Effect of air gap on the performance of circular sector antennas

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Theoretical analysis of a circular sector microstrip antenna having sector angle (α) and an air gap (h_a) between substrate material and ground plane is carried out by applying cavity-model-based modal expansion technique. Expressions for different antenna parameters are derived in TM_{i3} mode of excitation and the theoretical results are compared with simulated results. Average differences in computed and simulated resonance frequency, input impedance, directivity and bandwidth values for the sector geometry with air gap are found to be 1.3%, 10.25%, 0.75% and 1.9%, respectively. This validates the accuracy in proposed analysis technique for the present geometry.

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1 Introduction
The major limitation of a microstrip antenna is its narrow bandwidth. The typical bandwidths of patch antennas are roughly a few percent, which limits the antenna application in different fields. Many efforts have been made to increase the bandwidth. Since the bandwidth is determined by the patch physical size, one way to increase bandwidth is to increase antenna size. However, increasing the patch width is possible as long as it remains less than a wavelength in the dielectric. Beyond a wavelength width, higher order modes may be excited that creates patterns distortions and unpredictable impedance behaviour. An easier way to increase the patch bandwidth is the application of thick dielectric substrate, but the maximum recommended height of substrate may be h/\lambda_0 = 0.0324, where the surface waves are about 10% of the space waves. However, at the cost of lower efficiencies, even thicker substrates may be used for further higher bandwidths. Another way of increasing the bandwidth of patch antenna is by placing additional elements alongside the patch.

In addition to bandwidth enhancement, a substantial amount of efforts is devoted to increase the frequency agility of patch antennas. Application of varactor diode, tuning of antenna using shorting posts or pins and optically tuned patch antennas are some of the reported efforts. However, these methods suffer certain disadvantages including the following:
(a) The design of the patches gets complicated by adding components such as varactor diode, optically controlled pin diode and their associated biasing circuit.
(b) At higher frequency, the patch size becomes small that makes accommodation of diodes and shorting posts underneath each patch difficult.
(c) The added complication in designing gets multiplied for an array consisting of a large number of elements.

Considering these difficulties, Lee et al. and Guha introduced air gaps in microstrip antenna geometries. Introduction of air gap alters the effective permittivity of the microstrip geometry that, in turn, tunes the resonance frequency of the antenna.

Different workers have extensively analyzed circular patches of microstrip antennas under different conditions. However, geometrical constraints become vital when these radiators are needed for conformal mounting on pre-existing structures. In that case, utilization of other shapes, which are not circular becomes more advantageous. Annular ring, gap open
ring and ring slot antennas are some of those geometries, which are investigated in recent times due to above difficulties. In the present paper, mathematical modelling of yet another geometry, i.e. a circular sector microstrip antenna with and without air gap is carried out by applying modified cavity-model based modal expansion technique and the obtained results are compared with simulated results.

2 Theoretical formulation

A circular sector microstrip antenna (CSMA) with coordinate system is shown in Fig. 1. It consists of a planar circular sector with sector angle \( \alpha \) and radius \( a \) on a thin dielectric substrate having relative permittivity \( \varepsilon_r \) and loss tangent \( \tan \delta \). The total antenna thickness, \( h \), is considered as the sum of substrate thickness \( h_d \) and the thickness of air gap \( (h_a) \) between dielectric substrate and an infinitely large ground plane, i.e. \( h = h_d + h_a \). The z-axis is perpendicular to the plane of the patch. The region between the patch and ground plane is treated as a cavity bounded by magnetic wall along the edge of the patch and electric walls above and below. The electric field has only the \( z \) component and the magnetic field has only the transverse component in the region bounded by the conducting patch and the ground plane. The circular sector microstrip antenna is modelled as two-layered cavity instead of single layer cavity with upper layer as the dielectric substrate of thickness \( h_d \) with relative permittivity, \( \varepsilon_r \), and lower layer as an air gap of thickness \( h_a \) with relative permittivity equal to one. As a result, modified assumptions of Lee et al. are applied here for the analysis.

