Dynamics and the characteristics behaviour of soliton propagation in dusty plasmas

G C Das, Karabi Devi, Jnanjyoti Sarma & Nirupama Devi**

Mathematical Sciences Division, Institute of Advanced Study in Science & Technology, Khanapara, Guwahati 781 022
*Department of Mathematics, R G Barah College, Guwahati 781 025
**Department of Mathematics, Cotton College, Guwahati 781 001

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Formation of various solitary waves in a plasma contaminated with dust charged grain have been investigated. Based on pseudopotential analysis, nonlinear Sagdeev potential equation has been derived, from where acoustic modes describing the soliton propagation in dusty plasma have been highlighted. Moreover, for showing the formation of different features of solitons, sagdeev potential has been expanded due to which different nonlinear wave equations have been formed. It has been shown that the controlling of dust charge composition expects the coexistence of solitons, double layer or shock-like wave along with other nonlinear waves. The novelty of the present investigation has been emphasized to a modified formalism, called tanh-method, in finding the soliton solution from nonlinear wave equations. The success of using tanh-method derives soliton propagation from the Sagdeev potential equation in dusty plasma. Further, other nonlinear modes have been shown which enable to predict the collapse along with the conditions required for its existence.

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Email: gcdas44@sify.com; nirupama_cotton@rediffmail.com

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1 Introduction

Study on nonlinear ion-acoustic waves has attracted a good deal with a prime interest to show the evolution of solitary wave propagation in plasmas. The nonlinear wave phenomena have been established in fluid dynamics through the augmentation of a simple nonlinear wave equation, called Korteweg-deVries (K-dV) equation along with other nonlinear wave equations derived as Schrödinger equation, sinh-Gordon equation, Boussinesq equation in finding solitons and thus new techniques have been developed to solve them. The nonlinear wave dynamics has taken place in plasmas during sixties12, but stirred up with fast growing interest especially through the derivation of K-dV equation. The heuristic features of solitons, describing the nonlinear wave propagation, have found a great support to the experiments as well as to satellite observations and finally provided a basis in bridging well between theory and experiments6,9. Further, tremendous boosting has been taken up while studying the soliton dynamics in multicomponent plasmas especially when plasma has been contaminated with additional negative ions6-10. Many theoretical observations have proven in laboratory plasma as well as support the satellite observations in space.

In the present decade, a new motivation has been building up to endeavour the study on nonlinear acoustic waves in plasmas contaminated by the dust particles, which grow increasing interest not only in space plasmas but also in laboratory devices12. The study on the nonlinear wave in plasma, consisting of dust grains as one of the components, has given birth to a new class, called dusty plasma. Later dusty plasma interests as one of the threshold areas in different context of astrophysical problems13-17 have been developed. The theoretical approach, studying the small amplitude wave, claims the formation of a dust acoustic wave due to the presence of dust charged grains and has been successfully encouraged later by Barkan et al.18 to herald further in experiments on dusty plasmas. Mamun et al.19 have studied the formation of soliton propagation in dusty plasma through the derivation of Sagdeev potential equation. But model taken by them has been simplified by assuming that
all the electrons are to be attached to the finite sizes of dust particles due to which the negative dust charged grains have been formed and maintain the neutrality condition as \( n_\beta = z \mu_\beta \). Ma and Liu\(^2\) furthered the study on dust acoustic soliton by considering dust charge equation limit as \( I_1 + I_2 = 0 \) which yields, in small amplitude approximation, the rarefactive solitonic wave.

However, it might not be a practical model of dusty plasma, as it does not expect generally the cent percent absorption of electrons to dust grains and form the negative dust charged grains.

The present study considers the plasma model where some electrons are in the background in maintaining the charge neutrality condition of plasma. Based on the universal pseudopotential analysis\(^2\), the Sagdeev potential equation has been derived for analysing the arbitrary amplitude dust-acoustic modes. Later, under the small amplitude approximation, various nonlinear wave equations have been derived to show the possible existence of double layers or shock-like wave. The observations are supported by numerical computations to the formation of various solitary waves. In contrast to the steady state method known to nonlinear plasma waves during last several decades, tanh-method\(^21,22\) has been applied and follows as a main motto for finding the dust acoustic soliton propagation.

