Kinetic Alfven waves in inhomogeneous anisotropic dusty magnetoplasma with inhomogeneous electric field – Particle aspect analysis

R P Shandilya, P Varma & M S Tiwari
Department of Physics, Dr H S Gour University, Sagar (MP) 470 003

Received 31 March 2003; revised 4 June 2003; accepted 13 November 2003

Kinetic Alfven waves in the presence of an inhomogeneous electric field perpendicular to the ambient magnetic field in inhomogeneous dusty magnetoplasma are investigated. Expression for the field-aligned current, perpendicular current (to the Eo), dispersion relation, particle energy and growth-rate are derived and the effect of inhomogeneous electric field is discussed. The particle aspect approach is adopted to investigate the trajectories of charged particles in the electromagnetic field of kinetic Alfven wave in dusty magnetosphere. It is found that electric field enhances the frequency of kinetic Alfven wave for both the waves travelling towards the ionosphere and reflected by the ionosphere. Enhancement of growth rate at the particular wave number is predicted. Reduction in wave frequency and enhancement of growth rate by electric field inhomogeneity is observed. Minor effects of inhomogeneous electric field on perpendicular and parallel currents are noted. The applicability of the finding is discussed for auroral acceleration phenomena.

Keywords: Alfven waves, Magnetoplasma, Auroral acceleration, Inhomogeneous electric field

PACS No.: 94, 20.Yx

1 Introduction

In the past, several observational and empirical models have been developed for the electric and magnetic field distribution in the ionosphere and the magnetosphere. Early reviews concerning the electric field distribution observed in the ionosphere include the work of various workers. Early satellites in polar orbits equipped with an electric double probe such as OGO-6 and INJUN-5 have established the concept of polar cap in terms of electric field patterns across which the dawn-to-dusk electric field exists permanently. The most prominent feature of the electric field data from polar orbiting satellites is the persistent occurrence of steep reversal in the field. The electric field pattern under very quiet conditions is basically the same as that during disturbed conditions due to substorm. On the basis of 32 balloon flights, Mozer and Lucht have presented hourly average of electric field in the auroral zone as a function of local time which exhibits the reversal of electric field of inhomogeneous nature. These average values can be mapped in the equatorial plane of the magnetosphere. Thus, the inhomogeneity of electric fields may affect the wave emission around the auroral region. In the past, Sojka and Schunk have presented a model to study how electric field structures affect the polar cap F-region in which their electric field strength and polarity are varied. The variability of electric fields in the magnetosphere has been also reviewed by Mauk and Zanetti.

During the last decade, observations of electric field in the ionosphere and the magnetosphere using various techniques have led to important advances in the understanding of magnetosphere-ionosphere coupling. Electric fields of the order of hundreds of millivolts per metre have been predicted in the high latitude ionosphere, the auroral zone, magnetotail and the plasma-sheet. In a variety of situation, particularly at the time of substorm onset, the interplanetary magnetic field reverses its directions and two oppositely directed inhomogeneous electric fields are reported in the plasma-sheet and in the auroral zone. Studies of electric field and particle data from the S3-3 satellite at altitudes from 1000 to 8000 km have identified structures, called electrostatic shocks that represent the inhomogeneous electric field and are supposed to be responsible for auroral particle acceleration.

Most of the astrophysical and lower ionospheric atmosphere contain electrons, ions and charged dust grains which constitute the components of dusty plasmas. It is well known that21,22 the
presence of static and dynamic charged dust grains can give rise to the dust Alfvén waves, which are normal modes of a uniform, magnetized, fully ionized dusty plasma. Collective phenomena in dusty plasma are of great importance provided that the number density of charged dust grain is sufficiently high within a Debye sphere. The field of dusty plasma is recently growing rapidly because of its potential application in space and astrophysical systems (e.g. nebulae, from interstellar to intergalactic media, protostar and circumstellar disks, planetary magnetospheres, comet tail, asteroids zones, earth's lower ionosphere, etc.) and energy research\textsuperscript{21} (e.g. fusion devices, MHD generator, etc. as well as in the industrial and modern plasma processing technology). A magnetic field is almost invariably present in dusty plasma systems either in ambient condition or applied in the laboratory systems for processing or experimental purposes. The magnetic field may cause the presence of a number of new modes in a finite temperature dusty plasma.

The starting point of the model is the well known fact that in an auroral system, the electric field perpendicular to the magnetic field reverses direction across a very narrow latitude range which is usually close to the edges of the auroral oval\textsuperscript{42,43}. Frank and Gurnett\textsuperscript{42} have predicted that these field reversals occur on field lines along with inverted V-polarization. The field reversals are related to field aligned currents\textsuperscript{44}. In the magnetotail lobe, which has been identified as a region of open magnetic field lines\textsuperscript{45,46}, the plasma velocity distribution characteristics are qualitatively similar to those of the plasma sheet boundary layer\textsuperscript{47}. In this region, energetic ion tailward-directed velocities and anisotropic ion distribution signatures are often observed\textsuperscript{48-50}.

