Study of wave-particle interaction in the disturbed magnetosphere

Devendraa Singh
Indian Institute of Tropical Meteorology, Pune 411 008, India

and

Shubha Singh & R P Singh
Department of Physics, Banaras Hindu University, Varanasi 221 005, India

Received 11 October 2004; revised 23 February 2005; accepted 14 July 2005

Doppler-shifted cyclotron resonance interaction between whistler mode wave and counter streaming energetic electrons has been invoked to explain whistler triggered emissions recorded at low latitude station Varanasi (Geomagnetic latitude 14°55', L = 1.07) during moderate magnetic storm activity ($\Sigma Kp = 28$, $Kp$ index varies from 4 to 4' during the observation period) on 28 Feb. 1993. The mechanism of generation of triggered emissions is briefly discussed. Parallel resonance energy of participating electrons under normal and disturbed magnetospheric conditions have been evaluated, which is found to decrease with increase in L-value and wave frequency. Applying a simplified approach we have estimated the interaction length, wave magnetic field and transverse resonant current, which are found to increase during the disturbed magnetospheric conditions. However, the number of energetic electrons participating in resonance process under normal and disturbed magnetosphere remains approximately the same.

Keywords: ELF/VLF wave, Wave-particle interaction, Magnetosphere, Whistler mode, Whistler triggered emission, Resonance energy

PACS No: 94.20Rz; 94.20 Yx; 94.30Tz;

1 Introduction

The VLF (very low frequency) emissions generated in the Earth's magnetosphere and propagating in the whistler mode, are often received by the ground-based equipments. Their generation and propagation mechanism have been reviewed in the near past by a number of workers. Out of the many types of VLF emissions, triggered emissions exhibit a bewildering variety of dynamic spectral form (Singh et al., with references therein) and follow the apparent source, which could be a whistler or discrete emissions and signal from VLF transmitters. Sometimes triggering signals may not accompany the triggered emissions. Discrete emissions can be triggered by whistlers, VLF transmitters and power line radiation from the world's power grids. The observations in general support that strong VLF emissions may be triggered by very weak signals; sometimes trigger source may even be invisible. Sonwalker and Inan analyzing data from DE-1 satellite showed that lightning generated whistler often trigger hiss emissions. Their results indicated that the lightning generated whistlers may be an important embryonic source for hiss in the magnetosphere, which constitute dominant wave activity in the ~1.5-6 kHz range on L shells of 3.5-5 in the afternoon sector during geomagnetic quiet period.

A number of theories based on non-linear wave-particle interaction have been suggested to explain the triggering phenomena. The widely accepted mechanism is non-linear wave-particle interaction between finite amplitude wave train and particles that happen to be in resonance with it. The non-linear interaction produces a phase bunching of the resonant and nearly resonant electrons, thus giving rise to a current, which acts like an antenna. The de-trapped particles at the termination of triggering wave are phase organized and act coherently for a while, giving rise to an emission with either falling or rising tone, depending upon the sign of the inhomogeneity variation in the medium. Further, the spatially averaged part of the distribution function of these suddenly de-trapped electrons generates an instability, which amplifies the emitted waves. Helliswell and Inan have discussed a feedback model in which interaction region centered on the magnetic equator is treated like an unstable feedback amplifier with a delay time. Molvig et al. have developed self-consistent theory of triggered whistler emissions using kinetic approach, which is capable of predicting observed emissions and their frequency-time variations.
characteristics when several trapping periods occur
within the interaction region.

In this paper, we study whistler-triggered emissions
observed at low latitude Indian station Varanasi
during moderately disturbed magnetosphere.
Triggered emissions during disturbed magnetosphere,
and their probable generation mechanisms are briefly
discussed. We have numerically evaluated the
energies of resonating electrons and interaction
parameters for normal and disturbed magnetosphere.
The variation in these parameters with \( L \)-values and
wave frequencies are numerically studied.

