Importance of coherence length in explaining superconducting behaviour of cuprate superconductors

Bhaskar Gahtori, Ratan Lal & S K Joshi

Theory Group, National Physical Laboratory, Dr K S Krishnan Road, New Delhi 110 012

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It has been argued that, in explaining their data on cuprate superconductors some authors appear to have considered incorrect variation of the coherence length with impurity concentration. Considering two cases from the existing literature, namely the explanation of critical current density of Pr doped YBa$_2$Cu$_3$O$_7$ samples by Hodge et al.$^1$ [Phys Rev B, 48 (1993) 6465], and the explanation of the penetration depth of Fe doped YBa$_2$Cu$_3$O$_7$ samples by Hayani et al.$^2$ [Physica C, 221 (1994) 319], it has been clarified that, these proposed explanations may be misleading.

1 Introduction

In order to explain behaviour of their experimental data on impurity doped cuprate superconductors, several authors have used results of the standard Bardeen-Cooper-Schrieffer (BCS) theory. In doing so, they have considered variation of the coherence length with impurity concentration in the same manner as is done for the superconductors where impurity does not change the transition temperature. This is because, according to these authors, $d\xi(x)/dx < 0$ where, $\xi(x)$ is the coherence length corresponding to impurity concentration $x$. This behaviour of the coherence length may not be true in cuprate systems because, in these systems, an impurity can have both the magnetic as well as the electric effects associated with it and these cause a drop of the transition temperature. In view of this, the authors make here, an assessment of the variation of $\xi(x)$ with $x$ for cuprate superconductors. The importance of such a study shall be demonstrated by evaluating the role of the coherence length in the following two examples: (1) The first is the behaviour of critical current density $J_c(x)$ in the Pr doped YBa$_2$Cu$_3$O$_7$ samples (Ref. 1). (2) The second example is related to the behaviour of the in-plane penetration depth $\lambda_{ab}(x)$ of the Fe doped YBa$_2$Cu$_3$O$_7$ systems (Ref. 2).

2 Theory

In cuprate superconductors, an impurity has, in general, both the magnetic and electric characters. For example, in the Zn doped YBa$_2$Cu$_3$O$_7$ systems Zn shows magnetic$^3$ as well as electric$^4$ effects. In the superconducting state, the effects of the magnetic scattering and potential scattering will depend on the nature of the superconducting pairing. For the $s$-wave pairing, magnetic scattering will be pair-breaking$^5$, while, potential scattering will not change $T_c$ until the state of the system is near the metal-insulator transition$^6$. The roles of the magnetic scattering and potential scattering are reversed for the $d$-wave pairing$^7$.

Let $\xi_0(x)$ be the BCS coherence length, corresponding to the impurity concentration $x$. For cuprate superconductors, it may be written as$^7$:

$$\xi_n(x) = \frac{2h\nu_F}{nk_BT_c(x)G(x)}$$

where, $h$ is the reduced Planck’s constant, $\nu_F$ is Fermi velocity, $k_B$ is Boltzmann constant, and $G(x)$ is the gap ratio $2\Delta(x)/k_BT_c(x)$, for cuprate superconductors. Since, in general, an impurity has both the magnetic moment and electric charge, there will exist two different mean free paths, which can be denoted by $l_M$ and $l_P$, respectively. Then, the net coherence length for the $s$-wave pairing, say $\xi_s(x)$.
will be given, in the clean limit \( [h_0d_i'] > \xi_i(x) \) by Eq. (151) of Allen\(^2\). When recast in the form of \( l_M \) and \( l_P \), this equation of Allen reads as:

\[
\frac{1}{\xi(x)} = \frac{1}{\xi_0(x)} + \frac{5}{2l_M(x)} + \frac{1}{2l_P(x)} \quad \ldots (2)
\]

As already mentioned, magnetic scattering is pair breaking for a s-wave superconductor. So, \( T_c(x) \) will decrease due to finite \( l_M(x) \). Because of this, and according to Eq. (1) \( \xi_i(x) \) will increase. On the other hand, the term involving \( l_M(x) \) in Eq. (2) will tend to suppress this effect of increasing \( \xi_i(x) \) with \( x \). Because of this, for low \( x \), it may be assumed that:

\[
\frac{1}{\xi(x)} = \frac{1}{\xi_0(x)} + \frac{1}{2l_P(x)} \quad \ldots (3)
\]

