Particle tracking in rotating channel flow†

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The study deals with the numerical simulation of dilute solid-fluid flow in a two-dimensional straight rotating channel using Lagrangian particle tracking method. The developing carrier-phase flow, assumed to be steady in the mean, is computed using Galerkin finite element method. A rotation-modified $k-\varepsilon$ model is used to account for the effect of turbulence on the mean flow.

One-way coupling is assumed, i.e., the carrier-phase flow affects the particle motion, but not vice-versa. Model coefficients of restitution in the normal and tangential directions are used. The particle equations of motion include drag, pressure, centrifugal, Coriolis, virtual mass, and friction (in case the particle slides along the wall) forces. Mesh refinement and comparison with published numerical results are used to validate the code. Effects of such operating parameters as rotation rate, particle size, specific gravity and coefficients of restitution are discussed.

Keywords: Particle tracking, Rotating channel, Particle-wall collisions, One-way coupling, Forces on particle

Flow of solid-gas and solid-liquid mixtures are of interest in such wide areas of industry as slurry transportation, dredging, propulsion, power generation equipment, turbomachinery and pneumatic transport of food grains. Such two-phase flow handling systems are subject to significant erosion by particle impact and sliding\textsuperscript{1}. A clear understanding of the behaviour of dilute particulate flows in rotating and stationary passages and the attendant erosion are thus of fundamental, engineering, scientific and economic importance. These systems, in general, are characterized by complex passage geometry. In addition, they may involve a rotating reference frame as well. These complications make the numerical modeling of such two-phase flow systems difficult. As a first step, flow through straight rotating channel serve as a good benchmark problem for developing and testing robust numerical simulation techniques. It also provides a relatively simple-yet-adequate framework for understanding the physics of dilute two-phase systems. Indeed, several studies, e.g., Tsuji et al.\textsuperscript{7}, deal with dilute particulate flows in stationary channels for this purpose.

In predicting dilute two-phase flow, computation of the carrier-phase flow field is an important first step. A number of numerical and experimental studies\textsuperscript{3-6} have addressed single-phase flow in rotating channels. A significant common result of these studies is that with rotation, the velocity profile becomes asymmetric about the channel centerline; the pressure side flow becomes destabilized and the suction side flow becomes stabilized. Recently, inviscid\textsuperscript{7} and viscous\textsuperscript{8} free surface flows in rotating channels have also been addressed.

Once the carrier-phase flow field is determined, Lagrangian particle tracking\textsuperscript{9,10} is the preferred method for computing particle trajectories in dilute mixtures. In dilute two-phase flow, where the particle/fluid mass-loading ratio is relatively low and the particle number density is relatively small, particle-particle collisions are negligible\textsuperscript{11,12}. However, particle-wall collisions, which may considerably influence particle motion, are important\textsuperscript{11,12}. Particle-wall interactions also determine the surface erosion of the wall material by impact\textsuperscript{11} and sliding mechanisms.

With regard to particle-wall collisions, two characteristic regimes of particle motion may be distinguished\textsuperscript{13,14}. In the first regime, valid for relatively small particles, the fluid motion controls the motion of the particles. The predominance of the particle-wall collisions is less important since the particles promptly follow the carrier fluid. In the second regime, valid for relatively larger particles, the particle motion is influenced by its inertia. Such particles respond slowly to the changes in the mean

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carrier-phase flow (particularly near the wall surface, since the flow decelerates in that region), so that the wall-collision process in confined flows can considerably influence the particle motion. In the extreme situation, the particles may maintain their direction of motion for a long enough time (after collision with the wall) to result in collision with the opposite wall. The size of the particle is best characterized in terms of the settling velocity of the particle under the influence of gravity. For rotating channel flow, the Coriolis acceleration replaces the gravitational acceleration in determining the particle settling velocity. Thus, depending on the fluid viscosity and relative density of the particle, the same physical size of particle may be “large” or “small”.

In considering collisions with the wall, the wall roughness and the direction of irregularities on the wall might play a significant role. Since surface irregularities have a statistical character, some researchers\textsuperscript{13} introduce the concept of a virtual wall, which has a randomly distributed inclination with respect to the smooth plane wall. A Gaussian distribution for this randomness in inclination is found\textsuperscript{13} to give best correlation with experimental results.

