Some investigations on annular ring microstrip antenna

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The various parameters of annular ring microstrip antenna such as input impedance, VSWR, return loss, radiation pattern have been theoretically investigated as a function of frequency for different feed locations. It has been observed that resonance occurs at 3.285 GHz which is invariant with feed locations. The antenna behaves as RL network below resonance and as RC network above resonance and shows perfect matching at 3.35 cm feed location from the centre. The percentage bandwidth is found decreasing and the radiated power increasing with increasing b/a ratio.

1 Introduction
The microstrip antenna have several advantages over the conventional microwave antenna such as light weight, low volume, low cost, planar configuration and compatibility with integrated circuits associated with diversified applications. The major drawback of the microstrip antenna is very narrow bandwidth\(^1,2\). However, the annular ring microstrip antenna provides larger bandwidth and the separation of modes can be controlled by the ratio of the outer to inner radii. It has been reported\(^3,8\) that for thick substrate (\(h \geq 0.1 \lambda_0\)) the ring is excited in TM\(_{12}\) mode with excellent impedance match near resonance frequency at two optimum locations of the probe—one at an interior point at a distance of about \(R/3\) (\(R=\) Mean radius of the ring) from the inner edge with a slight decrease in the resonance frequency, and the other near outer edge at resonance frequency. For thin substrate (\(h \leq 0.02 \lambda_0\)), matching is excellent when the probe is placed near the outer edge and also at a distance about \(R/4\) from the inner edge. The usual method to find the input impedance of annular ring microstrip antenna by cavity Green’s function technique is critical and quite involved. Therefore, in the present work, for input impedance of the annular ring microstrip antenna (ARMSA), the radial transmission line concept\(^9,10\) is employed in which the wall admittance and the effect of mutual coupling between the inner and outer radiating slots are considered. Consequently, in the present investigation the input impedance, VSWR, return loss etc., are studied as a function of feed location (Fig. 1) and frequency with a view to accurately designing the ARMSA.

2 Theoretical considerations
Figure 1 shows a microstrip annular ring antenna with a feed point at \((c,0)\) by a coaxial line where \(a\) is inner radius and \(b\) is outer radius of the patch \((a<c<b)\). The substrate thickness \((h)\) is small as compared to wavelength. The feed current due to the coaxial line excitation is taken as \(I_0\).

2.1 Equivalent circuit
The equivalent circuit of ARMSA based on a transmission line model is shown in Fig. 2(a) in which the ring is considered as a radial transmission line terminated into a load admittance with radiating apertures at radii \(a\) and \(b\) for TM\(_{lm}\) mode. Here the transmission line is represented by the \(\pi\) network, where, \(g_1', g_2', g_3', g_1, g_2, g_3\) denote the conductance for inner and outer radiating apertures, respectively, \(y_n^m(a,b)=y_n^m(b,a)\) mutual admittance, and \(y_n^m(a)\) and \(y_n^m(b)\) are self-admittance at inner and outer radial apertures, respectively.

In Fig. 2(a) the value of \(Z_a, Z_b\) and \(Z_c\) can be obtained as

\[ Z_a = \frac{1}{[y_n^m(a)-y_n^m(a,b)+g_3']} \]  
\[ Z_b = \frac{1}{[y_n^m(b)-y_n^m(a,b)+g_3]} \]  
\[ Z_c = \frac{1}{[g_1+g_3]} \]
Figure 2(a) can be redrawn as Fig. 2(b) in which the points a, b, c are in delta form. Simplification with the help of delta to star transformation yields

\[ R_a = \frac{g_2}{g_2 y_n''(a,b) + \frac{g'_1}{g_2 y_n''(a,b)} + \frac{g_5}{g_2}} \]  \hspace{1cm} (4)

\[ R_b = \frac{\frac{g'_2}{g_2 y_n''(a,b) + \frac{g'_1}{g_2 y_n''(a,b)} + \frac{g_5}{g_2}}}{g_2 y_n''(a,b) + \frac{g'_1}{g_2 y_n''(a,b)} + \frac{g_5}{g_2}} \]  \hspace{1cm} (5)

\[ R_c = \frac{\frac{g'_2}{g_2 y_n''(a,b) + \frac{g'_1}{g_2 y_n''(a,b)} + \frac{g_5}{g_2}}}{g_2 y_n''(a,b) + \frac{g'_1}{g_2 y_n''(a,b)} + \frac{g_5}{g_2}} \]  \hspace{1cm} (6)

which are shown in Fig. 2(c).