Under such circumstances, Eq. (2) is reduced to:

\[
\frac{1}{\xi(x)} = \frac{1}{\xi_0(0)} + \frac{1}{2l_P(x)} \quad \ldots (4)
\]

This expression of \( \xi(x) \) is different from the usual clean-limit expression (cf. e.g. Ref. 7) in that here, \( 2l_P(x) \) appears in place of \( l_P(x) \). Let us see the effect of this difference. When \( l_P(x) \) decreases, a situation will arise when, the system will be in the dirty limit. For \( \xi_0(0) = l_P(x) \) the dirty-limit value of \( \xi(x) \) will be 0.85 \( l_P(x) \) (Ref. 5). On the basis of Eq. (4), the clean-limit value of \( \xi(x) \) for \( \xi_0(0) = l_P(x) \) will be 0.67 \( l_P(x) \). On the other hand, the clean-limit value of \( \xi(x) \) for \( \xi_0(0) = l_P(x) \) when \( 2l_P(x) \) in Eq. (4) is replaced by \( l_P(x) \) will be 0.51 \( l_P(x) \). This means that, the clean-limit value of \( \xi(x) \) for \( \xi_0(0) = l_P(x) \) is closer with the dirty-limit value when \( \xi(x) \) is expressed by Eq. (4). From this, it can be inferred that, Eq. (4) is valid for \( \xi(x) \), for a relatively larger value of \( x \) than the usual expression used (e.g. in Ref. 7).

3 Results and Discussion

In order to show the importance of the coherence length, the two examples mentioned above can be considered.

3.1 Example 1: \( l_P(x) \) of Pr doped YBa\(_2\)Cu\(_3\)O\(_y\) samples

Now, considering how coherence length \( \xi(x) \) varies with \( x \) in the Pr doped samples of YBa\(_2\)Cu\(_3\)O\(_y\), studied by Hegde \textit{et al.}\(^1\). According to these authors, the Pr ions do not cause magnetic scattering so that, \( l_M = \infty \). Moreover, the \( T_c \) degradation is stated to be due to the reduction of holes with increasing Pr concentration. Thus, according to Eq. (1), \( \xi_i(x) \) will increase with \( x \), and \( \xi(x) \) will be given by taking \( l_M = \infty \) in Eq. (2). That is to say:

\[
\frac{1}{\xi(x)} = \frac{1}{\xi_0(x)} + \frac{1}{2l_P(x)} \quad \ldots (5)
\]

Since \( \xi_0 \) increases with \( x \), while \( l_P(x) \) decreases with \( x \), the overall value of \( \xi(x) \) may increase or decrease with \( x \). From the data of Hegde \textit{et al.}\(^1\), it is not possible to estimate \( l_P(x) \). However, the advantage of the fact may be taken according to Fig. 1 of Hegde \textit{et al.}\(^1\). The \( R(T) \) versus \( T \) plots of the Pr doped YBa\(_2\)Cu\(_3\)O\(_y\) samples are metallic up to 35 \% Pr. This means that, there is no chance of weak localization up to \( x = 0.35 \). In this sense, the Mottn-Joffe-Regel condition, namely, \( k_F l_P(x) > 1 \), is satisfied. Here, \( k_F \) is the Fermi wave vector. A reasonable estimate of this condition requires that, \( k_F l_P(x) > 10 \). The value of \( k_F \) may be obtained by using \( k_F = m^* v_F / \hbar \) with \( m^* = 2.5 m_e \) (Ref. 10) and \( v_F = 2 \times 10^7 \) cm/sec (Ref. 11) for the YBa\(_2\)Cu\(_3\)O\(_y\) systems. Here, \( m^* \) is the effective mass of the carrier hole, and \( m_e \) is the bare mass of electron. On estimating \( k_F = 0.4 \AA^{-1} \) so that, \( l_P(0.35) > 25 \AA \).

According to Eq. 1 of Ref. 1, \( R(T=0) \) is about a half for \( x = 0.3 \) than that for \( x = 0.35 \). Thus, \( l_P(0.3) > 50 \AA \). On the basis of Eq. (5), the minimum value of the factor by which \( \xi(x) \) remains unchanged when \( l_P(x) \) decreases by a factor \( b \) is found to be given by:

\[
c = \frac{15}{16b} \quad \ldots (6)
\]