In reality, particles act as finite bodies during collision with the wall. They must be treated as rigid or deformable spheres. In some studies\textsuperscript{15}, particles are considered as point masses in handling collision to simplify the calculations. Empirically determined coefficients of restitution in the tangential and normal directions are introduced\textsuperscript{2,15} to compute the post-impact particle velocity. It is also possible that when the fluid is relatively viscous and dense, the particle may just slide along the wall after collision. In such a case, an empirical kinetic friction coefficient must also be introduced into the model.

In addition to particle-wall interactions, the particles and fluid experience centrifugal and Coriolis body forces in a rotating reference frame. The other predominant forces acting on the particle are the drag and pressure forces. Virtual mass force might also play a role when the density of fluid is comparable to that of the particle. Gravity may also be important, particularly if the flow rate and/or the rotation rate are small. Unless otherwise specified gravity effects are neglected in this study.

Although the particle motion might affect the carrier flow, this is not considered in the present study. In a study involving gas-solid dilute two-phase flow in a vertical pipe\textsuperscript{16} both experimental and numerical results indicate that fluid turbulence is suppressed by the presence of particles. Furthermore, the two-way coupling study\textsuperscript{16} attributed the decrease in turbulence intensity of the fluid-phase to the transfer of turbulent kinetic energy from the fluid to the particles, which was more in the case of small size particles since they can more easily follow the smaller turbulent eddies.

One-way coupling between fluid and particles is a suitable assumption particularly for relatively small particle\textsuperscript{16}. A recent study\textsuperscript{17} involving stationary channel shows reasonable agreement between results of computed particle trajectories using one-way and two-way coupling. Furthermore, it is shown that results of computed particle trajectories using simplified assumptions agreed well with experimental results in the open literature\textsuperscript{5}.

A detailed study on three-dimensional motion of spherical solid particles entrained in radial flow impeller having a low specific speed has been carried out\textsuperscript{15}. It discusses in detail the effect of particle size and specific weight on the positions and velocities of impact on the boundary surface of the impeller. However, the carrier-phase flow field is considered inviscid and irrotational, enabling the use of a potential function to describe the velocity field. Viscous effects, not included in Minemura & Uchiyama\textsuperscript{15} significantly alter the flow field (and hence the particle motion), particularly adjacent to the wall.

In the present study, viscous two-dimensional developing carrier-phase flow in rotating channel is considered. In general, rotating channel flow is three-dimensional\textsuperscript{5,6} due to the effect of Coriolis forces. However, when the aspect ratio of the channel is large, secondary flow is insignificant, and the flow may be approximated as two-dimensional\textsuperscript{3,6,18}. The two-dimensional developing flow problem may be treated using elliptic methods. In this study, Galerkin finite element method is used to determine the base flow. A rotation-modified $k−\varepsilon$ model is used to compute the turbulent viscosity in the Reynolds-averaged-Navier-Stokes equations governing the carrier flow. The details of this solution methodology are presented elsewhere\textsuperscript{19}.

The primary aim of the present work is to study the effect of channel rotation on particle motion in a viscous mean flow field. Therefore, gravity is neglected except in one case for comparing the results of the present work with those of others\textsuperscript{2} for a horizontal stationary channel. This is done as a means
of partially validating the present computer code. This study has direct relevance to the Coriolis wear tester channel\textsuperscript{19}. Traditionally, in the Coriolis wear tester, water-based slurries are used to determine the sliding wear coefficient necessary for wear prediction in slurry handling equipment. In the Coriolis wear tester channel, the slurry exits to the open atmosphere, so that a free surface can be avoided. The present study is directly applicable to such a situation. It is also possible to use some experimental studies\textsuperscript{7,8}, the formation of a free surface can be avoided. The present study is directly applicable to such a situation. It is also possible to use the observations of this study directly in the Coriolis wear tester, provided the carrier medium is air.

Mathematical Formulation

Carrier-phase flow

Consider a two-dimensional channel rotating about the z-axis at a constant angular velocity, \( \dot{\Omega} = \Omega \hat{\mathbf{k}} \), as shown in Fig. 1. The XYZ coordinate system is fixed to the rotating channel, while the xyz system is fixed. Let \( \dot{\dot{U}} = U \hat{i} + V \hat{j} \) be the mean flow velocity relative to the channel, \( \rho \) is the carrier density, \( \hat{r} = \hat{x} + \hat{y} \) is the relative position vector, \( p \) is the pressure, and \( \tau_{ij} \) is the stress tensor.