The parameter c (feed point) intervenes in the formulas for \( g_1, g_2, g_3 \) and \( g'_1, g'_2, g'_3 \) which have been derived using transmission line model involving \( y \)-parameters which do contain inner radius \( a \), outer radius \( b \) and feed point.

2.2 Input impedance

The impedance can be calculated from Fig. 2(c) as

\[ Z = \left[ \left( Z_a + R_a \right) + \left( Z_b + R_b \right) \right] \text{ } Z_c = \frac{E_{c_n}(c)}{I_{m}} \]  \hspace{1cm} (7)

where, \( E_{c_n}(c) \) is the electric field component on the feed point in \( z \) direction.

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Fig. 2—(a) Equivalent circuit of the antenna obtained by replacing the transmission by \( \pi \) networks, (b) Simplified forms of equivalent circuit of (a) and (c) Simplified forms of equivalent circuit of (b)
Hence, the impedance seen by the feed is given by

$$Z_n = \frac{hE_{cn}(c)}{I_0} = \frac{h}{\sigma_n \pi} Z$$

as $I_n = \frac{I_0}{\pi \sigma_n}$, where $\sigma_n = 1$ for $n > 0$.

**2.3 Wall admittance**

The wall admittance can be expressed in terms of self-admittance and mutual admittance. The self-admittance can be decomposed into self-conductance $g_n^s$ and self-susceptance $b_n^s$. The expression for self-conductance $g_n^s$ for the $TM_n$ mode in a circular microstrip disk is given as

$$g_n^s(a) = \frac{h}{2(1+\delta_n)\eta_0} \left[ (1+\delta_n)(k_0a)^2 I_1 + n^2(1-\delta_n)I_2 \right]$$

where,

$$I_1 = \frac{\pi}{6} [j_n^2(k_0a\sin\theta)]^2 \sin\theta d\theta$$

$$I_2 = \int_0^\infty \frac{\cos^2\theta}{\sin\theta} j_n^2(k_0a\sin\theta)d\theta$$

$\delta_n = 1$ for $n = 0$

$= 0$ for $n > 0$

$\eta_0 = \left( \frac{\mu_0}{\varepsilon_0} \right)^{\frac{1}{2}}$

Similar expressions for $g_n^s(b)$ can be obtained by replacing $a$ by $b$ in Eq. (9). The expression for wall susceptance $b_n^s(a)$ for the $TM_n$ mode has been derived from the magnetic wall model. The effective radii for a circular microstrip disk are used here to model the fringing field effect for the annular ring. The effective radii are given as

$$a_c = a \left( 1 - \frac{2hx}{\pi a e_r} \right)^{\frac{1}{2}}$$

$$b_c = b \left( 1 + \frac{2hx'}{\pi b e_r} \right)^{\frac{1}{2}}$$

where, $x = \ln \left( \frac{a}{2h} \right) + 1.41 e_r + 1.77 + \frac{h}{a} (0.268 e_r + 1.65)$

and $x'$ is obtained from Eq. (13) by replacing $a$ by $b$.

Thus, the value of wall susceptance can be written as

$$b_n^s(a) = -\frac{k_0 a j_n^1(k_0a) j_n(k_0a) y_n(k_0a) - y_n(k_0a) j_n^1(k_0a)}{\omega \mu_0 j_n^1(k_0a) y_n(k_0a) - y_n(k_0a) j_n^1(k_0a)}$$

The expression for mutual admittance is based on the near-field integration and can be written as

$$y_n^m(a,b) = \frac{jabh}{2\pi^2 \mu_0}$$

where,

$$r_1 = \left[ a^2 + b^2 - 2ab \cos(\phi - \alpha) \right]^{\frac{1}{2}}$$

The various parameters such as reflection coefficient, VSWR and return loss can be calculated as