Linear theory predicts that the kinetic Alfvén waves are excited around their resonance region with enhanced amplitudes by a drift wave\textsuperscript{51-54}. Thus, the plasma density inhomogeneity is considered as the main source of kinetic Alfvén wave instability and the influence of electric field inhomogeneity observed for the plasma sheet in the dusty magnetospheric plasma is discussed as the central theme of the present paper.

In the past, the method of particle aspect analysis\textsuperscript{55-64} was developed for the electrostatic and electromagnetic perturbations ignoring the effects of dusty plasma. In the present analysis, we extend the method for dusty magnetosphere and present a model for a magnetosphere-ionosphere coupling in view of the kinetic Alfvén waves excited in the plasma sheet by the density inhomogeneity. The advantage of this approach is its suitability for dealing with auroral electrodynamics involving the current system, acceleration and energy exchange by wave-particle resonance interaction.

2 Basic assumption

In this model, the plasma is divided into two groups of particles, namely, resonant and non-resonant. It is assumed that the resonant electrons participate in the energy exchange with the wave, whereas non-resonant particles support the oscillatory motion of the waves. A wave propagating obliquely to the magnetic field in a plane normal to the density gradient and applied electric field is considered in an anisotropic plasma. The ambient magnetic field is directed along the z-axis and the density gradient and perpendicular electric field are in the y-direction. The wave is propagating in the (x, z) plane.

The present analysis is limited to case of low-$\beta$ plasma and the electromagnetic mode. We have not considered the variation in ambient magnetic field and the wave propagates towards the auroral ionosphere which is a low-$\beta$ region. Therefore, the low-$\beta$ assumption is more suitable. The following physical conditions are imposed to study the behaviour of the kinetic Alfvén instability. The kinetic Alfvén wave is assumed to start at $t = 0$, when the resonant particles are undisturbed and

$$V_{\|}\ll V_{\perp} < \omega / k_z << V_{\parallel}; \text{then} \quad \omega << \Omega_{e}, \Omega_{d}, k_z^2 \rho_e^2 << k_z^2 \rho_d^2 << 1$$

... (1)

where, $V_{\|}$, $V_{\perp}$, $V_{\parallel}$ are the mean velocities of dust, ion and electron, respectively, along magnetic field; $\Omega_{e,d}$ are gyration frequencies and $\rho_{e,d}$ are the mean gyro-radius of the respective species.

In this paper, we extend particle aspect analysis to electromagnetic perturbations. Two potential representations are adopted, which are commonly used to express electromagnetic perturbations in the low-$\beta$ plasma\textsuperscript{52,55-57}. The idea is to decouple the compressional Alfvén mode by assuming the low-$\beta$ case, that is to take into account only the effect of field-line bending. Thus,

$$\left(\nabla \times E\right)_{\|} = -\frac{\partial B_{\|}}{\partial t} = 0$$
This enables us to use a scalar potential \( \varphi \) to express the perpendicular component of the wave electric field as

\[
E_{\perp} = -\nabla_{\perp} \varphi
\]

Since the wave is electromagnetic, we have to use an \( E_{\parallel} \) that is not equal to \(-\frac{\partial \varphi}{\partial z}\). Hence, for \( E_{\parallel} \) we use a different potential

\[
E_{\parallel} = -\nabla_{\parallel} \psi \quad \text{and} \quad \varphi \neq \psi
\]

The potentials \( \varphi \) and \( \psi \) must satisfy suitable field equations.

We begin by considering a wave electric field \( E \) of the form

\[
E = E_{\perp} + E_{\parallel}
\]

where

\[
E_{\perp} = -\nabla_{\perp} \varphi \quad \text{and} \quad E_{\parallel} = -\nabla_{\parallel} \psi
\]

and

\[
\varphi = \varphi_{1} \cos \left( k_{\perp} x + k_{\parallel} z - \omega t \right)
\]

\[
\psi = \psi_{1} \cos \left( k_{\perp} x + k_{\parallel} z - \omega t \right)
\]

... (2)

where, \( \varphi_{1} \) and \( \psi_{1} \) are assumed to be a slowly varying function of time \( t \), and \( \omega \) is the wave frequency, \( k_{\perp} \) and \( k_{\parallel} \) define the components of wave vector \( k \) across and along the magnetic field \( B_{0} \). Electrons streaming with their thermal velocity along the field lines are assumed to interact with the electric field of the kinetic Alfvén wave. Electrons whose velocity is slightly less than the parallel phase velocity \( \omega / k_{\parallel} \) of the wave cause Landau damping of the wave. The inhomogeneous applied electric field \( E(y) \) has the form

\[
E(y) = E_{0} \left( 1 - \frac{y^{2}}{a^{2}} \right)
\]

where, \( a \) is taken to be comparable to the mean ion gyro-radius, but much larger than the Debye length.