Beginning with the momentum equations for steady, incompressible flow in a rotating reference frame\textsuperscript{21}, and applying the usual Reynolds-averaging process\textsuperscript{22}, the equations governing mean flow may be written as

\[
\nabla \cdot \dot{\dot{U}} = 0, \quad \ldots (1)
\]

and

\[
\rho \left( \dot{\dot{U}} \cdot \nabla \right) \dot{\dot{U}} = -\rho \dot{\Omega} \times \left( \dot{\Omega} \times \dot{\dot{r}} \right) - 2 \rho \dot{\Omega} \times \dot{\dot{U}} - \nabla p + \nabla \cdot \tau_{ij}, \quad \ldots (2)
\]

where the total (laminar + turbulent) stress tensor is defined in terms of the laminar viscosity, \( \mu \), and eddy viscosity, \( \mu_t \), as

\[
\tau_{ij} = (\mu + \mu_t) \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right). \quad \ldots (3)
\]

The turbulent eddy viscosity, \( \mu_t \), of the carrier-phase is computed using the rotation-modified \( k-\varepsilon \) model\textsuperscript{13} as

\[
\mu_t = C_{\mu} \frac{k^2}{\varepsilon}, \quad \ldots (4)
\]

where, \( C_{\mu}, k \) and \( \varepsilon \), are the turbulence model constant, turbulent kinetic energy and its rate of dissipation, respectively.

Equations (1)-(4) along with the two-equations for \( k \) and \( \varepsilon \) form a closed system\textsuperscript{19}, which may be solved for the two components of velocity and the pressure with suitable boundary conditions. In the numerical calculations, these equations are used in non-dimensional form. Velocity components \( U \) and \( V \) are non-dimensionalized with respect to uniform inlet velocity, \( U_0 \); pressure with respect to \( \rho U_0^2 \); \( \Omega \) with respect to \( U_0 / L \); \( k \) with respect to \( U_0^2 \); \( \varepsilon \) with respect to \( U_0^2 / L \) and all lengths with respect to channel length \( L \). The final non-dimensional equations are presented elsewhere\textsuperscript{19}.

At the inlet, uniform parallel flow is assumed. Along the channel walls at \( y = \pm H / 2 \), wall functions\textsuperscript{19} are used. At the channel exit, the pressure and the velocity gradients \( \partial U / \partial x \) and \( \partial V / \partial x \) are forced to vanish. The computational length \( (1+\Delta) \) of the channel is greater than the physical length (= 1) of the channel, so that the flow inside the physical channel is more accurately predicted.

Motion of particles

The equation of motion for a single particle, \( P \), may be written in terms of the absolute coordinate system (XYZ in Fig. 1) as

\[ \ldots \]

Fig. 1 — Schematic of dilute particulate flow in two-dimensional rotating channel

\[ \ldots \]
\[ \rho \dot{V}_p = -\nabla p + \frac{C_D A p}{2} \| \dot{V}_p \| (\dot{V}_p - V_p) \]

\[ + (\rho - \rho_p) \nabla g (i \sin \theta + j \cos \theta) \]

\[ + \rho \gamma C_v \left[ \frac{d\dot{V}_p}{dt} - \frac{dV_p}{dt} \right] \] ... (5)

where \( \rho_p \) is the particle density; \( \forall \) is the particle volume; \( t \) is the time; \( \dot{V}_p \) and \( \ddot{V} \) are, respectively, the particle and fluid mean velocities with respect to ground; \( p \) is the fluid pressure; \( C_D \) is the drag coefficient; \( A \) is the projected area, \( \pi d_p^2 / 4 \), of the particle of diameter \( d_p \); \( \rho \) is the fluid density; \( C_v \) is the virtual mass coefficient. For spherical particles, \( C_v \) is taken\(^2\) as 0.5.

The first term on the right-hand side of Eq. (5) is the force on the particle due to fluid pressure. The second term describes the viscous force due to relative motion of the particle in the fluid. The third term describes the combined effects of gravity and buoyancy, and the fourth term describes the inertia of the added mass (entrained with the relative motion). This last term is negligible when \( \rho \ll \rho_p \) (as in gas-solid systems).