2 Triggered emissions during disturbed
magnetosphere

The experimental set-up for recording ELF/VLF
waves at low latitude ground station Varanasi consists
of T-type antenna, pre/main amplifier, and magnetic
tape/cassette recorder. The data are stored in analogue
form on magnetic tapes. The stored data are analyzed
by the Advanced VLF Data Analysis System
(AVDAS), which uses digital processing technique
and is capable of real time spectrum display. Typical
dynamic spectra of whistler triggered-emissions
observed on 28 Feb. 1993 are shown in Fig. 1: (a)
riser emission, (b) hiss, (c) oscillating tones, and (d)
riser followed by oscillating tones. The lower and
upper cut-off frequencies of the triggering whistler
along with its dispersion and derived \( L \)-values are
given in Table 1. In the same table we have also given
the lower and upper cut-off frequencies of triggered
emissions. The relative intensity and \( df/dt \) of triggered
emissions for all the four cases have been studied. Table 1 shows that the triggering whistlers have
propagated along the geomagnetic field line having \( L \)-values between 1.9 and 2.4. Thus, the triggered
emissions observed at Varanasi may have been
generated in the equatorial region of geomagnetic
field lines of higher \( L \)-values and propagated along
geomagnetic field lines either in ducted mode or pro-
longitudinal mode in the magnetosphere. After exiting
from the ionosphere they may have excited the Earth-
ionosphere wave guide, propagated towards the
equator and received at Varanasi. It may be noted that
we recorded\(^{17,18}\) a large number of whistlers on 8-9
Mar. 1991 (\( K_p \) index varied from 3\(^+\) to 4\(^-\)) and 17 Feb.
1997 (\( K_p \) index varied from 0\(^+\) to 1\(^-\)), which have
followed the \( L \)-values lying between 2.1 and 2.7.

The above events\(^{19}\) were recorded during weak
magnetic storm activity on 28 Feb. 1993 from 1413
hrs UT to 1418 hrs UT. The magnetic storm signature
is evidenced from hourly \( D_S \) variation plotted in
Fig. 2 for the period from 27 Feb. to 1 Mar., 1993. In
the same figure, we have plotted \( K_p \) index, which
varied between 4\(^-\) and 4\(^+\) during the event. The
emission time lies during the main phase of

![Fig. 1—A typical dynamic spectrum of whistler-triggered emission observed at low latitude ground station, Varanasi (Geomagnetic
Latitude 14°55', \( L = 1.07 \)) during the magnetic storm activity (\( \Sigma K_p = 28 \), \( K_p \) index varied from 4\(^-\) to 4\(^+\)) on 28 Feb. 1993. (a) riser
emission at 1413 hrs UT, (b) hiss like structure at 1416 hrs UT, (c) oscillating tone at 1416 hrs UT, and (d) riser followed by oscillating
tone at 1418 hrs UT](image-url)
geomagnetic storm. Thus, these events correspond directly to storm time. During the magnetic storm activity period, the conditions in the interaction region are drastically changed, which may lead to a suitable condition for the generation of triggered emissions. Singh et al. have reported hiss-triggered chorus emissions during the storm activity period of 7-9 Mar. 1986 ($\Sigma K_p = 34$) at low latitude station Gualmarg ($L = 1.2$). Based on statistical analysis of whistler-triggered VLF emissions recorded at Moshiri ($L = 1.6$), Hayakawa showed a good correlation between $K_p$ value and whistler activity. He showed that probability of occurrence of whistler-triggered emissions is rather small when $K_p$ value is less than 2, and the occurrence probability increases with increasing $K_p$ value, showing good correlation with geomagnetic activity when $K_p$ value lies between 3 and 7.

The injected plasma-sheet electrons into the inner magnetosphere during sub-storms influence the mid-latitude VLF emissions and hence it is possible to investigate the wave-particle interaction process, injection and drift process of particles, magnetospheric plasma structure, etc. during substorms. The intensity enhancement and frequency drift in dawn sector during magnetic disturbances could be interpreted in terms of combined effect of velocity dispersion during eastward longitudinal drift of energetic electrons injected near the mid-night sector and a quasi-linear electron cyclotron generation of VLF waves. Tsurutani et al. examined dependence of inner zone hiss activity on the level of geomagnetic activity using AE (to identify sub-storm) and $D_s$ (to identify storms) indices and reported that 92% of the hiss events occurred during active intervals containing a
sub-storm (AE > 100\gamma), a magnetic storm or in most cases both. As much as 55% of the events occurred during intense magnetic storms with peak $|D_s| > 45\gamma$. Most of the storm time events occurred during the recovery phase as had been earlier reported by Smith et al.\textsuperscript{26}. During low geomagnetic activity period hiss intensity was $10^2$ orders of magnitude lower than the average intensity during disturbed period. Analyzing Moshiiri data, Hayakawa et al.\textsuperscript{27} showed that the occurrence rate of ELF hiss abruptly increases when $K_p > 4$. There is a broad maximum in the occurrence rate for the $K_p$ range from 5 to 7. Assuming that ELF hiss are caused by the trapped electron fluxes, it is interesting to study variation of parameters related with generation and propagation of VLF emissions with $K_p$ values.