### 2 Basic Equations and Derivation of Sagdeev Potential Equation

To study the nonlinear plasma acoustic wave, we assume that the finite size dust grains contaminate plasma. We have considered the plasma conceiving warm electrons and ions as a neutral plasma background followed by the Boltzmann relations. The equations governing the dust charged grains are, in fluid description, the following basic normalized equations:

\[
\frac{\partial n_d}{\partial t} + \nabla (n_d u_d) = 0
\]

\[
\frac{\partial u_d}{\partial t} + (u_d \nabla) u_d = \nabla \phi
\]

where \( \phi \), the electrostatic potential, couples through the Poisson equation as:

\[
\nabla^2 \phi = Z n_d + (1 - Z) n_e - n_i
\]

To support the basic equations, the electron and ion distributions are assumed to be Boltzmannian and following the usual procedure\(^21,22\), the ion and electron densities respectively derive the following Boltzmann relations:

\[
n_i = n_i^0 \exp (-\gamma \phi) \quad \ldots (4)
\]

\[
n_e = n_e^0 \exp (\phi) \quad \ldots (5)
\]

where \( \gamma = \frac{e^2}{kT_i} \); \( Z = \frac{n_d}{n_i^0} \); \( \beta = \frac{\gamma}{e_d} \) represents the density normalised by \( n_d^0 \); the density at the equilibrium state \( n_d \) is the velocity of dust charged grains normalised by the dust acoustic speed \( c_d = \sqrt{\frac{Z T_i}{n_d^0}} \).

All other normalised parameters are defined elsewhere\(^23\).

The overall charge neutrality condition has been maintained throughout the plasma by the following relation:

\[
-Z n_d + (1 - Z) n_e = n_i \quad \ldots (6)
\]

Now, the variables in basic Eqs (1)-(5), in order to employ the pseudopotential analysis to derive Sagdeev potential equation, are assumed to vary functionally as \( \xi = \alpha (\nu - Mt) \); with respect to a frame moving with Mach number \( M \). The use of this transformation into basic equations, after some mathematical analysis, evaluate dust charge density, \( n_d \) as:

\[
n_d = \frac{n_i^0 M}{(M^2 + 2\phi)} \quad \ldots (7)
\]

where the appropriate boundary conditions \( \phi \rightarrow 0 \), \( n_i \rightarrow 0 \) and \( n_d \rightarrow n_d^0 \) at \( |\xi| \rightarrow \infty \) have been used. Substituting Eqs (4)-(7) into Eq.(3), and then integrating once along with the boundary conditions at \( |\xi| \rightarrow \infty \), the energy integral equation is derived as:

\[
\frac{1}{2} \left( \frac{d\phi}{d\xi} \right)^2 + V(\phi) = 0
\]

where \( V(\phi) \) is known as Sagdeev potential defined by:

\[
V(\phi) = \frac{1}{\alpha^2} \left[ Z M^2 (1-(1 + 2\phi M^2) \frac{d\phi}{d\xi}) \right] + (1 - Z)(1 - \exp(\phi)) + \frac{1}{\gamma} (1 - \exp(-\gamma \phi)) \quad \ldots (9)
\]

Eq.(8) regards as an analogy between the Sagdeev potential and motion of a classical particle moving with velocity \( \frac{d\phi}{d\xi} \) in a potential well. It derives the
conservation of energy to the formation of solitary wave indicating the reciprocation of particle between $\phi = 0$ and $\phi = \phi_m$ and thus a back and forth motion in potential well does play.

The condition for the existence of solitary wave solution requires:
(i) $V(\phi) = 0$ for $\phi = 0$ and $\phi = \phi_m$ means the particle moves in the potential well $V(\phi)$, with a velocity $\frac{d\phi}{d\xi}$.

