Extreme value analysis of meteorological elements at Trombay, Mumbai

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Extreme value analysis of meteorological elements at Trombay is presented in this paper. The elements analysed are annual rainfall, maximum monthly rainfall, maximum daily rainfall, maximum and minimum air temperature, minimum relative humidity and maximum wind speed. The observation period for the rainfall data is 1959-1996, while for the others the period is 1985-1997. It is found that all the elements except maximum daily rainfall obeyed Fisher-Tippett Type I distribution, while the maximum daily rainfall followed Fisher-Tippett Type II distribution which has been attributed to its bi-modal frequency distribution. Parameters of the distribution functions for each element are established. Extreme values corresponding to return periods of 50 and 100 years are derived. Extreme values corresponding to typical 50 years return period at Trombay are compared with corresponding values at other three nuclear reactor sites in India, namely, Tarapur, Kota and Kalpakkam obtained in an earlier study. In addition, the extreme value analysis of the concurrent rainfall data collected at two more locations within the Mumbai city, viz. Colaba and Santacruz, both within 15-20 kilometres from the Trombay location, is also carried out and the location-wise differences between the parameters of the distribution functions are discussed.

1 Introduction

Engineering structures have to withstand the stresses of weather and are, therefore, to be designed to withstand the extreme weather conditions that might occur during the expected lifetime of the structure. The relevant weather conditions in this respect are defined by the meteorological elements like rainfall, air temperature, relative humidity and wind speed. Specifically, the data to be considered for such application are: annual rainfall, maximum monthly rainfall, maximum daily rainfall, maximum and minimum air temperature, minimum relative humidity and maximum wind speed. These data base have to be necessarily local, i.e. collected within a few kilometres of the site of application. For example, daily or hourly data observed at different locations within a few kilometres of the site of application can be quite different from each other because of the influence of local weather transients, though on monthly or annual basis the data may be of comparable magnitude because of averaging effects on long term basis. This can have important bearing on the extreme value distribution parameters, since the basic data considered for the extreme value analysis are collected on short terms like daily, hourly or 3 min basis. In the past, many studies are done on the extreme value analysis of long term meteorological data collected by India Meteorological Department at different site observatories in India. However, studies dealing with the influence of local weather transients on the parameters of the extreme value distribution function are relatively sparse. This present study, apart from establishing the extreme value distribution parameters for all the meteorological elements measured at Trombay, also compares the extreme value distribution parameters, typically for the concurrent period rainfall data collected at three locations, namely, Trombay, Colaba and Santacruz separated by 15-20 km from each other within the Mumbai city, and discusses the location-wise differences. In addition, the extreme values for the return period of 50 years for the different meteorological elements observed at Trombay are compared with the similar values obtained for other reactor sites in India.

2 Data base and analysis procedure

Owing to the fact that the meteorological elements pass through a definite cycle, one can expect an absolute extreme value per year for each element. The annual extreme of a given element differs from year to year. In order to arrive at proper design-basis value, the set of extreme data is subjected to statistical analysis so that probability distribution of the extreme values can be obtained. The probability of exceedance of any given design value in a given time, say, the expected lifetime of the structure, can be worked out from this distribution.

Data on meteorological elements like wind speed, direction, screen-level air temperature, relative humidity are routinely collected at Trombay since 1985, while data on daily total rainfall are collected since 1959 till to date. Rainfall data on annual rainfall, maximum monthly rainfall and maximum daily rainfall for each of the 38 years from 1959 to 1996 are
used as basic data for the extreme value analysis. The maximum and minimum temperature, minimum relative humidity and maximum wind speed, all derived from hourly average values, for each of the 13 years from 1985 to 1997 form the basic data sets which are used for the extreme value analysis of the respective element. Rainfall data at Colaba and Santacruz locations for the concurrent period have been obtained from Colaba and Santacruz observatories operated by India Meteorological Department.

The basic data set was arranged in an ascending order (for minimum temperature and minimum relative humidity, in the descending order) and each data point was assigned a rank \( m \). Probability of non-exceedence of a particular magnitude \( x \) of the data point with rank \( m \) was obtained as

\[
P(x) = \frac{m}{M+1}
\]

where, \( M \) is the total number of data points.