Now the relations between relative and absolute velocities and accelerations are introduced as follows:

\[ V_p = \dot{U}_p + \Omega \times \vec{r}, \] \hspace{1cm} ...(6a)

and

\[ \frac{d\dot{V}_p}{dt} = \left( \frac{d\dot{U}_p}{dt} \right)_R + 2 \Omega \times \dot{U}_p + \Omega \times (\Omega \times \vec{r}). \] \hspace{1cm} ...(6b)

Similar expressions hold for the fluid velocity and acceleration. The subscript \( R \) denotes the rotating frame. The relative acceleration \( \left( \frac{d\dot{U}}{dt} \right)_R \) is given as

\[ \left( \frac{d\dot{U}}{dt} \right)_R = \frac{d\dot{U}}{dt} + \left\{ (\dot{U} \cdot \nabla) \dot{U} \right\}_R. \] \hspace{1cm} ...(6c)

Since the carrier mean flow is steady in the rotating frame of reference, \( \frac{d\dot{U}}{dt} = 0 \). In the sequel, the subscript \( R \) is dropped for convenience.

Using Eqs (6a)-(6c), Eq. (5) may be written in non-dimensional scalar form as

\[ \frac{dU_p}{dt} = -\frac{1}{S + C_v} \frac{\partial p}{\partial x} + \frac{3C_D}{4d_p (S + C_v)} \| \dot{U}_p \| (\dot{U}_p - U_p) \]

\[ + 2\Omega \left( V_p - \frac{C_v}{S + C_v} V \right) + \Omega^2 x + \frac{1 - S}{S + C_v} \frac{g}{U_0} \sin \theta \]

\[ + \frac{C_v}{S + C_v} \left( U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} \right), \] \hspace{1cm} ...(7)

and

\[ \frac{dV_p}{dt} = -\frac{1}{S + C_v} \frac{\partial p}{\partial y} + \frac{3C_D}{4d_p (S + C_v)} \| \dot{U}_p \| (\dot{V}_p - V_p) \]

\[ - 2\Omega \left( U_p - \frac{C_v}{S + C_v} U \right) + \Omega^2 y \]

\[ + \frac{1 - S}{S + C_v} \frac{g}{U_0} \cos \theta + \frac{C_v}{S + C_v} \left( U \frac{\partial V}{\partial x} + V \frac{\partial V}{\partial y} \right), \] \hspace{1cm} ...(8)

where \( S \) is the relative density of the solid, and \( U_p \) and \( V_p \) are the scalar components of the relative velocity of the particle. All quantities in Eqs (7) and (8) are in non-dimensional form. Time is non-dimensionalized with respect to \( L/u_0 \) and the non-dimensional quantity, \( \frac{g}{U_0} \), is defined as

\[ \frac{g}{U_0} = gL/U_0^2. \] \hspace{1cm} ...(9)

The drag coefficient, \( C_D \), is calculated from the empirical correlation\(^5\),

\[ C_D = \begin{cases} 0.44 & \text{when } Re_p > 1000, \\ \left( \frac{24}{Re_p} \right) \left( 1 + 0.14 Re_p^{0.7} \right) & \text{when } Re_p \leq 1000, \end{cases} \]

\hspace{1cm} ...(10)

where the particle Reynolds number, \( Re_p \), is defined as

\[ Re_p = \frac{|\dot{U}_p - U|}{v}. \] \hspace{1cm} ...(11)

For certain combinations of operating parameters (\( Re_{tr}, Re_{fl}, \rho_p \rho_f \), etc.), the particles may slide along the channel wall. In such a case, the particle experiences a kinetic frictional resistance \( \mu F_n \), where \( F_n \) is the
normal component (to the wall) of the net force acting on the particle due to drag, pressure, Coriolis, centrifugal and virtual mass effects. If the effective normal force is directed away from the wall, the particle is about to separate from the substrate, and then the friction force is zero.

Particles are treated as spheres of definite diameter while solving Eqs (7) and (8). A fourth order Runge-Kutta method is employed to integrate the initial value problem represented by Eqs (7) and (8). The initial position and velocity of the particle are specified at channel entry.

Special treatment is necessary for handling particle collisions with the channel walls. When determining the position of the particle at impact with the walls, the particles are treated as material points. Particle-wall collisions are treated as instantaneous. A coefficient of restitution is introduced in both the tangential and the normal directions to compute the post-impact velocity of the particle. The channel is assumed to be unaffected in the rotating reference frame by the collisions.