(ii) $\frac{d\phi}{d\xi} = 0$ at the edges $\phi = 0$ and $\phi = \phi_m$ means that the velocity $\frac{d\phi}{d\xi}$ should be zero otherwise the particle does not move back and forth and moving out of potential well.

(iii) $\frac{d^2\phi}{d\xi^2}$ at the edges $\phi = 0$ and $\phi = \phi_m$ must be equal to zero. There exists a nonzero $\phi_m$, the maximum (or minimum) of $\phi$ at which $V\phi = 0$, accordingly having solitons in compressive (or rarefactive) nature.

To highlight the existence of solitary wave from Eq. (8), it is customary to evaluate the nature of $V(\phi)$. At a glance, it concludes a condition $V(\phi) < 0$ under the variation of plasma parameter. Fig. 1 shows the variation of $V(\phi)$ with $\phi$ for different plasma compositions related through dust charge concentrations, $\chi$. It can be seen that increase of dust concentrations, for a typical Mach number $M$, derives the amplitude for rarefactive solitary wave at $\phi = \phi_m$ where $V(\phi) = 0$. However, the upper limit of $M$ evaluates $V(\phi_m) = 0$ with $\phi_m = -\frac{M^2}{2}$. The upper limit of $M$ evaluates $V(\phi_m) > 0$ with $\phi_m = -\frac{M^2}{2}$ which is the minimum value of $\phi$ and requires conditions on the existence of rarefactive solitons on Sagdeev potential $V(\phi)$ as to be $>0$. The conditions yield the maximum Mach number, $M_o$ at the zeroes of $F(M)$ where

$$F(M) = 2M^2 + \chi \left(1 - \exp\left(-\frac{M^2}{2}\right)\right) + \frac{1}{\gamma} \left(1 - \exp\left(-\frac{M^2}{2}\right)\right)$$

and bifurcated the region. $F(M) > 0$ relates the existence of rarefactive dust - acoustic solitons and restricted the variation of $M$ in $1 < M < M_o$ and shows the variation with $\chi$ in Fig. 2. $\chi > 1$ represents the case of plasma not having the dust charge grains while $\chi = 1$ represents the plasma which conceives ions while all the electrons are attached to dust grains forming the negative dust charge grains. Now the temperature variation plays also important role in soliton propagation. For chosen plasma with $\gamma < 1$, Fig. 2 shows clearly that the range of $M_o$ is shorten and solitary wave could not be observed for large Mach number $M$ in range $(1, M_o)$. Whereas plasma expects reasonably $\gamma < 1$, i.e., $T > T_c$, for which $M$ variation is widen and does give large $M_o$ for the existence of solitary waves. Overall scenario on the variation of $F(M)$ with $M$ and $\chi$ are shown in Figs 2 (a and b) which support the distinct variation of optimal Mach number. The observations conclude that the increase of dust charged grains increase the range of Mach number which causes the attachment of more electrons to dust grains. The range of Mach number plays the crucial role for having soliton behaviours in dusty plasma.

Now, to study the small amplitude nonlinear waves, $V(\phi)$ in Eq. (8) has been expanded in different powers of $\phi$. Some other small effects might play important role during the ordering effects in the dynamical system, but the present motto is to show the ordering $\phi$ effects in isolation in the formation of soliton propagation. Retaining the terms upto $\phi^3$, Eq. (8) evaluates the nonlinear differential equation as
\[ \alpha^2 \left( \frac{d\phi}{dz} \right)^2 = A \phi^2 - A_1 \phi^3 \] 

where \( A_1 = -\frac{\chi}{M^2} + (1 - \chi) + \gamma \) and

\[ A_2 = \frac{2}{3!} \left( -\frac{3\chi}{M^4} - (1 - \chi) + \gamma \right). \]

