High spectral purity cavity stabilized microwave oscillators

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Low noise microwave oscillators and novel techniques of reducing the close-in noise in these oscillators by using a high Q cavity as a frequency discriminator have been presented. In essence the discriminator converts noise-induced frequency fluctuations into corresponding phase variations of a signal reflected from it. An accurate model for the noise floor of the cavity discriminator has been described. The noise cancellation depends on factors such as—the cavity Q, microwave power into the cavity, the input coupling coefficient and the level of carrier suppression. Experimental results indicate reasonably good agreement with the model.

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1 Introduction

Microwave oscillators are crucial components of modern telecommunications, navigation and radar systems. They are typically used in these applications as the local reference or clock signal in secure communication protocols, very high speed jitter sensitive modulation-demodulation schemes and high-resolution digital signal processing applications. Significant advances in the performance of microwave oscillators, especially in reducing the magnitude of their phase and amplitude fluctuations, result in dramatic improvements in all the above applications. In addition to the commercial and military uses, microwave oscillators of high spectral purity play an essential role in metrology and precise physical experiments such as in atomic clocks, tests of general relativity and microwave spectroscopy. In the present paper the design of low noise microwave oscillators employing novel techniques of noise reduction by using high-Q cavity resonator as discriminator has been presented. It is perhaps appropriate to give a very brief description of the expressions that are commonly used for characterization of the noise or frequency instability in oscillator for benefit of readers who are not specialists in the area.

The general description of an oscillator output in the presence of noise is given by the standard expression:

\[ V(t) = (V_0 + \epsilon(t)) \sin[2\pi f_0 t + \phi(t)] \]  \hspace{1cm} (1)

where, \( V_0 \) and \( \nu_0 \) are the nominal amplitude and frequency respectively, \( \phi(t) \) and \( \epsilon(t) \) are the random components that represent phase modulation (PM) and amplitude modulation (AM) due to noise processes. To quantify the measure of PM noise we normally use the spectral density \( S_{\phi}(f) \) of the fluctuations of phase \( \phi(t) \). \( S_{\phi}(f) \) has the dimensions of \([\text{radians}^2/\text{Hz}]\). Another quantity very commonly used in describing the PM noise measurements is \( L(f) \) which is related to \( S_{\phi}(f) \) by:

\[ L(f) = 10 \log \left( \frac{1}{2} S_{\phi}(f) \right) = 10 \log S_{\phi}(f) - 3 \text{ dBc/Hz} \]  \hspace{1cm} (2)

Provided the RMS phase deviation due to PM noise is small, \( L(f) \) can be identified as the ratio of the single-sideband (SSB) noise-power to the carrier-power for a unit bandwidth at a Fourier frequency \( f \). It is common to refer to \( L(f) \) as SSB phase modulation noise or simply phase noise. It has been shown from both theoretical considerations and experimental measurements that the types of noise at the output of a high stability oscillator can be represented as a power law expression with the PM noise spectral densities varying as powers of \( f \). More specifically, we
can write the following expression:

$$L(f) = \sum_{a=0}^{1} \frac{1}{a!} f^a$$ ... (3)

The different values of $\alpha$ in the summation in Eq.(3) represent respectively, $\alpha=0$, white phase noise; $\alpha=1$, flicker phase noise; $\alpha=2$, white frequency noise; $\alpha=3$, flicker frequency noise; $\alpha=4$, random walk frequency noise. For short observation times, usually less than 1s, an oscillator is characterized using frequency domain description as given in Eq.(3).

2 Free-Running Resonator Loop Oscillator

Figure 1 shows the basic schematics of a microwave resonator loop oscillator. It contains an electromagnetic resonator, which predetermines the frequency of oscillation, and a sustaining stage (amplifier), which defines the stationary amplitude of oscillations. Both gain and phase conditions have to be met for the oscillator to operate. The former condition is met by choosing a small signal amplifier gain that is larger than the total loss in the loop. The latter condition is satisfied by setting the phase shift around the loop close to an integral multiple of $2\pi$. The frequency of oscillation is found from the condition:

$$f_{osc} = f_{res} + \frac{f_{osc}}{2Q_L} \tan \theta$$ ... (4)

where $\theta$ is the phase shift around the loop with the resonator's phase shift excluded. $f_{osc}$ and $Q_L$ are respectively the resonance frequency and loaded $Q$ factor of the resonator. $Q_L$ is given by $Q_L = Q_0/(1+\beta_1+\beta_2)$ where $Q_0$ is the unloaded cavity $Q$ and $\beta_1$, $\beta_2$ are the coupling coefficients. It follows from Eq. (4) that the fluctuations in the loop phase shift $\theta$, caused mainly by amplifier phase fluctuations, result in fluctuations of the oscillator frequency. The phase noise of the loop amplifier is usually characterized by a white phase noise component ($f^0$) and a flicker phase component ($f^1$) and is given by:

$$L_{amp}(f) = B_f \frac{f}{P_s}$$ ... (5)