Assuming point collision, the velocity components of the particle after impact are given as

\[ U_{p2} = e_x U_{p1}, \quad \ldots (12) \]

and

\[ V_{p2} = -e_y V_{p1}, \quad \ldots (13) \]

where \( U_{p1} \) and \( V_{p1} \) are x- and y-components of particle velocity just before impact, \( U_{p2} \) and \( V_{p2} \) are the components just after impact, and \( e_x \) and \( e_y \) are the coefficients of restitution in the two directions. The velocity at impact and the point of impact of the particle with the boundary wall are determined by interpolation (Fig. 2).

In Fig. 2, let \( U_{p1}, V_{p1}, U_{p2}, V_{p2} \) be the initial and final velocities of the particle in a time step \( \delta t \). Let \( A(x_1,y_1) \) and \( B(x_2,y_2) \) be the corresponding two positions of the particle. In the absence of the wall, the particle would trace out the path represented by the solid curve AB. The presence of the wall implies that the particle will collide before reaching point B.

The point of impact \( C(x_s,y_s) \) is computed by approximating the actual path AB by a straight (dashed) line AB. This approximation is valid if \( \delta t \) is chosen small enough. Linear interpolation is used between A and B to locate the point C of impact. The actual time \( \delta t_i \) from A to C is also computed by interpolation. Using this new time step \( \delta t_i \), Runge-Kutta method may again be used to compute the new coordinates for point B. This process of interpolation and new determination of point B may be repeated until \( y_B = y_{wall} \).

Alternately, at a slight loss in accuracy, point C may be approximated as the actual point of impact in Fig. 2. In that case, the particle velocity before the impact may be calculated by interpolation. The post-impact velocity of the particle at point C may be determined by using Eqs (12) and (13). In practice the “short-cut” method using interpolation gives practically the same results as the theoretically more accurate iterative method. Hence, the “short-cut” method is used in the numerical computations of this study.

Two more practical points are worth mentioning. First, it is possible that the post-impact normal velocity component may be quite small. In such a case, below a cut-off value, the y-component of the velocity at the wall may be set to zero, so that the particle just slides along the channel wall. Second, the center of the spherical particle can never touch the wall. In computing particle trajectory, therefore, the center of the particle is tracked. In other words, the wall is hypothetically shifted by an amount equal to the particle radius. This tactic is adopted in all simulations.

Numerical method

Galerkin finite element method is used to render Eqs (1)-(3) into weak (algebraic) form. The velocity components, \( u \) and \( v \), as well as \( k \) and \( \varepsilon \), are interpolated bilinearly using quadrilateral elements. Pressure is assumed to be a constant within each element. The velocity interpolation functions are used as weight functions for the momentum equations,
while for pressure the weight function is unity. Combined Newton’s method is used to simultaneously solve the system of finite element equations for the velocity components and pressure. The discontinuous pressure field obtained from the solution of the Galerkin residuals must be smoothed to obtain the nodal values of the pressure field. A least squares finite element technique is employed for this purpose. The convergence criterion used for the Newton’s iterations is that the infinity norm of the correction vector should be less than $10^{-6}$. A RAM-based frontal solution technique is used to solve the linearized matrix system.

Particle trajectory (beginning with a specified initial velocity and position at the channel entrance) is tracked by solving Eqs (7) and (8) by a fourth order Runge-Kutta method. A non-dimensional time step of 0.005 or less is used in the numerical computations.

**Code validation**

Carrier-phase flow field calculations were validated elsewhere using mesh refinement and comparison with classical experimental velocity profiles. Mesh independence test for the trajectories traced out by three particles released from different inlet positions in a stationary channel are compared for the 100x16 and 200x32 meshes, with both air and water as the carrier phases. The results shown in Fig. 3 are seen to match well for the two meshes. In case of water as the carrier, the results for the two meshes practically coincide. The calculations in Fig. 3 are for $Re_H=11500$, $Ro_H=0.069$, $d_p=1\text{ mm}$, $\rho_p=2700\text{ kg/m}^3$ and the channel aspect ratio, $H/L=0.05$.