A graph of \( x \) (or \( \ln x \)) versus \( P(x) \) with \( P(x) \) on a probability scale was examined for a straight line-fit for all the variables. Figure 1 shows typically three such plots for some of the variables studied. It was observed that all the variables except the maximum daily rainfall show a straight line-fit. In case of maximum daily rainfall, plot of \( \ln x \) versus \( P(x) \) on a probability scale showed a straight line-fit.

Equation of the straight line plot of \( x \) versus \( P(x) \) shown in Fig. 1 can be written as

\[
x = \alpha + \beta_1 Y_p
\]

where, \( Y_p \) is the reduced variate corresponding to \( P(x) \), defined as

\[
Y_p = - \ln \{- \ln \{P(x)\}\}
\]

Similarly, the equation for the straight line graph of \( \ln x \) versus \( P(x) \) is

\[
\ln x = \ln \beta_2 + \left(\frac{1}{\gamma}\right) Y_p
\]

Equations (1) and (3) can be easily transformed into the expressions for the two widely used extreme value distribution functions, viz.

**Fisher-Tippett Type I:**

\[
P(x) = \exp\{-\exp\{-\frac{(x-a)}{\beta_1}\}\}
\]

**Fisher-Tippett Type II:**

\[
P(x) = \exp\{-\frac{(x/\beta_2)^\gamma}{\gamma}\}
\]

In Eqs (4) and (5), \( \alpha, \beta_1, \) and \( \beta_2, \gamma \) can be identified as location and scale parameters of the type I distribution function and \( \beta_2, \gamma \) as the scale and shape parameters of the type II distribution function. Type I distribution can be obtained from Type II by a logarithmic transformation of the variable.

Thus, from the slope and intercept of the straight line obtained as shown in typical plots in Fig. 1, the distribution parameters \( \alpha, \beta_1 \) and \( \beta_2, \gamma \) can be determined. These parameters then define \( P(x) \) completely and enable one to obtain the probability that a given value is exceeded irrespective of whether the value lies within the observed range or not.

The parameter needed for actual application is a design value of the element with defined probability. Contrarily, when the design value is selected, the probability of exceedence of this value over a given period (say expected lifetime of the structure) can be worked out. For this, a mean recurrence interval (MRI) or return period is defined which is related to \( P(x) \) by the relation

\[
\text{MRI} = \frac{1}{1 - P(x)} \text{ years}
\]

Using the values of the distribution parameters, MRI for any value of the element or, conversely, the value of the element for any MRI can be obtained.

### 3 Results and discussion

The distribution parameters obtained from the slope and intercept of the straight line-fit for each of the elements studied are summarized in Table 1 which also shows the values of the extreme values of the respective element derived for typical MRIs of 50 and 100 years. These derived extreme values are,
particularly, useful for arriving at suitable design
basis values to ensure the safety of any civil structure
in and around Trombay with respect to stresses due
to weather conditions.

It is observed that maximum daily rainfall obeyed
type II distribution function, while all other elements
followed type I distribution function. Such feature
was also observed in an earlier study at the three other
nuclear reactor sites in India, viz. Tarapur, Kalpakkam
and Kota using data pertaining to
different periods. This feature can be attributed to
the bi-modal frequency distribution observed for this
element, whereas for the other parameters, single
mode frequency distribution was observed. The
bi-modal frequency distribution for the maximum daily
rainfall arises from the fact that the maximum daily
rainfall is mainly governed by extreme phenomena
of two types, viz. transient peak convective activity
during the monsoon and some migratory events like
passage of severe storm or cyclone. Because of the
bi-modal frequency distribution for the maximum daily
rainfall, the whole extreme value data set for this
element can be classified roughly into two ranges,
name, (i) higher magnitude range and (ii) lower
magnitude range. The mode in the higher range can
be ascribed to severe migratory event, while the mode
in the lower range to peak convective activity.