In obtaining Eq. (6), the authors have used the fact that, for \( x = 0, \xi_0(0) = 10 \AA \) (Ref. 1) and \( l_P(0) = 75 \AA \) (Ref. 1). In order to utilize Eq. (6), it is noticed that, the above arguments suggest, \( l_P(0.3) > 50 \AA \). Even if, \( l_P(0.3) \) is smaller than this, say \( l_P(0.3) = 25 \AA \), Eq. (6) demands \( c > 1.15 \). if, \( \xi_i(x) \) were to increase with \( x \). Since, \( T_c(0)/T_i(x) \) is about 1.6 for \( x = 0.3 \) (Ref. 1), and \( G(x) \leq G(0) \) (Ref. 7), on the basis of Eq. (1) it is expected, \( c < 1.6 \). This is much larger than 1.15, thereby, implying that \( \frac{d\xi(x)}{dx} > 0 \). This is contrary to the assumption of Hegde \textit{et al.}, who considered \( d\xi(x)/dx < 0 \). In fact, on the basis of \( c > 1.6 \) against the requirement of \( c = 1.15 \), it is expected, \( \xi(x) = 12.0 \AA \) for \( x = 0.3 \). It may be noted that, if the interpolation formula of Kim &
Overhauser\textsuperscript{12} is used, the value of \( \xi(x) \) comes out to be 15.8 Å for \( x = 0.3 \).

It can now be seen, what impact \( \xi(0.3) = 12 \) Å will have on the interpretation of results of Hegde \textit{et al.}\textsuperscript{1}. Putting this value of \( \xi(0.3) \) in Eq. (3) of Ref. 1 it is obtained, \( J_c(0.3)/J_c(0) = 6 \times 10^{-7} \). This reduction of \( J_c(0.3) \) is too large to be comparable with the observed reduction of \( 3 \times 10^{-1} \). In fact, the situation will be even worse if, the value of \( \xi(x) \) is used on the basis of the interpolation formula of Kim \& Overhauser\textsuperscript{12}. This, therefore, raises doubts about the explanation of the observed reduction of \( J_c(x) \) with \( x \).

3.2 Example 2: \( \lambda_{ab} \) of Fe doped YBa\textsubscript{2}Cu\textsubscript{4}O\textsubscript{7} samples

Now, considering the role of the coherence length in determining the behaviour of the \( ab \)-plane penetration depth \( \lambda_{ab}(x) \) measured by Hayani \textit{et al.}\textsuperscript{2} for the Fe doped YBa\textsubscript{2}Cu\textsubscript{4}O\textsubscript{7} samples. One of the main results of Hayani \textit{et al.}\textsuperscript{2} is that, initially, for low Fe concentration \( x \) \( \lambda_{ab}(x) \) shows a rapid increase. In order to explain this behaviour, Hayani \textit{et al.}\textsuperscript{2} have considered expression of \( \lambda_{ab} \) in the BCS dirty limit. The variation of \( \lambda_{ab} \) is given by [cf. Eq. (3) of Ref. 2].

\[
\lambda_{ab}^2 \approx \left[ 1 + \frac{\xi_{ab}(x)}{l_p(x)} \right] \quad \text{(7)}
\]

Hayani \textit{et al.}\textsuperscript{2} appear to have assumed \( d\xi_{ab}(x)/dx < 0 \) so that, for a rapid increase of \( \lambda_{ab} \), for low \( x \), \( l_p(x) \) should decrease to un-physical lower values. This picture is misleading since, according to Lan \textit{et al.}\textsuperscript{13} \( d\xi_{ab}(x)/dx > 0 \), for Fe doped YBa\textsubscript{2}Cu\textsubscript{4}O\textsubscript{7} single crystals. Thus, according to Eq. (7) \( \xi_{ab}(x) \) will increase rather than decreasing it. Quantitatively, for 4 \% Fe, \( \xi_{ab}(x) \) will increase by a factor of about 1.5. This means, a much less reduction of \( l_p(x) \) with \( x \) will be required to explain the rapid increase of \( \lambda_{ab} \) for low \( x \). Therefore, \( l_p \) is not expected to be un-physically low. Consequently, the observed increase of \( \lambda_{ab} \) for low \( x \) may be understood in the BCS dirty limit.

4 Conclusion.

In conclusion, the authors have made an effort for clarifying the importance of the coherence length in explaining various properties of cuprate superconductors. Considering the measurements of Hegde \textit{et al.}\textsuperscript{1} and of Hayani \textit{et al.}\textsuperscript{2} the authors have demonstrated how the knowledge of variation of coherence length in cuprate superconductors with impurity concentration help in deciding whether, an explanation of the observed behaviour of a physical property is reasonable or not.

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References