Single injection in a spherical insulator in CDMM mode

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The electrical behaviour of an insulator has been studied for a single injection (electrons) current flow containing significant density of thermally generated free carriers under density dependent mobility mode. The current injection in CDDM mode with spherical geometry has been further studied. In this problem, it is found that at low current injection, the pure Ohm’s law for current-voltage characteristics has finally been converted into the half power law at high injection level under space charge regime.

**Keywords**: Spherical insulator, Electron injection, Point contact, Density

**IPC Code**: HOIB

1 Introduction

Insulators have very vital and important role in electronics in order to fabricate electrical devices. Conventionally, they do not conduct charge under applied voltage but as per energy band theory, it is possible to inject the electrons into the desired geometry with a suitable contact as in Fig. 1. The space-charge-limited single injection current in various geometries i.e., rectangular, cylindrical, spherical is well known 1-3. When the thermally generated carriers become evident in the operation, the problem becomes highly complex. The spherical geometry can be arranged to inject the electrons from the central point contact as a cathode and the surface be a collector (anode) the mobility, which is the quality of the carrier movement, becomes a function of carrier density. In this case, carriers are electrons. The spherical geometry is divided into different regions as per the injection level, space charge and thermal free carriers.

2 Analysis

If we consider single injection spherical diode containing significant density of free carriers $n_0$, which are thermally generated. The diode is working under CDDM mode as shown in Fig. 2.

The general equations characterizing the current flow and Poisson’s law are given by,

$$I = 2\pi e \mu n r^2 E$$

$$\frac{\varepsilon}{e} \frac{1}{r^2} \frac{d}{dr} (r^2 E) = n - n_0$$

where $\varepsilon$ is the permittivity of the material of diode (spherical), $I$ the total current, $\mu$ the carrier density dependent mobility, $N$ the concentration of free carriers, $e$ the electronic charge, $r$ the one dimensional spherical co-ordinate ranging from $r_c$ to $r_a$, $r_c$ the cathode radius, $r_a$ the anode radius, $E$ the electric field, $n$ is the carrier concentration.
In low mobility insulators as shown by Wintle\(^2\), that the mobility is varying with the carrier concentration. Also the variations in mobility are subjected to boundary condition for ohmic contact.

The approximate regional investigation of current and carrier concentration can be applied to the general equations as follows from the work of previous researchers\(^1,3,4,7\). The insulator is divided into two virtual regions by a transition plane at spherical radius \(r_x\). The region I is space charge limited and region II is ohmic.

**Region I** — \((r_c < r < r_x)\), the current and carrier concentration for this region are:

\[
I = 2 \pi e \mu n r^2 E \quad \text{...}(3)
\]

\[
\varepsilon \frac{1}{e r^2} \frac{d}{dr} (r^2 E) = n \quad \text{...}(4)
\]

The transition plane \(r_x\) is defined by:

\[
n(r_x) = n_0 \quad \text{...}(5)
\]

Using Eqs (3-5) and using boundary condition with mobility considerations, the electric field intensity in this region will be as:

\[
E(r) = \frac{e_1^{1/3}}{16 \pi e^2 h} \left| \frac{r^3 - r_x^3}{r^3} \right| \quad \text{...}(6)
\]

where \(h\) is Planck’s constant.

From Eqs (4) and (6), the carrier concentration in region I is given by:

\[
n = \left| \frac{e_1^{1/3}}{2 \pi h^2 e} \left| \frac{1}{(r^3 - r_x^3)} \right| \right| \quad \text{...}(7)
\]

The transition plane can be deduced from Eqs (5) and (7) as:

\[
r_x = \left| \frac{e_1^{1/3}}{2 \pi h^3 e n_0^3} \right| \quad \text{...}(8)
\]

From Eqs (6) and (7), the voltages across region I is given by:

\[
V_{c,x} = \int_{r_c}^{r_x} E(r) \, dr = \left| \frac{l^2}{32 \pi^2 e_0 e h^2 n_0^3} \right|^{1/3} \quad \text{...}(9)
\]

where \(r_x \gg r_c\)

**Region II** — \((r_s \leq r \leq r_a)\), the current and carrier concentration for this region are:

\[
I = 2 \pi \mu n_0 r^2 E \quad \text{...}(10)
\]

\[
\varepsilon \frac{1}{e r^2} \frac{d}{dr} (r^2 E) = 0 \quad \text{...}(11)
\]

The relation between mobility and thermally generated carriers which is available in literature is given by:

\[
\mu = h n_0 \quad \text{...}(12)
\]

The electric field intensity in ohmic region II is given by Eqs (10) and (12) as:

\[
E(r) = \frac{1}{4 \pi^2 e^2 h^2 n_0^3 r^2} \quad \text{...}(13)
\]

From Eqs (8) and (13), the voltage across region II is given by:

\[
V_{x,a} = \int_{r_x}^{r_a} E(r) \, dr = \left| \frac{1}{32 \pi^2 e_0 e h^2 n_0^3} \right|^{1/3} \quad \text{...}(14)
\]

where \(r_x \ll r_a\)

Eqs (9) and (14) are exactly equal and hence, the voltage under these regions are equal where \(r_x\) is far away from both anode and cathode which further signifies the situations of a linear current flow during middle of the operation.

The total voltage developed across the insulator is observed as:

\[
V = V_{c,x} + V_{x,a} = 2 V_{c,x} \quad \text{...}(15)
\]

\[
= \left| \frac{l^2}{4 \pi^2 e^2 h^2 n_0^3} \right|^{1/3} \quad \text{...}(16)
\]

**3 Discussion**

It is found that due to the current-voltage analysis, the single injection spherical insulator in CDDM mode was started by ohmic regime in the beginning where the injected current carriers were very small or negligible. The region II was only present inside the
insulator. In medium injection level, the transition regime occupies the insulator with both region I and II. At high injection level, the insulator fills with space charge and region I spreads throughout the diode. The intersection levels of the current and voltage can be found from Eq. (15), which will be self-explanatory in term of the transition regime.

It will be evident from Eq. (17) and regionwise treatment of insulator that if the applied voltage increases, the Ohm’s law which was valid over the low injection levels turn phase. The injected space charge spreads essentially along with transition region with $r_x$ and this relation leads the consequence of $3/2$ power law.

A graph is drawn in Fig.3 to give more explanation to the resultant Eq. (17). The equation is self-explanatory to understand the current-voltage characteristics.

4 Conclusion

The current-voltage characteristics for a space-charge limited single injection spherical solid state insulator operating in carrier density mobility mode have been analysed with a regionwise treatment. It is evident from above analysis that the characteristics vary as per the injection level of the injected carriers, viz., electrons and duration of operation for the thermal reasons. This reveals that the characteristics in the beginning were pure Ohm’s law type but finally convert into space-charge limited cube law via $3/2$ power law with a transition mode. The study has wider applications in solid state electronics as to understand the insulating materials and also to fabricate new ones.

References