When \( \frac{y^{2}}{a^{2}} \ll 1 \), \( E(y) \) becomes a constant uniform field. In the case \( y > a \), the electric field changes sign and oppositely directed.

The solution for the particle orbit in terms of the field \( E(y) \) and \( B_{0} \) is obtained by the Bogoliubov Mitropolskii method as

\[
x(t) = x_{0} + \Delta + \frac{V_{\perp}}{\Omega} \left[ 1 - \frac{3}{4} \frac{E'(y)}{\Omega^{2}} \right] \sin \left( \Omega t - \frac{\pi}{4} \right)
\]

\[
y(t) = y_{0} - \frac{V_{\parallel}}{\Omega} \left[ 1 + \frac{1}{4} \frac{E'(y)}{\Omega^{2}} \right] \cos \left( \Omega t - \frac{\pi}{4} \right)
\]

... (3)

and

\[
z(t) = z_{0} + V_{\parallel} t
\]

where, \( \theta \) is the phase of \( V_{\parallel} \) at \( t = 0 \) and

\[
\Delta = -\frac{E(y)}{\Omega} \left[ 1 + \frac{E'(y)}{\Omega} \left( \frac{V_{\perp}}{\Omega} \right)^{2} \right]
\]

... (4)

It should be noted that \( \frac{d \Delta}{dt} \) represents the drift velocity and the second term in the square brackets on the right hand side of Eq. (4) represents the finite gyro radius correction.

We consider a zeroth-order distribution \( N(y, V) \) of the form

\[
N(y, V) = N_{0j} \left[ 1 - \varepsilon \left( y + \frac{V_{\perp}}{\Omega} \right) \right] f_{j}(V_{\parallel}) f_{j}(V_{\perp})
\]

... (5)

where, \( \varepsilon \) is a small parameter of the order of the inverse density gradient scale length and \( j \) represents i.e and d for respective species.

\[
f_{i}(V_{\perp}) = \frac{m}{2 \pi T_{\perp}} \exp \left( -\frac{m V_{\perp}^{2}}{2 T_{\perp}} \right)
\]

\[
f_{j}(V_{\parallel}) = \left( \frac{m}{2 \pi T_{\parallel}} \right)^{\frac{3}{2}} \exp \left( -\frac{m V_{\parallel}^{2}}{2 T_{\parallel}} \right)
\]

where, \( N_{0j} \) is the equilibrium density of the \( j^{th} \) species, \( T_{\parallel} \) and \( T_{\perp} \) are the parallel and perpendicular temperatures with respect to the ambient magnetic field.

3 Particle trajectory

The equation of motion of a particle is given as
\[ \frac{d\mathbf{v}}{dt} = q \left( \mathbf{E} + \frac{1}{c} \mathbf{v} \times \mathbf{B} \right) \]

where

\[ \mathbf{E} = E(y) + E_1 \text{ and } \mathbf{B} = B_0 + B_1 \quad \cdots (6) \]

and \( E_1 \) and \( B_1 \) are the electromagnetic components of the wave. The velocity \( \mathbf{v} \) has an unperturbed component \( \mathbf{V} \) and a perturbed component \( \mathbf{u} \), i.e., \( \mathbf{v} = \mathbf{V} + \mathbf{u} \). Expressing the wave magnetic field \( B_1 \) in terms of the wave electric field \( E_1 \) with the help of Maxwell's equations and substituting in Eq. (6), the perturbed velocity \( \mathbf{u} \) is determined by the following set of equations.

\begin{align*}
\frac{d\mathbf{u}}{dt} + i\Omega \mathbf{u}_r &= \frac{q}{m} \left[ \phi k \left( \frac{V}{k} - V \frac{k}{m} (\phi - \psi) \right) \right] \\
& \quad \times \sin (k_x x + k_z z - \omega t) \\
\frac{d\mathbf{u}}{dt} &= \frac{q}{m} \left[ k \psi + \frac{V}{k} \left( \frac{\psi - \psi}{\omega} \right) \cos (\Theta - \Omega t) \right] \\
& \quad \times \sin (k_x x + k_z z - \omega t) \quad (7)
\end{align*}

where \( \mathbf{u}_r = \mathbf{u} + i \mathbf{u}_y \), \( \Omega = qB_d/mc \), \( \mathbf{u}_x \) and \( \mathbf{u}_y \) are the perturbed velocities in the \( x \) and \( y \) directions, respectively. Equations (7) are solved under the approximation of replacing the co-ordinates of particles on the right-hand sides of Eq. (7) by those of free gyration Eq. (3) and treating \( q \) and \( \psi \) as constants. This process is similar to that of linearization of the Vlasov equation.

