thermally insulated cover which could be used as a booster mirror during the day; while during the off-sunshine hours, the same was used as an insulation cover, providing a unit with a means of overnight storage. Various designs of this system for different applications have also been studied by the scientists at the Lawrence Livermore Laboratory.\textsuperscript{3} Bansi developed an analysis of the system which was utilized further by Sodha et al.\textsuperscript{4} to compare the results of experimental measurements with the corresponding numerical calculations. They also performed an economic analysis in terms of the cost of useful energy and compared it with that of conventional systems. Garg et al.\textsuperscript{5} reviewed the work on shallow solar ponds and concluded that the performance of the system could be improved by the proper choice of the material and by optimizing the design and the modes of operation.

The analyses of shallow solar pond water-heaters presented by various authors\textsuperscript{6-16} have not taken into account the need of withdrawing hot water form the proposed solar water-heater. Sodha et al.\textsuperscript{17} presented a simple transient analysis for the behaviour of a solar collector/storage water-heater from which water is withdrawn at a constant flow rate. An analytical model for predicting the performance of such a solar water-heater was also developed by Husain et al.\textsuperscript{18} The analytical model presented by Husain et al.\textsuperscript{18} neglected the contribution of the heat energy required by the elemental water mass to raise its temperature per unit time so as to make the solution of the key differential equation easy. However, this approximation does not appear valid because one cannot neglect this heat energy, owing to the time-dependent behaviour of the input solar energy and ambient temperature. Further, the expression for water temperature obtained is a function of the space coordinate only, and hence, one cannot estimate the water temperature from this expression at any instant of time and at any position of the collector along its length.
Yadav et al.\textsuperscript{18} obtained solution for water temperature as a function of time and space coordinate. The shallow solar pond water-heater loses heat from the water surface to the glass cover via evaporative mode of heat transfer. This usually occurs as the glass cover does not remain exactly in contact with the water. Suppression of this evaporative loss will improve the performance of the system. Yadav et al.\textsuperscript{18} however, have not taken into consideration the evaporative heat losses from the water surface to the cover. This chapter presents the performance analysis of a shallow pond solar water-heater incorporating the evaporative heat losses from the water surface to cover. The water temperature has been obtained as a function of time and space coordinates. This is useful for determining the water temperature at any instant of time and at any position along the length of the collector. The effects of collector-length, water-depth and the flow velocity, and the performance of the system have also been studied. Explicit expressions are also derived for the temperatures of the glass-cover and the absorber. For appreciation of the analytical results, numerical calculations were made employing meteorological parameters for a typical winter day in Delhi.

2 Analysis

A schematic of a collection-cum-storage solar water-heater is shown in Fig. 1(a), while the cross-section view along the flow direction is shown in Fig. 1 (b). The assumptions with regard to the thermal analysis are as follows:

(a) The fractions of solar radiation scattered, reflected and absorbed by the transparent top cover and the water mass are negligible in comparison to that absorbed by the blackened bottom surface of the system.
The heat capacities of the material of the transparent top cover and the absorbing surface are negligible as compared to that of the water mass in the system.

c) There does not exist a temperature gradient along the thickness of the transparent top cover and the water mass.

d) The system is completely filled with water and, therefore, the evaporative heat transfer from the water surface to the inner surface of the cover is completely inhibited.

e) The water surface and the transparent top cover are parallel.

f) The surface areas of the cover, water and the absorber are the same.

g) The proposed system is insulated from the sides and the bottom to reduce conduction losses.

Energy Balances—With these assumptions, the energy balances at the various components of the proposed system can be written as:

For the transparent top cover:

\[ h(T_c - T_q) = h(T_c - T_a) \]  \hspace{1cm} (1)

Case-I When the glass-cover is in contact with the water of the shallow solar pond water-heater, the total heat transfer coefficient is equal to the convective heat transfer coefficient from the water surface to the glass-cover. Hence

\[ h_c = h_{wc} \]  \hspace{1cm} (2)

Case-II In case there is a gap between the water surface and the glass-cover the total heat transfer coefficient from the water surface to the glass-cover is equal to the sum of the convective, evaporative and radiative heat transfer coefficients from the water surface to the glass-cover of the shallow solar pond water-heater, that is,

\[ h = h_{wc} + h_{ev} + h_{rad} \]

Ignoring the contribution due to \( h_{rad} \), one can have:

\[ h = h_{wc} + h_{ev} \]  \hspace{1cm} (3)