To find the soliton solution of Eq.(10), the proposed tanh-method has been employed in finding the exact nonlinear wave solution. This method was first initiated in fluid dynamics as well as extended successfully in the astrophysical problems. It has been developed later to plasma dynamics. The simple wave solution technique, based on the knowledge on K-dV soliton profile, has been modified by tanh-method by putting \( Z = \tanh(\xi) \) with \( \phi(\xi) = W(Z) \) and the nonlinear wave Eq. (10) writes as:

\[ 2\alpha^2 (1 - Z^2)^2 \frac{d^2W}{dZ^2} - 4\alpha^2 Z (1 - Z^2) \frac{dW}{dZ} - 2A_1 W + 3A_1 W^2 = 0 \]  

Eq. (11) becomes an ordinary differential equation of Fuchsian-type and thus a series solution is desirable in the following form:
Putting the series of Eq.(12) in Eq.(11) gives usually a lengthy and tedious procedure to find out the solution of the differential equation. However, the catchy of the tanh-method for obtaining the solution, in contrast to usual Frobenius method, modifies the procedure to a truncated series with finite terms (N) in Eq.(12). Thence balancing the higher order nonlinear terms with the leading order of linear terms determines N equals to 2 and due to which the Eq.(12) writes as:

\[ W(Z) = \sum_{r=0} a_r Z^r \]  

Again the nature of the differential equation ensures that the series solution in Eq.(13) should not be with any odd terms and finally W(Z) takes the form:

\[ W(Z) = a_0 (1-Z^2) \]  

which ultimately derives recurrence relation for Eq.(11) as:

\[ 2a_0^2 (6Z^2-2) - 2A_1 + 3A_2 a_0 (1-Z^2) = 0 \]  

and reduces to a problem of finding \( \alpha \) and \( a_0 \). Equating the different powers in \( Z' \) and \( Z'' \) determines \( a_0 \) and \( \alpha \) as:

\[ a_0 = \frac{A_1}{A_2} \quad \text{and} \quad \alpha = \sqrt{\frac{A_1}{4}} \]

and correspondingly the soliton profile derives as:

\[ \phi(\xi) = \left( \frac{A_1}{A_2} \right) \sec h^2 \left( \sqrt{\frac{A_1}{4}} (x - Mt) \right) \]  

which represents the dust-acoustic wave and is nothing but the well-known small amplitude K-dV soliton solution derived in plasma. The value of \( A_1 \) and \( A_2 \) determine the nature representing either compressive or rarefactive soliton. For the prescribed values of plasma parameter, \( A_1 \) and \( A_2 \) are in opposite signs and resulted always in a rarefactive dust-acoustic solitary wave Fig. 3. This analysis has already shown the existence of rarefactive solitary wave with dust charge compositions. The variation of \( A_2 \) plays the controlling role to exhibit the qualitative nature of soliton. The
result concludes that, because of the dust charged grains, there is rarefactive acoustic mode what exactly could be observed in plasma with additional negative charges. Further observations in multicomponent plasma with negative ions or multi-temperature electrons find $A_2 = 0$ due to which soliton profile grows to infinitely large amplitude causing soliton explosion or collapse. So it is necessitated to control the plasma parameters to prevent $A_2$ to be zero. But the region at which $A_2$ is positive, maintains the profile of compressive solitons, while the negative $A_2$ derives a rarefactive soliton propagation in dusty plasma. Further these nature of soliton have been highlighted by Sagdeev potential equation [Figs (1,2)]. The explosive soliton propagation appears due to the large growth of amplitude wherein the energy conservation fails.

Otherwise the case of conserving the energy leads to a collapse. Now to avoid the unusual behaviours in soliton propagation, $V(\phi)$ approximated with next higher order term i.e., $V(\phi)$ has been expanded up to the fourth order terms in $\phi$ and Eq. (8) becomes

$$\alpha^2 \left( \frac{d\phi}{dx} \right)^2 = A_3 \phi^3 - A_4 \phi^4$$  \hspace{1cm} (17)

where $A_3 = \frac{2}{41} \left[ \frac{15\chi}{41} - (1 - \chi) - \gamma^2 \right]$.