Computed particle trajectories are also compared for a stationary horizontal channel with numerical predictions. There is hardly any experimental study reported in the literature that deals with particle tracking through rotating channel. For stationary channels, the following data are used: width of channel ($H$) = 25 mm, length of channel ($L$) = 5 m, density of particle, $\rho_p=1000\text{ kg/m}^3$, and coefficient of restitution = 0.8. Air is used as the carrier phase. In the previous study, lift forces were included (neglected in the present study). In the absence of spin rate information, particle lift cannot be incorporated. It is also shown in an earlier study that lift force is significant only at very high particle rotation rates. However, that study also showed that particle lift force is considerably influenced by the choice of lift coefficient. Since in the present study, particles are considered as point masses during impact, it is not possible to compute the spin rate of the particle. Except for the exclusion of lift, the equations of particle motion of this study (with $\Omega=0$) are similar to those of Tsuji et al.². In Fig. 4, the computed particle trajectories of the present study are compared with the simulated results of Tsuji et al.². The qualitative trends are similar. The quantitative difference is partially due to the negligence of lift forces in the present study. In addition, note that in the simulation of Tsuji et al.² at the entrance into the channel, the particles have a positive y-component of velocity. In the present study, lift forces are included and the particle trajectories are compared with those of Tsuji et al.².
study, particles are assumed to enter parallel to the channel centerline.

In Fig. 5a, the successive positions at which particles impact the base (pressure side wall) of the channel are shown for the present study and Tsuji et al.\textsuperscript{2} Similarly, in Fig. 5b, the successive peak positions attained by the particle are shown after each bounce. The results are shown for three particles released respectively at $\frac{\varphi}{H} = 0.25, 0.5, 0.75$. The general trends predicted by the present study are physically possible and agree reasonably with the results of Tsuji et al.\textsuperscript{2} The quantitative difference is about 10-20% for the various cases.

A further test of the code was done for the case of $\Omega = 0$ with the channel rotating in a horizontal plane (Cases 1 and 2 of Table 1). In this case, particle trajectories are found to be (the theoretically expected) straight lines parallel to the channel walls.

**Results and Discussion**

The effects of various operating parameters on particle trajectories are of practical interest. The important operational parameters are $Re_H, Ro_H, d_p, S$, and the model parameters (in addition to turbulence model) are $\mu_k, e_x$ and $e_y$. Note that the two dimensional quantities, $d_p$ and $\mu$ (fluid viscosity) may be expressed equivalently as a non-dimensional particle diameter ($d_p^*$) and a non-dimensional settling velocity $v_{ts}^*$. For settling under the influence of gravity, these are defined as:\textsuperscript{24}

\[
d_p^* = d_p \left[ \frac{\rho (\rho_p - \rho)}{\mu^2} g \right]^{\frac{1}{3}} \quad \ldots \text{(14)}
\]

and

\[
v_{ts}^* = 1.73 \left[ \frac{g d_p (\rho_p - 1)}{\rho} \right] \left[ \frac{\rho^2}{\mu (\rho_p - \rho)} g \right]^{\frac{1}{3}} \quad \ldots \text{(15)}
\]

For flow in a rotating channel, the gravitational acceleration ($g$) is replaced by a representative Coriolis acceleration, $2\Omega U_0$. Strictly speaking, Eq. (15) is based on empirical studies with water. It is used here only to get a ballpark idea of $v_{ts}^*$ in air.

In Table 1, the cases for which particle motion was simulated in this study are shown. In all cases, the channel is assumed to rotate in a horizontal plane, so that effect of gravity (acting normal to the plane of the channel) may be considered to be only of secondary importance. In Fig. 3, the marked difference in particle trajectories in water and air is now readily explained. These correspond to Cases 11 (air) and 7 (water) of Table 1. The dimensionless particle diameter (Eq. 14) and dimensionless settling velocity (Eq. 15) are both much larger for air than for water. With air as the carrier (Fig. 3a), the particles rebound to a considerable distance from the base due to the relatively low drag force, indicated by the large dimensionless settling velocity. Also note that for the

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**Fig. 5** — Comparison of (a) positions of particle impact along channel base (pressure side), and (b) peak positions of particles across the channel.
same $Re_H$, air has a much larger inlet velocity.  When water is the carrier (Fig. 3b), the particles more or less follow a straight trajectory till they contact the pressure side of the channel.  Due to the small drag force, coupled with the Coriolis force, the particles barely rebound off a small distance.  After a few feeble bounces, they tend to slide along the channel base.  Thus, in case of water, particles may be expected to concentrate near the channel base.  