The computation of resonance frequency of antenna with air gap is carried out in TM\(_{\alpha \alpha} \) mode of excitation by considering arbitrary sector angle \( \alpha \). The effect of air gap below the substrate is accounted by considering an equivalent dielectric constant of medium below the patch and given by Abboud et al.\(^{10} \)

\[
\varepsilon_{eq} = \frac{\varepsilon_r (h_d + h_a)}{(h_d + h_a) \varepsilon_r} \quad \cdots (1)
\]

The resonance frequency of antenna in TM\(_{\alpha \alpha} \) mode is calculated analytically by applying the relation\(^{5} \)

\[
f = \frac{\left( \varepsilon_{eq} a \right) c}{2 \pi \alpha \sqrt{\varepsilon_r}} \quad \cdots (2)
\]

Here \( a_e \) is the effective radius of the patch\(^2 \) and taken as,

\[
a_e = a + \left( \frac{2 h_a}{\pi \varepsilon_r} \right) \left[ \tan^{-1} \left( \frac{a}{2 h_a} \right) + \left( \frac{0.41 \varepsilon_r + 1.77}{a} \right) + \left( 0.268 \varepsilon_r + 1.65 \right) \right] \quad \cdots (3)
\]

\( K_{vm} \) is equivalent wave number and \( \nu \) is the mode number given by

\[
\nu = \frac{\pi n}{\alpha} \quad \cdots (4)
\]

The dynamic dielectric constant \( \varepsilon_d \) used above is calculated following Wolff and Knoppik with a change that in the expressions of \( C_{\alpha \alpha \alpha} \) and \( C_{\alpha \alpha \alpha} \), area of patch \( (\pi a^2) \) is replaced by \( \frac{a d^2}{2} \) and \( C_{\alpha \alpha \alpha} \) is taken as

\[
C_{\alpha \alpha \alpha} = \frac{1}{\alpha_0} \int C_{\alpha} \cos(\nu p)^2 dp \quad \cdots (5)
\]

In this way, the resonance frequency of antenna becomes a function of sector angle. The parameter \( \varepsilon_d \) is already a function of \( \alpha \) in addition to patch radius and substrate thickness. The resonance frequencies of different antenna geometries with air gap \( h_a \) are calculated. The validation of computed results is carried out with EM simulator IE3D (version 10.1). Both computed and simulated resonance frequencies are shown in Table I.

The modelling of feed location is done by considering that the patch is excited in such a way that the input filamentary current at feed location \( (d, \phi) \) is...
Table 1—Comparison of calculated and simulated resonance frequencies with air gap $h_a$

<table>
<thead>
<tr>
<th>Sector angle ($\alpha$)</th>
<th>Resonance frequency (GHz) when $h_a = 0$ mm</th>
<th>Resonance frequency (GHz) when $h_a = 0.5$ mm</th>
<th>Resonance frequency (GHz) when $h_a = 1$ mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>60°</td>
<td>3.958</td>
<td>3.94</td>
<td>4.408</td>
</tr>
<tr>
<td>90°</td>
<td>2.90</td>
<td>2.95</td>
<td>3.254</td>
</tr>
<tr>
<td>180°</td>
<td>1.769</td>
<td>1.76</td>
<td>2.033</td>
</tr>
</tbody>
</table>

$J = J_s(\phi') \frac{\delta(\rho - d)}{d} \hat{z}$ ... (6)

$J_s(\phi') = J_s(\phi' - w < \phi' < \phi' + w) = 0, \text{ elsewhere}$ ... (7)

Here $2w$ is the width of current ribbon considered on the patch centred on the feed axis at a distance $d$ from the centre of the patch.