The use of tanh-method straightway might not be simple as done earlier. We simplify the Eq.(17), after differentiation, as:

$$2\alpha^2 \left( \frac{d^2\phi}{dx^2} \right) = 2A_2 \phi - 3A_3 \phi^2 - 4A_4 \phi^3$$  \hspace{1cm} (18)

In order to use tanh-method, we further modify the Eq.(18) to a desired equation through the following transformation $F = \tanh(\chi)$ with $v = 1$ and $\mu = -\frac{A_2}{4A_4}$ due to which the equation takes the form:

$$a \frac{d^2F}{dx^2} - bF + cF^3 = 0$$  \hspace{1cm} (19)

where $a = 2\alpha^2$, $b = 2A_2 - 6A_3 \mu = 12A_3 \mu^2$ and $c = 4A_4$.

The modified equation is known as Duffing-like equation and has its special characteristics in exhibiting the acoustic mode. The stable solution of the mode depends fully on the values of $a$, $b$, $c$ and specially on the variation of $c$, i.e., $\phi$; the nonlinear effect introduced by the higher order in $\phi$. To employ the tanh-method, we following the earlier transformation, $Z = \tanh(\xi)$ and $\phi(\xi) = W(\chi)$, modify the Duffing Eq. (19) as:

$$a(1-Z^2)^2 \frac{d^2W}{d\chi^2} - 2a(1-Z^2) \frac{dW}{d\chi} - bW + cW^3 = 0$$  \hspace{1cm} (20)

which is an ordinary differential equation and nature of the equation needs a finite series of Eq.(12). Thereafter, the task to find $N$ needs the balancing of power order analysis, which evaluates $N=1$. It renders a non-acceptable solution, due to which the series has been taken up to terms as $W(\chi) = \sum_{i=0}^{\infty} a_i Z^i$. Finally, with a straightforward mathematical manipulation derives the solution of Eq. (20) as:

$$W(\chi) = a_0 \sqrt{(1-Z^2)}$$  \hspace{1cm} (21)

leaving behind the evaluation of $a_0$ and $\alpha$. These are determined by the substitution of Eq. (21) into Eq.(20), by a recurrence relation of the form:

$$a(2Z^2 - 1) - b + c a_0 (1-Z^2) = 0$$  \hspace{1cm} (22)

has been derived.

Balance in different orders in $Z'$ and $Z''$ evaluates $a_0$ and $\alpha$ respectively as:
Putting these in Eq.(21) along with the original coordinate Eq.(17), the solution evaluates in $\phi$ as:

$$\phi(x,t) = \frac{A_2}{4A_3} + \frac{A_1 - 3A_2 \mu - 6A_2 \mu^2}{A_3} \frac{(x - \lambda t)}{\sqrt{A_1 - 3A_2 \mu - 6A_2 \mu^2}}$$

where the variables $A_1, A_2, A_3$ are functions of plasma parameters and play the role to control the existence of dust-acoustic solitary waves in plasma. Because of having the possible negative values of the term under the radical sign, the existence of acoustic mode depends crucially on the variation of $A_1, A_2, A_3$. The expression under the square root should be positive to get the real solitary wave otherwise solution yields a shock-like wave. For the typical value of $\psi$ number, $M$ and the temperature ratio $\gamma$, the nonlinear $A_4$ along with $A_1, A_2$ has been plotted for different $\psi$.

Its necessary requirements, for having the shock-like wave, are $A_1 > 0, A_2 > 0$ along with the condition $A_3 < 3A_2 \mu + 6A_2 \mu^2$ otherwise, the solution yields the solitary acoustic mode. But from the plot it is clear that, even though $A_1 < 0$, the condition for $A_2$ might not be satisfied for observing the real acoustic mode in plasma. In every case, $A_1, A_2$ or $A_3$ control the role in exhibiting the features of shock-like wave. The negative values of $A_3, A_2$ will exhibit compressive solitary wave and consequently possibility of having shock-wave disappears. Like earlier, $A_1$ could play in showing the explosion or collapse in soliton solution. To avoid such nature, a better approximation with next higher order effect in Sagdeev potential Eq. (8) has been taken i.e., up to $\phi^5$ for the higher order nonlinear correction and the potential equation writes as:

$$\left( \frac{d\phi}{d\xi} \right)^2 = A_1 \phi^2 - A_2 \phi^4 - A_3 \phi^6 - A_4 \phi^8$$

with $A_4 = \frac{2}{5} \left[ -\frac{105 \phi^2}{M^3} - (1 - \psi) + \psi \right]$. The Eq.(24), following the procedure described in Eq.(18), derives following form

$$2A^2 \left( \frac{d\phi}{d\xi} \right)^2 = 2A \phi - 3A_3 \phi^4 - 4A_2 \phi^6 - 5A_1 \phi^8 \quad \ldots (25)$$