For the elemental water mass: [Fig. 1(b)]

\[ b \lambda c \left( T_c - T_a \right) = b \lambda c \left( T_c - T_a \right) \]

\[ + m_c C_w \left( \Delta T_c / \Delta \tau \right) \text{d}x + b \lambda c h(T_c - T_a) \]  \hspace{1cm} (4)

The LHS of Eq. (4) stands for the energy available to the elemental water mass, while the first and second term on the RHS depict the energy absorbed by the elemental water mass and lost of the transparent top cover respectively. The energy absorbed by the elemental water mass consists of two parts; the first part denotes the energy absorbed by the elemental water mass per unit time and the second part stands for the energy absorbed by the elemental water mass over an elemental length \( d \), of the collector along the flow direction.

For the absorber surface:

\[ \alpha_c H = h_c(T_c - T_a) + h_c(T_c - T_a) \]  \hspace{1cm} (5)

Eqs (1) and (5) give:

\[ T_c = \left[ \frac{h_c(h_c + T_a)}{h_c(h_c + T_a)} \right] T_a + \frac{h_c(h_c + T_a)}{h_c(h_c + T_a)} T_a \]  \hspace{1cm} (6)

and

\[ T_c = \left[ \frac{h_c(h_c + T_a)}{h_c(h_c + T_a)} \right] T_a + \frac{h_c(h_c + T_a)}{h_c(h_c + T_a)} T_a \]  \hspace{1cm} (7)

Substituting the expressions for \( T_c \) and \( T_a \) from Eqs (6) and (7), respectively, in Eq. (4), one can obtain:

\[ \left( \frac{d T_c}{d \tau} \right) + \left( \frac{U_m}{C_w} \right) \Delta T_c + \left( \frac{U_m}{C_w} \right) \Delta T_c = f(t) \]  \hspace{1cm} (8)

where

\[ U = \frac{U_c + U_t}{C_w} \]

\[ U_t = \frac{h_c(h_c + T_a)}{h_c(h_c + T_a)} \]

\[ U_m = \frac{U_c}{h_c(h_c + T_a)} \]

\[ f(t) = \left( \frac{b \lambda c C_w}{h_c(h_c + T_a)} \right) \frac{U_j J_c}{C_w} \alpha_c H + \left( \frac{U_c + U_t}{C_w} \right) T_a \]

In order to solve the partial differential Eq. (8), the transformation relations, initial and boundary conditions to be used are as follows:

The transformation relations are:

\[ x + u_c \tau = \xi \]  \hspace{1cm} (9)

and

\[ x - u_c \tau = \eta \]  \hspace{1cm} (10)

The initial and boundary conditions are:

\[ T_c(x = 0, \tau = 0) = T_{in} \]  \hspace{1cm} (11)

Using Eqs (9-11), the solution of Eq. (8) can be written as:

\[ T_c = T_{in} e^{\frac{u_c}{x^2} \pm \sqrt{\frac{2 \lambda}{x^2} \left( \Delta T_c / \Delta \tau \right)}} \left[ \left( U_j J_c / C_w \right) \alpha_c H + T_a \right] \]

\[ + \left( 1 - e^{\frac{u_c}{x^2} \pm \sqrt{\frac{2 \lambda}{x^2} \left( \Delta T_c / \Delta \tau \right)}} \right) \frac{U_c}{C_w} T_a \]  \hspace{1cm} (12)
Putting $x = L$, $L$ being the collector length, in Eq. (12), one can obtain the water temperature at the outlet of the collector as:

$$T_{o} = T_{o}e^{-\frac{\alpha_{L}T_{o}+h_{b}}{1-e^{-\frac{\alpha_{L}T_{o}+h_{b}}{1-h_{b}}}}} + \left[\frac{U/h_{c}+T_{o}}{1-e^{-\frac{\alpha_{L}T_{o}+h_{b}}{1-h_{b}}}}\right]$$  \hspace{1cm} (13)

The useful heat obtained from the system and the collection efficiency $\eta$ can also be expressed as:

$$q_{u} = \Sigma m_{w}C_{w}(T_{oc}-T_{o}) \, dt \hspace{1cm} (14)$$

and

$$\eta \% = \frac{q_{u}}{H_{o} \, \Delta t} \times 100 \hspace{1cm} (15)$$

3 Results and Discussion

Eqs (6) and (7) are explicit expressions for the temperatures of the transparent top cover and the absorbing surface of the system. An explicit expression for the water temperature has been presented by Eq. (12). This expression depicts the profile of the water temperature in the system as a function of the time and space coordinates. However, it also shows the dependence of the water temperature on the system parameters, namely, flow velocity, water depth and inlet water temperature, and the meteorological parameters, namely, the solar intensity and the ambient temperature. Eq. (13) is the explicit expression for the water temperature at the outlet of the proposed system. This expression has been used further to obtain explicit expressions for the useful heat and the collection efficiency of the system, expressed by Eqs (14) and (15) respectively.