where $B$ is the amplifier noise figure, $T_a$ the amplifier's physical temperature, $k_B$ the Boltzmann's constant, $P_s$ the power at the input of the amplifier and $b$ is the flicker noise component at 1Hz. The Fourier frequency at which the flicker and white noise components are equal is known as the flicker corner, $f_l$. The relationship between the $L_{amp}(f)$ and the oscillator output was first derived by Leeson and are given below for the oscillator outputs 1 and 2 as shown in Fig. 1:

$$L_1(f) = L_{amp}(f) \left[ 1 + \left( \frac{f_{osc}}{2Q_L} \right)^2 \right]$$ ... (6)

$$L_2(f) = L_{amp}(f) \left( \frac{f_{osc}}{2Q_L} \right)^2$$

It is clear from the Leeson's model that the $1/f$ flicker-phase noise of the amplifier is converted to $1/f^2$ flicker-frequency noise by the oscillator loop. Likewise the amplifier's white phase noise gets converted to $1/f^2$ white-frequency noise. Thus, the oscillator phase noise usually has a $1/f^2$ behaviour close to the carrier changing to $1/f^4$ beyond the flicker corner of the amplifier in the loop. This is typically what is experimentally observed in a variety of microwave oscillators that use some kind of a resonator in the configuration as shown in Fig. 1.

As follows from Eq.(6), the obvious way to improve the phase noise of the free running oscillator is to implement a low-noise amplifier and a high $Q$ resonator. The lowest-noise amplifiers are constructed using Si bipolar transistors, which unfortunately do not operate in the X-band and beyond. Thus most common amplifiers at these microwave frequencies are constructed using GaAs FET's, which exhibit large $1/f$ flicker component when operated with high signal level. A typical phase noise behaviour for one of the
low noise GaAs FET amplifiers, used in a low-noise oscillator at 9.7 GHz, is given by:

$$L(f) = -102.5 - 12.5 \log_{10}[f] \text{ dBc/Hz} \quad (7)$$

The highest Q values at microwave frequencies are realized using sapphire dielectric resonators operating in very high order modes or the so-called whispering gallery modes. Typical values of $Q_L$ obtained with sapphire loaded cavities at room temperature are about $10^5$, about $10^7$ at 77 K and about $10^9$ at liquid He temperatures of 4 K. A free running X-band oscillator with one of the lowest phase noises has been constructed using a cryogenic sapphire loaded cavity with $Q_L=2.5 \times 10^9$ and a GaAs FET amplifier with noise as in Eq.(10) has been found to have $L(f) = -140$ dBc/Hz at 1 kHz from the carrier. This is consistent with the Leeson’s model in Eq.(6) for the output 1.

### 3 Phase Noise Reduction using a Cavity Discriminator

The extremely high Q resonators are often required in order to obtain acceptable low phase noise performance of free running oscillators. Accordingly, the oscillator expense can be effectively dictated by the high cost of sapphire dielectric resonators and set up for cryogenic operation.

A novel way to reduce the close in phase noise of an oscillator was first proposed by Galani et al. The system basically consists of stabilizing the microwave oscillator with a frequency control system as shown in Fig. 2. Here the high-Q cavity is used both as a band pass filter in the loop oscillator and as a dispersive element. Due to dispersion, the phase noise induced frequency fluctuations of the oscillator are transformed into corresponding phase fluctuations in the signal reflected from the cavity. The mixer acts as a phase detector that compares the phase of the reflected signal with a reference derived from the oscillator output and so the entire arrangement works like a frequency discriminator. Feeding the phase detector output back, with suitable low-pass filtering and amplification, to control the loop phase shift results in suppression of the oscillator phase noise. For a high gain frequency control loop, the phase noise suppression is primarily limited by the intrinsic noise in the discriminator. These arise mainly due to the phase noise in the mixer and to a lesser extent in the ferrite circulator. Walls used a high Q cavity as an external discriminator to lower the phase noise of an oscillator locked to it. In this case of a Cavity Stabilized Oscillator (CSO), the discriminator feedback modulates the oscillator frequency using a voltage controlled tuning port. Subsequently, Dick et al. improved the basic cavity discriminator technique by making the input coupling coefficient very close to critical coupling ($\beta \sim 1$) so that there is a deep notch in the reflected signal at the carrier frequency. The level of the suppressed carrier is then brought up by introducing an amplifier before the mixer. Introduction of the amplifier serves to increase the loop gain. By virtue of the fact that even at X-Band it is possible to get amplifiers with much lower phase noise than mixers, the amplifier also helps to swamp the mixer noise. Ivanov et al. have further improved the discriminator performance by introducing interferometric carrier suppression. This consists of vectorially combining a reference signal having the same amplitude and exactly opposite phase with the reflected output. The reference is also derived from the oscillator output by appropriate attenuation and phase shifting. The resultant carrier suppression thus obtained can be made so high that the amplifier almost sees no carrier. This greatly suppresses the 1/f flicker noise and by using an amplifier with low noise figure, it is possible to get an effective amplifier phase noise that is close to thermal noise floor. Recently, it has been shown by Sen Gupta et al. that even using a conventional air-dielectric cavity with a modest Q of about 70,000 (compared to the more esoteric and expensive sapphire loaded systems) as frequency...
At discriminator it is possible to get very high noise suppression by amplifying the oscillator output before feeding it to the cavity. Figs 3(a) and 3(b) show the final noise reduction arrangements using a cavity discriminator, both within the oscillator as well as in the CSO mode, by incorporating the improvements mentioned above.