Using the transformation $F = v \phi + \mu$ with $v = 1$ and $\mu = -\frac{A_1}{5A_4}$, Eq.(25) has been simplified as:

$$\frac{d^2F}{d\xi^2} - bF + cF^4 = 0$$

where $a = 2A^2$, $b = 2A_2 - 6A_1 \mu - 12A_2 \mu^2 - 20A_3 \mu^3$ and $c = 5A_4$, supported by two additional conditions $3A_3 \mu^2 + 4A_2 \mu^3 + 5A_3 \mu^4 - 2A_1 \mu = 0$

and $A_2 + 4A_2 \mu^3 + A_3 \mu^4 = 0$.

Eq. (26), with higher order nonlinearity, resembles very much to Painleve equation. To solve this Painleve-like equation by the proposed tanh-method, the transformation, as defined earlier for $Z$ and $W(Z)$, modifies the Eq. (26) as:

$$a(1-Z^2) \frac{d^2W}{dZ^2} - 2a(1-Z^2) \frac{dW}{dZ} - bW + cW^4 = 0 \quad \ldots (27)$$

The well defined procedure for obtaining a finite series solution fails as the balance of linear and nonlinear terms evaluates $N = \frac{2}{3}$. This value of $N$ does not ensure to derive the solution and thus the series has been expanded to an infinite series as $W(Z) = \sum_{n=0}^{\infty} a_n Z^n$ and thereafter, the similar procedure derives solution as:

$$F(Z) = a_n Z^{1/3} \quad \ldots (28)$$

To find $a_n$ and $\alpha$, we substitute the solution into Painleve Eq. (26) and derive the recurrence relation as:

$$\frac{2a_4}{9} (5Z^2 - 3) - b + ca_4 (1-Z) = 0 \quad \ldots (29)$$

The recurrence relation, based on the ordering analysis of $Z^0$ and $Z^1$, finds $a_n$ and $\alpha$ in the following forms:

$$a_n = \frac{(A_1 - 3A_2 \mu + 6A_2 \mu^2 - 10A_3 \mu^3)}{A_4}$$

and

$$\alpha = \frac{2}{3} \sqrt{A_3 - 6A_2 \mu^2 + 10A_3 \mu^3}$$

The Eq.(18), following the procedure described in Eq.(18), derives following form
Consequently solution of Eq. (25) finds

\[ \phi(x,t) = \frac{A_1}{5A_4} \pm \left( \frac{A_1 - 3A_2\mu - 6A_3\mu^2 - 10A_4\mu^3}{A_3} \right)^{1/3} \]

\[ \text{sech}^2 \left( \frac{x - \lambda t}{\delta} \right) \]

with \( \delta = \frac{2}{3} \sqrt[3]{A_1 - 3A_2\mu - 6A_3\mu^2 - 10A_4\mu^3} \)

Now, because of higher order nonlinearity, plasma acoustic mode derives different nature of solitary wave in the dynamical system. Solitary waves depend completely on the controls of nonlinear effects described from the expansion of \( V(\phi) \) i.e., on the functional variation of dust charged grains expressible through \( \lambda \)'s. The numerical support for the variation of \( \lambda \)'s are plotted in Fig. 4 from which the salient