Substituting Eq. (3) in Eq. (7) and following the procedure as described by Tiwari and Varma, we get the oscillatory solution \( \mathbf{u}(r, t) \) for the non-resonant particles. For resonant particles, we take into account the initial condition \( \mathbf{u}(r, t(0)) = 0 \) inferred from our basic assumptions. Thus,

\[ \mathbf{u}_r(r, t) = \frac{q}{m} \left[ k \phi + \frac{V}{k} \left( \frac{\phi - \psi}{\omega} \right) \right] \\
\times \frac{\sum J_n(\mu) \sum J_\mu(\mu) \frac{1}{\Lambda_n^2 \sin (\xi_n \delta)} \cos (\xi_n \delta - \eta_{n+1} t)}{2 \eta_{n+1}} \\
- \frac{\delta}{2} \cos (\xi_n \delta - \eta_{n+1} t) - \frac{\delta}{2} \cos (\xi_n \delta - \eta_{n-1} t) \quad \cdots (8) \]

Here, \( \delta = 0 \) for non-resonant particle and \( \delta = 1 \) for resonant particle and

\[ \eta_n = (\Omega + V) k + k \Delta - \omega \]

\[ \Delta = -\frac{E(y)}{\Omega} \left[ 1 + \frac{E''(y)}{E(y)} \frac{1}{4 \Omega^2} \right] + \ldots \]

\[ \xi_n = k_x x + k_z z + \omega t + n \cdot (2\pi \omega t) \]

\[ \Lambda_n^2 = \eta_n^2 - \Omega^2 \]

\[ \mu = \frac{k \Omega}{V} \left[ 1 + \frac{3 \hat{E}(y)}{4 \Omega^2} \right] + \ldots \]

Also use has been made of the relations

\[ \exp \left[ -i \mu \sin (\Theta - \Omega t) \right] = \sum J_n(\mu) \exp \left[ -i n (\Theta - \Omega t) \right] \]

\[ \cos \phi \exp (-i \mu \sin \phi) = \frac{n}{\mu} \sum J_n(\mu) \exp (-i \phi) \]

where, \( J_n(\mu) \) and \( J_\mu(\mu) \) are the Bessel functions. The appearance of the Bessel function weakens the wave field owing to the finite-Larmor radius effect. The true trajectory of the particle to first order is obtained by integrating \( \mathbf{u}(r, t) \) and then adding the component from Eq.(3) to it. In view of the approximation introduced at the beginning, the dominant contribution comes from the term with \( n = 0 \). The resonance condition is defined by \( k \Delta + k \| - \omega = 0 \).
4 Density perturbation

In order to find the density perturbation associated with the velocity perturbation \( u(\mathbf{r},t) \), let us consider a group of particles with the same initial condition and let the number density be

\[
n(\mathbf{r},t,V) = N(y,V) + n_1(\mathbf{r},t,V) \quad \ldots \quad (11)
\]

where, \( N \) is the zeroth-order distribution and \( n_1 \) is the perturbed density, which is determined by the equation\(^{68}\)

\[
\frac{dn_1}{dt} = -(\nabla \cdot u) N - \nu \frac{dN}{dy} \quad \ldots \quad (12)
\]

Substituting the values of \( u(\mathbf{r},t) \) to evaluate \( \nabla \cdot u \) and then transferring the right hand side of Eq.(12) in terms of \( t \) and after integration we obtain \( n_1 \) for non-resonant particles as

\[
n_1(\mathbf{r},t) = N(V) \sum_j f_j(\mu) \frac{q_j}{m_j} \left[ \left( \frac{V_j}{\omega} - \frac{k_j^2}{\omega} \right) \cos \xi_{0s} \right] \cos \xi_{0d} \quad \ldots \quad (13)
\]

and for the resonant particles

\[
n_1(\mathbf{r},t) = N(V) \sum_j f_j(\mu) \frac{q_j}{m_j} \left[ \left( \frac{V_j}{\omega} - \frac{k_j^2}{\omega} \right) \cos \xi_{0s} \cos \xi_{0d} \right] \quad \ldots \quad (14)
\]

Here, \( V_{d,j} \) is the diamagnetic drift velocity of respective species which is defined by

\[
V_{d,j} = \frac{T_{d,j}}{m_j \Omega_j} \cdot \frac{1}{N} \frac{\partial N}{\partial y} \quad \ldots \quad (15)
\]