The study also reported different return period
values of various meteorological elements for different
return periods derived from the long term
meteorological data collected at the respective reactor
sites. It will be interesting to compare the extreme
values at the three reactor sites with corresponding
values at Trombay obtained in this study. Table 2
shows such comparison. It is seen that the values for
Trombay and Tarapur are somewhat comparable, while
for Kota and Kalpakkam, they are quite different. The
extreme value of any element at a particular site,
generally, depends on the climatological as well as
topographical features of the site. Tarapur and
Trombay, both being the coastal sites on the western
coast separated by about 100 kilometres only, are
expected to be climatologically and topographically
similar and hence the relevance of the comparison.
For Kota and Kalpakkam sites, the former being an inland
and northern region site and the latter being a coastal
and southern region site, climatological and
topographical features are quite different and hence
the comparison itself may not be relevant.

In order to examine the local influence on the
parameters of the extreme value distribution function
and on the extreme values of the specific return
period, similar analysis has also been made typically
for the rainfall data collected concurrently at two
other locations, viz. Colaba and Santacruz, and the
results are compared with those for Trombay (Table 3).
All the three locations are within the Mumbai city
and are separated hardly by 15-20 kilometres from
each other. As expected, at all the three locations, the
maximum daily rainfall only follows the type II

<p>| Table 1 — Distribution parameters for extreme value probability functions and 50 and 100 years return period values for different meteorological parameters at Trombay |
|-----------------|-----------------|-------------------|-----------------|</p>
<table>
<thead>
<tr>
<th>Element</th>
<th>Observed variate form used</th>
<th>Function type</th>
<th>α</th>
<th>β₁</th>
<th>Return period (years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period and No. of obs. years : 1959-1996; 38</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Annual rainfall (mm)</td>
<td>Linear</td>
<td>FT-1</td>
<td>2138.8</td>
<td>376.9</td>
<td>3609.0 3872.5</td>
</tr>
<tr>
<td>Maximum monthly rainfall (mm)</td>
<td>Linear</td>
<td>FT-1</td>
<td>886.8</td>
<td>163.9</td>
<td>1522.9 1636.7</td>
</tr>
<tr>
<td>Maximum daily rainfall (mm)</td>
<td>Logarithmic</td>
<td>FT-II*</td>
<td>5.145</td>
<td>0.294</td>
<td>540.3 663.4</td>
</tr>
<tr>
<td>Period and No. of obs. years : 1985-1997; 13</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maximum air temperature ℃</td>
<td>Linear</td>
<td>FT-1</td>
<td>39.6</td>
<td>0.742</td>
<td>42.5   43.0</td>
</tr>
<tr>
<td>Minimum air temperature ℃</td>
<td>Linear</td>
<td>FT-1</td>
<td>15.5</td>
<td>-1.650</td>
<td>9.1    7.9</td>
</tr>
<tr>
<td>Minimum relative humidity (%)</td>
<td>Linear</td>
<td>FT-1</td>
<td>24.7</td>
<td>-4.105</td>
<td>8.7    5.8</td>
</tr>
<tr>
<td>Maximum wind speed (km/h)</td>
<td>Linear</td>
<td>FT-1</td>
<td>29.5</td>
<td>-6.86</td>
<td>56.2   61.0</td>
</tr>
</tbody>
</table>

Note: (i) FT-I means Fischer-Tippett type I distribution function of the form \( P(x) = \exp \left( - \exp \left( - \frac{y_1}{\alpha} \right) \right) \) or \( x = \alpha + \beta_1 y \) with \( y_1 = -\ln(-\ln P(x)) \) and \( P(x) = m(x)(M+1) \), where \( m(x) \) is the rank of variate \( x \) and \( M \) the total no. of data points.

(ii) FT-II means Fischer-Tippett type II distribution function of the form \( P(x) = \exp \left( - \exp \left( - \frac{y_1}{\beta_1} \right) \right) \) or \( x = \beta_1 y \) with \( y_1 = -\ln(-\ln P(x)) \) and \( P(x) = m(x)(M+1) \)

*These values are the parameters of FT-II distribution function, i.e. \( \beta_1 = \exp (\alpha) \) and \( y = 1/\beta_1 \).
Table 2 — Comparison of extreme meteorological elements at Trombay with those at other nuclear sites for 50 years return period