Fig. 4- Variation of nonlinearities with concentrations, \( \chi \).
features of solution can be highlighted. The nonlinear effects can be studied for formation and propagation of solitons in dusty plasma. Apart from the solitons in dusty plasma, study has been furthered to find the other modes of plasma acoustic waves and accordingly \( \phi \) in Eq.(24) has been defined as a new variable \( \psi \) by the relation \( \psi (\xi) = -\frac{1}{4}(A_1 \phi + A_1 \frac{\phi}{3}) \) and due to which the Eq. (24) reduces to

\[
\frac{d\psi}{d\xi} = g_3 (\psi - g_3) (4g_3^2 - g_3^2 \psi - g_3^2) \psi \frac{1}{2} \quad \ldots (31)
\]

where \( g_3 = \frac{4}{A_1}, g_4 = \frac{12}{A_1}, g_5 = \frac{12}{A_1} - \frac{4}{A_1} \) and \( g_6 = \frac{8}{A_1} \frac{1}{2} - \frac{4}{A_1} \frac{4}{A_1}
\]

The solution of Eq. (31) depends on the discriminant of \( \psi \) i.e., \( 4\psi^3 - g_3 \psi - g_3 \), which functionally depends on the nonlinearity. In case of trivial degenerate solution of having two roots equal to \( \psi_0 \) and \( \psi_0 \), hence Eq. (31) leads to the following integral:

\[
I(\psi) = \frac{1}{2} \int \frac{d\psi}{(\psi - \psi_0)(\psi - \psi_1)(\psi - \psi_1)} = \pm g_3 \frac{1}{2} \quad \ldots (32)
\]

where \( \psi_0 = -\frac{3g_3}{2g_2} \) and \( \psi_1 = \frac{3g_3}{2g_2} \).

The evaluation of integral in Eq (32) finds the solution of nonlinear Eq.(24) in the following form:

\[
\frac{1}{\sqrt{\psi_0 - \psi_1}} \tanh^{-1} \left( \frac{\psi - \psi_0}{\psi_0 - \psi_1} \right) - \frac{1}{\sqrt{\psi_1 - \psi}} \tanh^{-1} \left( \frac{\psi - \psi_1}{\psi_1 - \psi_0} \right) = \pm (g_0 - \psi_0) g_3 \xi \quad \ldots (33)
\]

This leads to determine \( \psi (\xi) \) from the Eq. (33), the actual solution \( \phi \), evaluated from the expression:

\[
\phi(\xi) = \frac{16}{A_1^4} \psi(\xi)^2 + 8A_1 \psi(\xi) + \frac{A_1^2}{9A_1} \quad \ldots (34)
\]

The plasma-acoustic wave solution of Eq.(34) expects unstable mode and that too it collapses as and when the discriminant of cubic expression in \( \psi \) i.e., \( 2g_3g_5 + 3g_3g_6 \) is equal to zero. The condition can be regarded as a twin conditions as the following form:

\[
g_3^2 - 27g_3^2 - 2g_3g_4 + 3g_5 = 0 \quad \ldots (35)
\]

The above conditions are satisfied to exhibit the collapse of plasma-acoustic wave and the reason for such behaviour may be attributed to find various nonlinear and dispersive effects taking place in plasma medium.

3 Conclusion

Based on the pseudopotential analysis, Sagdeev potential equation has been derived to study the soliton propagation in dusty plasma. The acoustic modes have been described with the variation of dust charge composition in plasmas, that too with the effectiveness of nonlinearity as well as on various Mach number. The considered dusty plasma exhibits the effects of higher order nonlinearity as the functions of dust grain compositions, due to which they exhibit different nature of dust acoustic waves. The possible excitation of various solitons and shock-like waves along with general acoustic modes has been discussed. The highlighted observations could be of interest in space and laboratory plasmas as the plasma could be contaminated by dust grains and many results are yet to be known. The main emphasis has been given how to use the tanh-method to get soliton solutions from Sagdeev potential equation. It has also been shown that though the wave equation is derived with different higher order nonlinear effects but the soliton propagation has been obtained by tanh-method. The tanh-method has shown convincingly a success in derivation of soliton propagation in dusty plasma, and, in addition, other progressive modes have been derived.

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References

4. Ikezi H, Phys Fluids, 16 (1973) 1668.