5 Dispersion relation

To evaluate the dispersion relation of the kinetic Alfvén wave (KAW) in a dusty plasma we calculate the integrated perturbed density for non-resonant particles as

\[
\bar{n}_j = \int_0^\infty 2\pi V_{d,j} dV_{d,j} \int_0^\infty dV \ n_1(\mathbf{r},t) \quad \ldots \quad (16)
\]

where, \( n_j \) is the average density of the \( j \)th species \( (j = e, i \) and \( d \) for the electron, ion and dust, respectively). With the help of Eqs (5), (8) and (13) we find the average perturbed density \( \bar{n}_j \) for inhomogeneous dusty plasma as

\[
\bar{n}_j = \frac{\omega_{pe}^2}{4\pi e} \left[ -\frac{k_j^2}{\Omega_j^2} + \frac{k_j^2}{\omega_{de}^2} + \frac{V_{d,j}^2}{T_{d,j}(\omega - \omega_{de})} \right] P
\]

\[
\bar{n}_d = \frac{\omega_{pe}^2}{4\pi e} \frac{V_{d,e}^2}{\Omega_d^2}
\]

\[
\bar{n}_d = \frac{\omega_{pe}^2}{4\pi e} \frac{V_{d,e}^2}{\Omega_d^2} \left[ \frac{k_j^2}{\Omega_j^2} + \frac{k_j^2}{\omega_{de}^2} + \frac{V_{d,j}^2}{T_{d,j}(\omega - \omega_{de})} \right] Q
\]

where

\[
P = \left( 1 - \frac{k_j^2 P^2}{2} \right)
\]

\[
Q = \left( 1 - \frac{k_j^2 P^2}{2} \right)
\]

\( Z_d \) is the charge residing on the dust grain.

In order to obtain the relation between \( \Phi \) and \( \psi \), we apply the charge neutrality condition at \( x = 0 \)

\[
\bar{n}_i = \bar{n}_e + Z_d \bar{n}_d \quad \ldots \quad (18)
\]

and get the relation between \( \Phi \) and \( \psi \) as

\[
\Phi = \frac{\Omega_d^2}{k_j^2} \left[ \frac{\omega_{pe}^2}{\omega_{pe}^2 V_{d,e}^2 Q} - \frac{k_j^2}{\omega_{de}^2} \left( 1 + \frac{\omega_{de}^2 P}{\omega_{pe}^2 Q} \right) R^{-1} \psi \right]
\]

where,

\[
\omega_{pe,d}^2 = \frac{4\pi N_{e,d} e^2 q^2}{m_{e,d}}
\]

\[
R = \left[ \left( 1 - \frac{V_{d,e}^2}{\Omega_d^2} \right)^{1/2} + \frac{SP}{\Omega_d^2} \right] \left[ \left( 1 - \frac{V_{d,e}^2}{\Omega_d^2} \right)^{1/2} \right] \]

\[
S = \frac{N_0 m_d}{N_0 m_d Z_d}
\]
Using perturbed ion, electron and dust particle densities \(n_i, n_e, n_d\) and Ampere’s law in the parallel directions we obtain the equation

\[
\frac{\partial}{\partial z} \nabla_{\parallel}^2 (\phi - \psi) = \frac{4\pi}{c^2} \frac{\partial J_z}{\partial t} \quad \text{... (20)}
\]

where,

\[
J_z = q \int_0^{2\pi} V_d dV_e \int dV_i \left[ \{N(V_i)u_i(r,t) + V_i n_i(r,t)\}_d - \{N(V_i)u_i(r,t) + V_i n_i(r,t)\}_s \right] \quad \text{... (21)}
\]

With the help of Eqs (19), (20) and (21) we obtain the dispersion relation for the kinetic Alfven wave in inhomogeneous dusty plasma as

\[
(1 - \frac{(\omega - \omega_i e)^2}{c_d^2 k_{\parallel}^2 Q}) \left(1 - \frac{(\omega - \omega_d e)^2}{c_d^2 k_{\parallel}^2 S}\right) = \frac{k_{\parallel}^2 (\omega - \omega_i e)^2 R}{\Omega_d^2 k_{\parallel}^2 S} - \frac{\omega_d^2 \rho_0 (\omega - \omega_d e)}{\Omega_i c^2 k_{\parallel}^2} \frac{T_{\parallel i}}{m_i} \quad \text{... (22)}
\]

where, \(c_d = \omega_{pd} / \omega_p\) is the square of dust-acoustic speed and \(V_A^2 = c^2 \Omega_i^2 / \omega_{pd}\) is the square of dust Alfven speed.