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Annual rainfall (mm)</td>
<td>3609.3</td>
<td>1510.1</td>
<td>3210.0</td>
<td>3188.0</td>
</tr>
<tr>
<td>Maximum daily rainfall (mm)</td>
<td>540.3</td>
<td>313.9</td>
<td>462.9</td>
<td>539.9</td>
</tr>
<tr>
<td>Maximum air temperature (°C)</td>
<td>42.5</td>
<td>46.7</td>
<td>45.2</td>
<td>39.2</td>
</tr>
<tr>
<td>Minimum air temperature (°C)</td>
<td>9.1</td>
<td>2.3</td>
<td>15.7</td>
<td>8.5</td>
</tr>
<tr>
<td>Maximum wind speed (km/h)</td>
<td>56.2</td>
<td>65.0</td>
<td>81.6</td>
<td>61.0</td>
</tr>
</tbody>
</table>

Table 3 — Values of estimates for probability distribution functions for various extreme rainfall parameters at Trombay, Colaba and Santacruz

<table>
<thead>
<tr>
<th>Element</th>
<th>Location</th>
<th>Observed variate form used</th>
<th>Function type</th>
<th>( \alpha )</th>
<th>( \beta )</th>
<th>Return period (years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum daily rainfall (mm)</td>
<td>Trombay</td>
<td>Logarithmic</td>
<td>FT-II</td>
<td>5.146</td>
<td>0.294</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td>Colaba</td>
<td></td>
<td></td>
<td>5.058</td>
<td>0.401</td>
<td>100</td>
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<tr>
<td></td>
<td>Santacruz</td>
<td></td>
<td></td>
<td>5.160</td>
<td>0.269</td>
<td></td>
</tr>
<tr>
<td>Maximum monthly rainfall (mm)</td>
<td>Trombay</td>
<td>Linear</td>
<td>FT-I</td>
<td>886.8</td>
<td>163.9</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Colaba</td>
<td></td>
<td></td>
<td>758.5</td>
<td>199.6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Santacruz</td>
<td></td>
<td></td>
<td>832.7</td>
<td>178.4</td>
<td></td>
</tr>
<tr>
<td>Annual rainfall (mm)</td>
<td>Trombay</td>
<td>Linear</td>
<td>FT-I</td>
<td>2138.8</td>
<td>376.9</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Colaba</td>
<td></td>
<td></td>
<td>1861.6</td>
<td>415.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Santacruz</td>
<td></td>
<td></td>
<td>2064.1</td>
<td>365.3</td>
<td></td>
</tr>
</tbody>
</table>

Note: FT-I and FT-II are the same as defined in Table 1.

*These values are the parameters of FT-II distribution function, i.e. \( \beta = \exp(\alpha) \) and \( \gamma = 1/\beta \).

distribution function, whereas the annual and maximum monthly rainfall obey type I distribution function. The values for 50 and 100 yr return periods at the three locations for all the rainfall elements are comparable with each other, barring the maximum daily rainfall for which significant location-wise differences are observed. Generally, the data evaluated on short term basis (like, on daily total basis for the rainfall data and hourly average basis for the other elements) at different locations within a few kilometres of the site of application can be quite different from each other, while the data evaluated on long term basis (e.g. monthly or annual total basis for the rainfall, and monthly or annual average basis for the other elements) can be of comparable magnitude. This is reflected in the extreme values of the elements as well. One should take cognisance of such location-wise differences in the design-basis extreme values while considering the safety of structures in the respective locations, mainly, from the point of view of the stresses due to severe weather elements.

4 Conclusions

Except for the daily maximum rainfall, all other meteorological elements followed type I distribution function. The daily maximum rainfall obeyed the type II distribution function which has been ascribed to its observed bi-modal frequency distribution. The extreme value distribution functions for all the routinely collected meteorological elements at Trombay are established. The derived extreme values for different return periods as tabulated in the Table 1 are particularly useful for arriving at suitable design values to ensure the safety of any civil structure in and around Trombay, Mumbai, with respect to stresses due to weather conditions. Also significant location-wise differences in the extreme values of the elements, especially, for the data evaluated on the short term basis, can exist even though the data are collected at the same site but at different locations separated by a few kilometers from each other within the site.
Acknowledgements

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References