The resonant mode eigen function in dielectric medium is taken as

$$\left(\psi_{vn}\right)_d = J_s(K_{vn} \rho) \cos(\nu \phi') \cos[\beta_{vn}(h-z)]$$ ... (8)

Here, $\beta_{vn} = \sqrt{K_d^2 - K_{vn}^2}$

while in air gap,

$$\left(\psi_{vn}\right)_a = J_s(K_{vn} \rho) \cos(\nu \phi') \cos[\beta_{avn}(z)]$$ ... (9)

with $\beta_{avn} = \sqrt{K_a^2 - K_{vn}^2}$

Assuming eigen functions to be orthogonal, the solutions of wave equation for $E_z$ for dielectric substrate region and air gap region of antenna are obtained following Lee et al.

For dielectric region

$$E_{zd} = E_0 J_s(K_{vn} \rho) \cos(\nu \phi') \cos[\beta_{vn}(h-z)]$$ ... (10)

with

$$E_{za} = E_0 \frac{\sin(\nu \rho w)}{\sin(\nu \beta_{avn} h_a)} J_s(K_{vn} d)$$ ... (11)

while for air gap region

$$E_{za} = E_0 J_s(K_{vn} \rho) \cos(\nu \phi') \cos[\beta_{vn}(z)]$$ ... (12)

with

$$E_{za} = E_0 \frac{\sin(\nu \rho w)}{\sin(\nu \beta_{avn} h_a)} J_s(K_{vn} d)$$ ... (13)

Here $J_s(K_{vn} \rho)$ is the cylindrical function of first kind of Bessel’s function of order $v$. The resonance occurs when $K = K_{vn} = K_{vn}/\alpha$

By applying equivalence principle and image theory, the components of equivalent magnetic current sources $M$, i.e. $M_{o}$ which represents outer $M$ source associated with curved edges and equivalent source associated with linear edges ($M_{p1}$ and $M_{p2}$), are calculated for dielectric substrate and for air gap.
separately. These components of M are used to compute the far-field radiation patterns in E (φ = 0°) plane and H (φ = 90°) plane.

The radiated power by the antenna in free space is obtained by evaluating complex poynting vector which, in turn, is applied to obtain dielectric and conductor losses inside the cavity. In the expression of quality factor of a single layer microstrip antenna, the contribution of energy stored in air is also added. Following Lee et al., at resonance, the total energy stored in the cavity under the modified condition will be

\[
W_c = W_d + W_f = \left[ \frac{E_0 E_f}{2} \int_{a}^{r} \int_{0}^{\alpha} |E|^2 \, d\phi \, d\zeta \right] \quad \ldots \quad (14)
\]

The feed point (d, φ') on antenna geometry is obtained so that it may be fed with a 50 Ω line. The expression for input impedance of antenna is derived following Garg et al.

\[
Z_{in} = j \omega \mu \sum \left( \frac{K^2}{(1-j\beta_{eff})-K_{m}^{-2}} \right) \int_{0}^{\alpha} \left[ \alpha^2 - \frac{\alpha^2}{K_{m}^{-1}} \right] \int_{0}^{2\pi} \frac{\sin(2\alpha\phi)}{2\pi} \, d\alpha
\]

A proper feed point D along arm OA (shown in Fig. 1) is obtained by computing above expression.

3 Results and discussion

The resonance frequency of antennas excited in TM_{11} mode, increases on increasing air gap thickness as shown in Table 1. The composite dielectric between the patch and the ground plane due to presence of air gap will support the surface waves. For three values of h ( = h, +h), considered in this paper, the ratio h/λ₀ varies from 0.021 to 0.04. With h₀ = 0 and h₀ = 0.05 mm, the calculated values of h/λ₀ is less than the recommended value (h/λ₀ = 0.0324). Therefore, the contribution of surface waves will be less than 10% of space waves and hence may be ignored. However, on increasing air gap further (h₀ = 1 mm), the contribution of surface waves will increase. Better radiation performance of antenna is expected till the air gap thickness is less than 0.95 mm.