The dispersion relation for kinetic Alfven waves reduces to that derived by Hasegawa53 under the approximation \(V_A = 0\), \(k_{\parallel} \rho_i = 0\) and \(k_{\parallel} \rho_d = 0\), i.e.

\[
\omega_{ie} = \omega_e (1 - \delta)
\]

\[
\delta = \frac{\rho_i^2}{2a}, \quad \omega_e = \frac{k_e E_0}{B} \quad \text{... (23)}
\]

6 Current density

To evaluate the perturbed current density per unit wavelength we use the following set of equations:

\[
J_x = \int_0^{2\pi} V_d dV_e \int dV_i \left[ N_i (V_i) u_i (r,t) + V_i n_i (r,t) \right] \left[ \{N_i (V_i) u_i (r,t) + V_i n_i (r,t)\}_d - \{N_i (V_i) u_i (r,t) + V_i n_i (r,t)\}_s \right] \quad \text{... (24)}
\]

and

\[
J = J_i - J_e - J_d
\]

With the help of Eqs (5),(8) and (13), we obtain

\[
J_x = \frac{k_e k_i^2 e^2 \lambda \psi_i}{8 \pi} \frac{\omega_{pe}^2}{\Omega_i^2} \left[ \{\psi_1 - \psi_i\} - \frac{(\omega - \omega_i e)}{2} \right] \quad \text{... (25)}
\]

and

\[
J_z = \frac{c k_e k_i^2 \lambda \psi_i}{8 \pi} \frac{\omega_{pe}^2}{\Omega_i^2} \left[ \{\psi_1 - \psi_i\} + \frac{(\omega - \omega_i e)}{2} \right] \quad \text{... (26)}
\]

where it has been assumed that the currents are driven by the kinetic Alfven wave and the contribution of the diamagnetic current has been ignored. These expressions [Eqs (25) and (26)] represent the average values of the current densities associated with the
kinetic Alfvén wave, which are the central features of auroral electrodynamics and can be related to current drive by kinetic Alfvén waves in tokamak plasmas also.

7 Energy balance and growth rate

We now define wave energy density per unit wavelength $W_w$ as the sum of pure field energy and the changes in energy of the non-resonance particles as

$$W_w = \frac{\lambda k_i^2 q_i^2}{8\pi} + W_i + W_e + W_d \quad \ldots \quad (27)$$

and

$$W_j = \left[ \int_0^\infty 2\pi \nu_j d\nu_j \int dV_j \frac{m_j}{2} [(N + n_i)(V + \nu)]^2 - \nu_j V_j^2 \right]_{\nu_j}$$

where

$$\frac{\lambda k_i^2 q_i^2}{8\pi} \equiv \left( \frac{T_{le}}{m_e} \right)$$

and

$$W_e = \frac{\lambda k_i^2 q_i^2}{16\pi} (\omega - \omega_{ie}) \left[ \frac{T_{le}}{m_e} \right]$$

After substituting the values from Eqs (8) and (10) with $\Delta = 0$ and with the help of Eq. (5), we get the energy associated with the ion, electron and dust components of the non-resonant particles as

$$W_i = \frac{\lambda k_i^2 q_i^2}{16\pi} \left( \omega_{pe}^2 \right) \left[ (1 - k_i^2 p_i^2) \right] \left[ 1 - \frac{4k_i V_i^2}{(\omega - \omega_{ie}) m_i} \right] \psi_i^2$$

$$W_e = \frac{\lambda k_i^2 q_i^2}{16\pi} \left( \omega_{pe}^2 \right) \left[ (1 - k_i^2 p_i^2) \right] \left[ 1 - \frac{4k_i V_e^2}{(\omega - \omega_{ie}) m_e} \right] \psi_i^2$$

where

$$k \cdot r = k_i r_i$$

Now we calculate the resonance energy $W_i$ of the electrons per unit wavelength, that is

$$W_i = \int_0^\infty 2\pi \nu_i d\nu_i \int dV_i \left[ \frac{1}{2} \left( N m_i u_i^2 + n_i m_i u_i V_i \right) \right] dV_i$$

with the help of Eqs (5), (8), (14) and (30). Expanding the integrand around $V_i = \omega/k$ and following the procedure as discussed by Tiwari and Varma and Terashima in the limiting case of $k_i p_i << 1$, we obtain

$$W_i = -\frac{\lambda k_i^2 q_i^2}{8\pi} \left[ \frac{1}{k_i^2} \frac{V_i}{k_i T_{le}} \right] = f_{le}^{\omega_e} \left( \frac{\omega_{pe}}{k_i^2} \right) f_{le}^{\omega_e} \left( \frac{\omega_{pe}}{k_i^2} \right)$$

where, $\omega_{pe} = \omega - \omega_{ie}$ and $f_{le}$ represents the first derivatives of $f_{le}$ with respect to its argument. This expression can be utilized for electron acceleration process by kinetic Alfvén waves in the auroral acceleration region.