Intuitively, for a stationary channel, as $Re_H$ is increased, the particles should be more effectively carried by the fluid.  Thus, as $Re_H$ increases, a particle released at a given $y$-location from the channel inlet should traverse a longer distance before hitting the pressure-side wall.  For a rotating channel, however, results contrary to this intuitive expectation are obtained.  In order to bring out this effect, particle trajectories in rotating channel for air and water are shown in Figs. 6a and 6b, respectively, for $Re_H=11500$ and 35000.  These correspond to Cases 5, 6, 7 and 8 of Table 1.  As $Re_H$ is increased, particle trajectories become steeper and the particles cover a smaller distance before hitting the pressure-side wall.  This is because as the inlet velocity $U_0$ increases (for increasing $Re_H$), the rotation rate must increase as well to keep $Ro_H$ constant.  Now, the Coriolis force on the particle is proportional to $2\Omega U_0$.  Thus, for the higher $Re_H$, the particle experiences a greater force in the cross-stream direction, and hence follows a steeper trajectory.  This effect is more pronounced when the carrier phase is water.  This is because of the relatively low density-ratio $\rho_p/\rho$ for water in comparison with that for air.

In Figs 7a and 7b the effect of rotation rate on the particle trajectories is shown for Cases 9, 10, 7 and 12 of Table 1.  In all four cases, $Re_H$ is held constant at 11500.  Thus, an increase in $Ro_H$ corresponds to an increase in the rotation rate.  This has a direct implication on the Coriolis force acting on the particle.  Thus, the particle trajectories become steeper.  For air, the peaks of trajectories decrease considerably with increasing rotation rate.  For water, the particles impact the pressure-side wall closer to the inlet with increasing $Ro_H$.  After touching the pressure side, the particles essentially slide along.  Thus, as the rotation rate increases, we would expect a larger portion of the channel wall to experience sliding wear.  Furthermore, as $x$ increases, a greater number of particles would slide past a given $x$ location.  In other words, it is reasonable to expect sliding wear rate to increase with $x$.  This is exactly what is observed\textsuperscript{20,25,26} in the Coriolis wear tester.

Figures 8a and 8b show the particle trajectories for different diameters of the particle, corresponding to Cases 13-16 of Table 1.  In Fig. 8a, the peaks reached by particles of $d_m=2$ mm are greater than the peaks reached by particle with $d_m=0.5$.  This is due to the higher inertia of the larger particles.  In Fig. 8b, the
Fig. 6 — Effect of $Re_H$ for Cases 5-8 of Table 1: (a) $Re_H = 11500$, (b) $Re_H = 35000$ for air (top) and water (bottom)

$Ro_H = 0.069$, $H/L = 0.05$, $d_p = 1\text{mm}$, $\rho_p = 2700\text{ kg/m}^3$, $e_x = 1$, $e_y = 1$, $f = 0.1$

$Re_H = 11500$, $H/L = 0.05$, $d_p = 1\text{mm}$, $e_x = 1$, $e_y = 1$, $f = 0.1$

Fig. 7 — Effect of $Ro_H$ for Cases 9, 10, 7 and 12 of Table 1: (a) air and (b) water for $Ro_H = 0.069$ (left) and $Ro_H = 0.21$ (right)
larger particles exhibit a greater settling effect under the action of Coriolis force. The smaller particles more closely follow the carrier fluid. In conjunction with Fig. 7(b), the effect of particle size is clearly seen. For \(d_m=2\) mm, the particles mildly bounce off the pressure side a few times. For smaller size, the bounce is almost non-existent. Depending on the range of operating parameters used, for particles larger than a certain limit, no significant increase in the peak was found.

Figure 9 (corresponding to Cases 17-19 of Table 1) shows the effect of coefficient of restitution on particle trajectory with air as the carrier phase. A distinct difference in the particle trajectories can be seen for the different cases in Figs. 9a, 9b and 9c. This is attributed to the loss in energy of particle after successive impacts. Obviously, \(e_x=1\) and \(e_y=1\) corresponds to the case of perfectly elastic collision. When \(e_x\) is reduced to 0.8 from 1, the peaks of the trajectories decrease considerably. A surprising result arises when \(e_x\) is decreased from 1 to 0.8 to account for friction during impact. In contrast to Fig. 9b, the particles in Fig. 9c tend to bounce to a somewhat greater distance from the wall. In Fig. 9b, the particles tend to slide along the wall (with apparently no bounce) for \(x>0.75\).