For evaluation of the various heat transfer coefficients, Duffie and Beckmann and Wong have been followed. Values of the relevant parameters used for the numerical calculations are:

- $h = 1.0 \, m$
- $h_{r} = 50 \, W/m^2^\circ C$ from Eq. (2)
- $h_{r} = 58.55 \, W/m^2^\circ C$ from Eq. (3)
- $h_{f} = 43.70 \, W/m^2^\circ C$
- $h_{s} = 185.8 \, W/m^2^\circ C$
- $h_{b} = 0.79 \, W/m^2^\circ C$
- $C_{w} = 4190.0 \, J/kg^\circ C$

Fig. 2 shows the hourly variation of solar intensity and ambient temperature.

The effects of space coordinate i.e. length of the

![Fig. 2 — Variation of solar intensity and ambient temperature](image-url)
collector, depth of the water and flow velocity are presented graphically in Figs 3 to 5. The dashed curves stand for the results calculated while ignoring evaporative heat losses from the water surface to the glass-cover. It is found from the Figs 3 to 5, the calculated and observed results approximately 0.5% errors are obtained with basis functions. From the Figs 3 to 5 it is clear that the calculated and observed results show excellent agreement.

On the other hand, the evaporative heat losses from the water surface to the glass-cover are included in the results depicted by the continuous curve.

![Graph 1](image1)

**Fig. 3 — Effect of space coordinate on hourly variation of water temperature**

![Graph 2](image2)

**Fig. 4 — Effect of water depth on hourly variation of water temperature**
4 Conclusions

On the basis of the above discussed results, the following conclusions have been drawn:

The hourly water temperature decreases with an increase in flow rate. The smaller the flow rate, the higher the water temperature. This happens because, with a smaller flow rate, the water mass in the system remains in contact with the absorbing surface for a comparatively longer time. Thereby taking more heat from the absorbing surface as compared to water flowing with a greater velocity.

As the depth of water flowing inside the system increases, the hourly water temperature decreases. As the length of the system increases, the hourly water temperature increases because of the increased absorber area. The performance of the system is improved marginally when the evaporative heat loss suppressed.

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References

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Nomenclature

- \( b \) = Breadth of collection-cum-storage solar water-heater (m)
- \( C_w \) = Specific heat of water (J/kg °C)
- \( d_w \) = Water depth (m)
- \( h_w \) = Convective heat transfer coefficient from water surface to the glass cover (W/m² °C)
- \( h_e \) = Evaporative heat transfer from coefficient from water surface to the glass cover (W/m² °C)
- \( h_r \) = Radiative heat transfer coefficient from water surface to the glass cover (W/m² °C)
- \( h_\text{ts} \) = Heat transfer coefficient from water surface to transparent cover (W/m² °C)
- \( h_t \) = Total heat transfer coefficient from transparent cover to ambient (W/m² °C)
- \( h_a \) = Heat transfer coefficient from absorbing surface to water (W/m² °C)
- \( h_\text{ra} \) = Overall heat transfer coefficient from absorbing surface to ambient through insulation (W/m² °C)
- \( H \) = Solar intensity (W/m²)
- \( L \) = Length of collection-cum-storage solar water-heater (m)
- \( L_i \) = Thickness of insulation (m)
- \( t \) = Time (s)
- \( T_a \) = Ambient temperature (°C)
- \( T_c \) = Cover temperature (°C)
- \( T_p \) = Temperature of absorbing surface (°C)
- \( T_{in} \) = Inlet temperature (°C)
- \( T_{out} \) = Outlet temperature (°C)
- \( T_w \) = Water temperature (°C)
- \( U_a \) = Flow velocity (m/s)
- \( X \) = Space coordinate (m)
- \( \alpha \) = Absorptivity of absorbing surface
- \( \rho_w \) = Density of water (kg/m³)