Using the law of conservation of energy, we calculate the growth rate of the drift kinetic Alfvén wave by

$$\frac{d}{dt} (W_w + W_i) = 0 \quad \ldots \quad (32)$$

Substituting the expressions for the wave and particles energies, we obtain the growth rate as,

$$\gamma = \frac{1}{\varphi_i} \frac{d\varphi_i}{dt} = \frac{\omega_{pe}^2}{\left( \frac{T_{le}}{m_e} \right)} \left[ \frac{V_i}{\omega_{pe}} \right]$$

or

$$\gamma = \frac{1}{\omega_{pe}^2} \left( \frac{T_{le}}{m_e} \right) \left( \frac{T_{le}}{\omega_{pe}^2} - 1 \right) \exp \left( \frac{-\omega_{pe}^2}{\left( \frac{T_{le}}{m_e} \right)} \right) \quad \ldots \quad (33)$$

8 Results and discussion

In the numerical evaluation of the growth rate, current and dispersion relation for the kinetic Alfvén waves in a dusty magnetized plasma, we have used the following dusty plasma parameters which may be suitable for auroral acceleration region.

$$B_0 = 4300 \text{ nT}; \quad \Omega_i = 412 \text{ s}^{-1}; \quad \Omega_i = \frac{Z_a e B_0}{m_e c} = 6.88 \times 10^{-16} Z_a$$

$$N_0 = 10 \text{ cm}^{-3}; \quad m_d = 10^{-12} \text{ g}; \quad kT_{le} = 100 \text{ eV}; \quad kT_{le} = 10 \text{ keV}; \quad E_d = 100 \text{ mV m}^{-1}; \quad E_d = 200 \text{ mV m}^{-1}.$$

The results are presented in Figs 1-8.

Figure 1 shows the relation between wave frequency ($\omega$) rad sec$^{-1}$ versus $k_i$ at the fixed values of $Z_{ad}, N_{ad}$ and $m_d$ for different values of electric field.
The effect of electric field on the frequency $\omega$ is seen in the form of Doppler shift. The negative values of $\omega$ may be represented by a negative phase velocity wave travelling in the reverse direction. Thus, the electric field contributes to the frequency of reflected wave also. It is also observed that wave frequency is smaller in dusty plasmas as compared to the waves supported by ions and electrons only. The higher charge and higher mass particles support lower frequency waves. The electric field enhances the frequency and phase velocity of wave travelling towards the ionosphere and reflected by the ionosphere by ionospheric conductivity. The acceleration of charged particles along the magnetic field is predicted as the energy is transferred to the particles by Landau damping process. The formation of discrete aurora may require longer time in dusty plasmas due to reduced phase velocity of the wave since the onset of the substorm.

Figure 2 shows the variation of normalized growth rate $(\gamma/\omega)$ with $k_\perp$ for different values of electric field $E_0$ at fixed $Z_\infty$, $N_\infty$, and $m_\infty$. It is seen that the wave is excited by the plasma density inhomogeneity. The electric field may influence the kinetic Alfvén wave in a narrow frequency band. A particular perpendicular wave number $k_\perp$ is suitable to generate the wave as depicted in Fig. 2. The higher wave number $k_\perp$ weakens the wave electric field by averaging over Larmor orbit and may not be suitable for exciting the wave. Thus, it is predicted that the inhomogeneous electric fields of plasma-sheet region and auroral acceleration region may generate the kinetic Alfvén waves which have been observed as broadband extremely low frequency emissions by Freja satellites and interpreted as kinetic Alfvén wave. The dust particles may be responsible for
supporting the lower frequency emissions. The kinetic Alfven wave observed on Freja satellite and the high latitude electrostatic shock observed on S3-3 satellite are reported to be of the common origin and are likely to be closely related entities. The electrostatic shocks resemble the inhomogeneous electric field and may generate kinetic Alfven wave as observed on Freja satellite.

Figures 3 and 4 show the relation between perpendicular current $J_x$ and parallel current $J_z$ versus $k_\perp$ at different values of electric field at the fixed values of $Z_d$, $N_0$, and $m_d$. Here, it can be seen that the
perpendicular current is increased due to the decreasing value of electric field at higher values of \( k_1 \). The perpendicular current \( J_x \) is more affected by the applied electric field than the field-aligned current \( J_z \). The decreasing values of \( J_z \) with the increase of \( k_1 \) is seen. The magnitude of the field-aligned current depends upon the \( k_1 \). The enhancement of \( J_x \) and reduction in \( J_z \) with respect to \( k_1 \) are due to the maintenance of current continuity condition. The minor effects of electric field on perpendicular and parallel currents are observed in Figs 3 and 4. Thus, the currents driven by kinetic Alfvén wave are little affected by the presence of inhomogeneous electric fields.