4 Model of Noise Cancellation using Cavity Discriminator

Assuming that the gain in the feedback loop is high enough, the extent of phase noise suppression is limited by the noisy components within the discriminator. We characterize the noise suppression of a discriminator by stating its Noise Floor $L_{D0}$ ($f$) which is the lowest phase noise that an oscillator can reach by using it. A detailed model computation of the noise floor has been performed by Sen Gupta et al. who have shown that for the setups in Fig.3 (a and b) it is given by:

$$L_{D0}(f) = k_B(T_{amp} + T_0) + \frac{(1 + \beta_c)^2}{4\beta_c} HLB^2 f + \left[1 + \frac{1}{4\beta_c^2 HLB^2 f} \right]^{\frac{L_{circ}}{L_{D0}}}(f)$$

where $HLB = V_{res}/(2Q_0)$ = Half loaded bandwidth of the cavity. $V_{res}$ and $Q_0$ are respectively the centre frequency and loaded Q of the cavity. $\beta_c = \beta / (1 + \beta_c)$ = effective input coupling coefficient

of the cavity. $P_i$ = power input into the cavity; $T_{amp}$ = noise temperature of the microwave amplifier; $T_0$ = amplifier’s physical temperature and $E_{th}/f$ = phase noise of the ferrite circulator.

In Eq.(8) it has been assumed that since the carrier is very highly suppressed the flicker noise of the amplifier is very small and also the noise of the mixer is swamped by the amplifier gain. The first term in Eq.(8) corresponds to the thermal noise of the amplifier and its contribution can be reduced by (a) increasing the cavity Q, (b) making $\beta_c$ approach unity or critical coupling and (c) increasing the power input into the cavity. The second term in Eq.(8) is due to the noise of the circulator. Although the circulator noise is very small, its contribution becomes appreciable at low Fourier frequencies (or close to the carrier), unless $\beta_c$ is very nearly equal to unity.

5 Ultra-Low Noise Cavity Stabilized Oscillators

Using a setup as in Fig.3(a) Ivanov et al. have constructed an ultra-low noise Sapphire loaded cavity oscillator (SLCO). We have computed the expected noise floor using Eq.(8) for this SLCO using the experimental parameters: $\nu_{os} = 10$ GHz; $Q_0 = 190,000$; $\beta_1 = 0.75$; $\beta_2 = 0.15$; $P_i = +17$ dBm and also include the noise of the circulator and the VCP. This is shown in Fig. 4 by the curve marked Model Computation. For Fourier frequencies less than 2 kHz the noise floor falls off at $1/f^3$ beyond which it tapers off to $1/f^5$. Beyond about 50 kHz the expected noise floor of the discriminator falls below the thermal noise level of -190 dBm at this output power. The experimental noise
Free Running Oscillator

Model Computation

Noise Reduced Using Discriminator

Thermal Noise Floor

Fig. 4—Experimental results of Ivanov et al. for a Sapphire loaded cavity oscillator compared with model computations in Eq. (8).

Fig. 5—Experimental results of Sen Gupta et al. for a air dielectric cavity stabilized YIG oscillator compared with model computation in Eq. (8).

measurements of Ivanov et al. have been superimposed on this graph. At 1 kHz the measured phase noise is -150 dBc/Hz, which corresponds to a noise reduction of about 50 dB due to the cavity discriminator compared to the free running oscillator. The agreement between the reduced noise values and the model computations is observed to be quite good. The deviations at frequencies higher than about 10 kHz are due to inadequate servo gain of the frequency control loop. Also, the somewhat higher experimental values at lower Fourier frequencies can be explained due to resonance frequency fluctuations of the cavity due to inadequate temperature control.

We also consider the case of a Cavity Stabilized YIG oscillator as discussed by Sen Gupta et al. The experimental set-up is similar to that given in Fig.3(b) and the different parameters are: \(v_0 = 10\) GHz; \(Q = 73,000\) (TE025) and 59,000 (TE023); \(\beta_1 = 0.95; \beta_2 = 0.02; P_i = +33\) dBm. Also shown in Fig. 5 are the measured values of the phase noise of the free running YIG oscillator and for the reduced noise cavity discriminator setup. The noise reduction due to the cavity discriminator in this case is as high as about 100 dB at 1 kHz. The best agreement between the measured values and model computations is within 6-9 dB and it happens between 1-10 kHz. At higher and lower Fourier frequencies the cause of disagreement is about the same as discussed in the previous case.

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