The far-field E plane (φ = 0°) and H plane (φ = 90°) radiation patterns of circular slot microstrip antenna with air gap thickness h₀ = 1.0 mm are compared with the simulated results in Figs. 2 and 3. It may be noted that in both E (φ = 0°) and H (φ = 90°) planes, computed and simulated patterns are nearly identical in shape. The comparison between computed relative radiation intensities in E plane and H plane are shown in Figs. 4 and 5, respectively. These Figs 4 and 5 indicate that on increasing air gap thickness, the relative radiation intensity of antenna increases.

Figure 6 shows the computed directivity for different air gaps as a function of resonance frequency. The curves indicate that in the presence of air gap, sufficiently large directivity may be obtained for antenna operating in lower range of frequencies. The computed results for directivity of antenna at resonance frequency are compared in Table 2 for TM_{11} mode of excitation. It can be observed that the

![Fig. 2 - E (φ = 0°) plane patterns of antenna with α = 60° and h₀ = 1 mm](image)

![Fig. 3 - H (φ = 90°) plane patterns of antenna with α = 60° and h₀ = 1 mm](image)
directivity of each antenna geometry with air gap is higher than the antenna geometry without air gap ($h_a=0$).

Figure 7 shows the variation of total quality factor ($Q_t$) of antenna geometry as a function of resonance frequency with different air gap thickness. It can be seen that irrespective of resonance frequency of antenna, its $Q$ factor decreases on increasing air gap thickness. The variation of quality factor of different CSMA geometries as a function of air gap thickness ($h_a$) is shown in Fig. 8. It indicates that among the considered geometries, antenna with sector angle 60°

<table>
<thead>
<tr>
<th>Sector Angle ($\alpha$)</th>
<th>Directivity (dB) when $h_a=0$</th>
<th>Directivity (dB) when $h_a=0.5$ mm</th>
<th>Directivity (dB) when $h_a=1$ mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>60°</td>
<td>7.17</td>
<td>7.842</td>
<td>8.232</td>
</tr>
<tr>
<td>90°</td>
<td>7.084</td>
<td>7.81</td>
<td>8.188</td>
</tr>
<tr>
<td>180°</td>
<td>6.98</td>
<td>7.708</td>
<td>8.109</td>
</tr>
</tbody>
</table>

Fig. 6—Variation of directivity with frequency for antenna geometry ($\alpha = 60°$)
is the most efficient radiator even in the presence of air gap since it has lowest $Q$ factor.

The bandwidth values of CSMA elements shown in Fig. 9 for [$\text{VSWR} < 2$], indicates that with large air gap, higher bandwidth antenna may be obtained in comparison to the case where narrow gap or no gap is applied. For the antenna geometry considered in this paper, the rise in bandwidth on applying air gap thickness $h_a = 0.5$ mm is about 35% of that with no air gap. This bandwidth increases further to 54% when 1 mm air gap is applied. With small size antennas, presence of air gap provides much higher bandwidth.

The computed and simulated input impedances of the considered geometry as a function of antenna frequency are shown in Fig. 10((a) and (b)) for two air-gap thicknesses. These variations are obtained in the range between 4 GHz and 4.6 GHz for both the cases. The difference in computed and simulated values of input impedances at resonance frequencies are shown in Table 3. In Table 4, the variation of feed point of antenna as a function of air gap thickness is shown. All the parameters except feed location are kept uniform for each antenna while calculating the feed point. On increasing air gap, the feed point shifts away from the origin of the patch.

The return losses of CSMA geometry ($\alpha = 60^\circ$) having air gap are also calculated theoretically near the resonance frequency and are compared with the simulation results in Fig. 11. The curves in Fig. 11 indicate a small difference in the resonance frequency of desired antenna. A difference of 1.14% in the computed and simulated resonance frequencies is obtained from these figures. An agreement between the computed and simulated input impedance and return loss results indicates that proposed model is sufficiently capable in designing CSMA geometries with an air gap.
In this study, the presented formulism is tested for three antenna geometries and a good agreement is found between the computed and simulated results. The present results indicate that this formulism may also be applied with larger accuracy for designing similar geometries having different slot angles.

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