Figure 5 shows the variation of the wave frequency \( \omega \) with \( k_1 \) at different values of \( \delta = \frac{\rho_1^2}{2\alpha^2} \) at fixed \( Z_0, N_0, \text{ and } m_d \). It is seen that the wave frequency decreases with the increasing degree of inhomogeneity. At \( \delta = 1 \), an electric field of any magnitude has no effect on the wave frequency. It is noticed from Fig. 5 that electric field is effective through its inhomogeneity in the determination of wave frequency. Thus, the frequency of kinetic Alfvén wave generated in the plasma-sheet region by density inhomogeneity and travelling towards the ionosphere is also determined by electric field inhomogeneity of the region.

Figure 6 shows the relation between \( \gamma/\omega \) with \( k_1 \) at different \( \delta = \frac{\rho_1^2}{2\alpha^2} \) and fixed values of \( Z_0, N_0, \text{ and } m_d \). It is found that the electric field inhomogeneity contributes to the wave growth. At the higher values of \( k_1 \) the wave growth is decreased. The wave is generated due to inverse Landau damping process. It is observed that electric field inhomogeneity is also a source of free energy to excite kinetic Alfvén wave at the particular wave number. However, the kinetic Alfvén wave is generated by density inhomogeneity of the plasma-sheet region and electric field inhomogeneity merely contributes to wave growth.

![Fig. 6 — Growth rate (\( \gamma/\omega \)) versus perpendicular wave number (\( k_1 \)) for different \( \delta \)](image)

![Fig. 7 — \( J_z \) versus perpendicular wave number (\( k_1 \)) for different \( \delta \)](image)
Figures 7 and 8 show the variation of perpendicular current $J_x$ and parallel current $J_z$ with different values of $\delta$ and for fixed values of $Z_0$, $N_{do}$, and $m_d$. It is observed that $J_x$ is increased and $J_z$ is decreased due to the inhomogeneity of the electric field, which may be the effect of weakening of the electric field due to finite Larmor radius effects. The electric field inhomogeneity has effect on perpendicular and parallel currents driven by kinetic Alfven wave in the dusty plasma and major effects on wave frequency and growth rate are noted.

9 Summary and conclusion

Here, it may concluded that field-aligned and perpendicular currents associated with kinetic Alfven waves in the magnetosphere-ionosphere coupling system depend upon perpendicular wavelength and ion gyroradius effect\textsuperscript{62,63}. The wave may be generated by the pressure gradient on magnetic flux tube in the distant magnetosphere and propagates towards the ionosphere constituting field-aligned and closure currents. The field-aligned currents may reverse its directions while propagating towards the ionosphere\textsuperscript{62,63} as the magnetic flux tube converges. The frequency of the wave is determined by the perpendicular wave number and the ion gyroradius in the presence of electric field inhomogeneity. The wave characteristics are controlled by perpendicular electric field in the system. From this study, we may conclude that the plasma density inhomogeneity alone is the fundamental source of free energy to excite kinetic Alfven waves and the electric field inhomogeneity merely enhances the growth of pre-existing waves.

This paper has been devoted to the current central problem of cosmic plasma physics with Dawson's approach to Landau damping, namely, the influence of kinetic Alfven wave or particle motion in the auroral zone of the dusty magnetosphere. Charged particle trajectories can be used to obtain a detailed understanding of the ion pickup process, which has been extensively studied in the literature by means of both quasi-linear theory and numerical simulations. The theory and results of the investigations are applicable to the magnetosphere-ionosphere coupling, where the field-aligned current and auroral acceleration are explained in terms of kinetic Alfven waves\textsuperscript{62,63} with dusty environments.

Acknowledgements

One of the authors (RPS) is thankful to the Department of Science and Technology, New Delhi, and the other (PV) to ISRO, Bangalore, for financial assistance.

References

33 Zhao G H & Kelley M C, J Atmos & Terr Phys (UK), 59 (1997) 239.
42 Frank L A & Garnett D A, J Geophys Res (USA), 76 (1971) 5202.
43 Carlson C W & Kelley M C, J Geophys Res (USA), 82 (1977) 2349.
52 Hasegawa A & Chen L, Phys Fluids (USA), 19 (1976) 1924.
68 Terashima Y, Prog Theor Phys (Japan), 37 (1967) 775.