This anomalous behavior may be explained as follows. When \(e_x=0.8\), there is a slight decrease in the post-impact tangential (x-component) velocity of the particle. This implies that the local Coriolis force (=2Ω\(U_p\)) on the particle decreases just after the impact, indicating that the particle should rise to a higher peak. In the case of \(e_x=1\), post-impact \(U_p\) remains unchanged, and hence there is no decrease in the Coriolis force. On the contrary, as the particle traverses the channel, the centrifugal force (proportional to \(\Omega^2 x\)) increases, resulting in an increase of \(U_p\), and hence the Coriolis force, 2Ω\(U_p\). Thus, in combination with the drag, the net effect is that the particles rise to successively smaller heights until they begin to slide along.

Note that the kinetic friction coefficient, \(\mu_k\), enters into the picture only after the particle begins to slide, if at all. Until then, friction is handled through \(e_x\).

The effect of particle density, for water as the carrier, is shown in Figs 10a and 10b. These correspond to Cases 4 and 20 of Table 1. A drastic difference in the particle trajectory as well as in the residence time is seen. For \(S=2.7\) (Case 4 of Table 1, Fig. 10a), the particles follow an initial steep path, hit the channel pressure side, feebly bounce a few times, and then slide along. The particle released near the suction side of the channel (at \(y=0.25H\)) takes the minimum time (or angle \(\theta\) rotated by the channel) while the particle released at \(y=-0.25H\) (near the pressure side of the channel) takes the maximum time.
Fig. 9 — Effect of coefficients of restitution for Cases 17-19 of Table 1: (a) $e_x = 1, e_y = 1$, (b) $e_x = 1, e_y = 0.8$, (c) $e_x = 0.8, e_y = 0.8$

Re$_H$ = 11500, Ro$_H$ = 0.21, $d_p = 1$ mm, $e_x = 1, e_y = 1$, H/L = 0.05, $\rho_p = 1000$ kg/m$^3$, $f = 0.1$

This difference is due to the greater distance of sliding (against friction) of the particles released at $y = -0.25H$. For $\rho_p = 1000$ kg/m$^3$, (Case 20, Fig. 10b), the particles are neutrally buoyant, and hence they more or less closely follow the carrier-phase streamlines. The trajectories are straight without touching the channel base. Thus, there is no sliding friction on the particles. Due to these reasons, the particle residence time in the channel is significantly reduced, being 50% of that in Fig. 10a.

Finally, we note that particles exit faster (less residence time) in air (as the carrier) than in water for the same Re$_H$. For the same Re$_H$=11500, air has a larger inlet velocity than water by a factor of more than 15. However, since the particles in air bounce off the channel wall many more times than in water, the residence times are not directly proportional to the bulk velocity of the carrier.

Conclusions

Some key factors in the simulation of particle trajectories in a viscous fluid medium flowing through a rotating channel have been addressed. The results of this work have a direct relevance to the Coriolis wear tester. The particle trajectories show fundamentally different behaviour in air and water. In air, the particles have a much higher representative dimensionless diameter, and hence repeatedly bounce along the channel. In sharp contrast, for the same rotation and bulk Reynolds numbers, in water, the particles have a much smaller representative dimensionless diameter ($d_p^*$). Thus, the particles tend to hit the channel base (pressure side wall), feebly bounce a few times, and thereafter slide along the channel base. The implication of this observation is
that in air, the erosion of the channel base is likely to predominantly due to impact. In water, the predominant erosion mechanism is likely to be sliding, except perhaps in the near entrance. The paper also discusses the effect of several important operating parameters such as particle diameter and density, bulk Reynolds number and rotation number.

In order to apply the present study to the Coriolis wear tester, it would be necessary to track the trajectories of a fairly large number of particles (within the dilute mixture assumption). A “steady state” may be attained when the number of particles inside the channel remains constant. Knowing the positions and velocities of particles during such a “steady state”, especially along the channel base, the erosion rate along the channel base may be determined via a suitable erosion model.

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References