Reflection coefficient $(\Gamma) = \frac{Z_{in} - Z_0}{Z_{in} + Z_0}$

where,

$Z_0$ = Characteristic impedance of the feed line

$Z_{in}$ = Input impedance of patch at feed point

$\text{VSWR}(s) = \frac{1 + |\Gamma|}{1 - |\Gamma|}$

The effective loss tangent, comprising the dielectric, copper and radiation losses is given by

$$\delta_{\text{eff}} = \delta + \frac{1}{\eta_0 \sqrt{\varepsilon_0 \sigma_0 \pi f}}$$

$$+ \frac{2 \varepsilon_0 \mu_0 h l_i}{\eta_0 e_r} \left[ \frac{j_n^2(k_0a)}{j_n^2(k_0b)} \left( \frac{1}{k_1^2 b^2} - 1 \right) \left( \frac{1}{k_1^2 a^2} - 1 \right) \right]$$
The bandwidth (BW) is defined as

\[ BW = \frac{100(s-1)}{Q_T \sqrt{s}} \text{ for } s \geq 1 \]  

where, \( s = \text{VSWR} \)

The radiation fields of an annular ring microstrip antenna may be obtained by calculating the far-field components which may be expressed as

\[ E_\theta = k_1 E_0 j \frac{e^{-jkr}}{r} \cos \theta \cos \phi \]
The radiation pattern is calculated using the above equation.

### 3 Design data

The ARMSA investigated were designed with following details:

- Relative permittivity of the substrate ($\varepsilon_r$) = 2.2 ± 0.002
- Effective relative permittivity ($\varepsilon_e$) = 2.085
- Thickness of the dielectric substrate ($h$) = 1.59 mm
- The inner radius of annular ring (a) = 3 cm
- Effective inner radius of annular ring ($a_e$) = 2.8287 cm
- The outer radius of annular ring (b) = 6 cm
- Effective outer radius of annular ring ($b_e$) = 6.1784 cm
- Design frequency ($f$) = 3.29 GHz

### 4 Calculations

The input impedance of ARMSA for a different feed location was calculated as a function of frequency using Eqs (7) and (8), and the data thus obtained are shown in Fig. 3. The VSWR was also calculated as a function of frequency with the help of Eq. (17) for different values of feed locations, i.e. the distance from centre to feed point ($c$). The data thus obtained are shown in Fig. 4. The inner and outer radii of the annular ring patch are in the range of 3-6 cm. Therefore, the feed point can be varied from 3 to 6 cm. However, the calculations show that the important information can be obtained by varying the feed point between 3 and 4 cm. Hence the feed points ($c$) were selected between 3 and 4 cm in Figs 3 and 4. The quality factor and bandwidth were also calculated as function of $b/a$ ratio with the help of Eqs (18) and (19). The data thus obtained are shown in Fig. 5. The radiation patterns calculated from Eq. (20) are shown in Fig. 6.

### 5 Discussion of results

Variation of input impedance with frequency is shown in Fig. 3 indicating real and imaginary part separately for different feed locations. It is observed that resonance occurs at 3.285 GHz for each feed location and it is invariant with the feed location. Similar observations were also made by Dahele and Lee. The antenna behaves as RL network below resonance, whereas it behaves as RC network above resonance. It is also observed that, at resonance, real part of impedance increases with decreasing value of feed location from the centre of the annular ring microstrip antenna.
The VSWR is shown as a function of frequency for different feed location in Fig. 4. It is found that around the resonance, the value of VSWR increases with increasing feed location. This further shows that the band of satisfactory operation decreases with increasing feed location, i.e. the distance from centre to feed point.

The variation of percentage bandwidth and quality factor as a function of \( b/a \) are shown in Fig. 5 for VSWR =1.5:1 and 2:1. It is found that the value of bandwidth decreases with increasing value of \( b/a \). This is also corroborated from the quality factor data which increases with increasing value of \( b/a \). It is further observed that the percentage bandwidth for VSWR 2:1 is higher than that for 1.5:1. From radiation pattern (Fig. 6) it is clear that the radiated power increases with the \( b/a \) ratio. Further, it is observed that radiated power in the main lobe increases and side lobe level decreases with \( b/a \).

6 Conclusions

From the above analysis it is found that the matching of antenna with the feed depends on feed location and the resonance behaviour of patch is invariant with the feed location. The radiation power in the main beam increases and level of the sidelobe decreases with \( b/a \) ratio.

References