3 Generation mechanism

Triggered emissions are supposed to be generated during non-linear interaction between energetic electrons and narrow band whistler mode waves in the Earth's magnetosphere. Because of inhomogeneity in the geomagnetic field, resonance condition breaks as the interaction zone moves away from the equator and the second order resonance effect is proposed\textsuperscript{11,12}, which is useful in determining the frequency spectrum of emissions\textsuperscript{28}. The role of spatial inhomogeneity of the magnetic field in the generation mechanism and evolution of wave amplitude remains to be analyzed. Attempts have been made to solve this problem using different numerical models such as sheet current model\textsuperscript{15}, non-linear resonant current model\textsuperscript{29}, electromagnetic full particle model\textsuperscript{30}, fluid particle (hybrid) model\textsuperscript{31}, Vlasov hybrid model\textsuperscript{32}. These models have been developed to simulate wave-particle interaction and explain different types of triggered emissions. No single theory could explain all the observed features of triggered emissions.

In the wave-particle interaction, the exchange of energy between energetic electrons and waves maximizes when the interacting particles and waves are in resonance, i.e. gyrating charged particles and rotating wave-electric fields are in phase. The resonance velocity of the particle parallel to the geomagnetic field direction is obtained as

$$V_{r_{||}} = \frac{\omega^2 (\omega_p^2 - \omega^2)^{1/2} \{ c \omega_p \pm (c/\omega_p^2) \} \omega^2 \omega_p^2}{\omega_p^4 \omega^2 + \omega^4 \omega_p^2}$$

where $\omega$ is wave frequency, $\omega_p$ the electron gyrofrequency, $\omega_f$ the plasma frequency, and $c$ the velocity of light in free space. For whistler mode propagation $\omega < \omega_f$, the resonant interacting electrons move in the opposite direction to that of wave propagation. Hence, out of the two solutions of Eq. (1), only one solution is valid for resonance interaction. The parallel component of energy of resonant electron [$W_{||} = mv_{||}^2/2$] as a function of frequency and $L$-value is computed and shown in Fig. 3 for the normal and disturbed geomagnetic conditions. The equatorial electron density under these conditions is taken as 1200, 550, 400, 80 cm$^{-3}$ and 315, 25, 6, 1 cm$^{-3}$ for $L = 1.9$, 3, 4, and 5, respectively\textsuperscript{33}. The latter values are for the disturbed conditions. Figure 3 shows that the resonance electron energy decreases with increase in wave frequency and $L$-value.

Further, the decrease in the equatorial electron density (disturbed condition) causes an increase in resonant electron energy for the same conditions of wave frequency and $L$-value by an order of magnitude. However, the change is less perceptible at lower $L$-values (inside the plasmasphere, $L < 3$) than the outside plasmasphere ($L > 4$). Rycroft et al.\textsuperscript{34} have shown that energetic electrons ($W_{||} \sim 0.5$ keV, and

![Fig. 3](image-url) Variations of computed parallel resonance energy of electrons with the wave frequency for the normal and disturbed conditions of the magnetosphere (N for normal condition, D for Distributed condition) at $L = 1.9$, $L = 3$, $L = 4$, $L = 5$. 
temperature anisotropy ~1.0) resonating with the upper cut-off frequency of nose whistler can trigger VLF emissions. This mechanism has also been invoked to explain electron precipitation associated with discrete riser type of emissions. Singh and Singh have studied the effect of inhomogeneity on the resonance energy of interacting electrons for suitable VLF emissions. Molvig et al. have estimated minimum energy required for emission process in the presence of inhomogeneity. For the lower wave frequency (5-10 kHz) and L = 1.9, the required electron energy is ~200 keV. In the present case, we have not considered relativistic effect. If we take into account relativistic effect, then the required electron energy decreases. This decrease is identical both for normal and disturbed conditions of the magnetosphere. Thus the reported electron energy is on the higher side.

The energetic electrons initially uniformly distributed in phase space, during resonance interaction are phase bunched by the \( qV \times B \) force, where \( q \) is the electronic charge and \( B \) is wave-magnetic field of the interacting wave. The transverse current caused by phase-bunched electrons is written as

\[
J(z) = q \int_0^1 \int_0^{2\pi} \nu(z) f(\nu, \alpha) \nu^2 \sin \alpha \, d\alpha \, d\nu \, d\phi \n\]

where \( \alpha \) is pitch angle, and volume integral is computed over the initial values of the parameters \( \phi \), \( \alpha \) and \( \nu \) (or \( \nu_l \)). The term \( \nu(z) \) represents the perpendicular velocity of each particle and is derived from total velocity \( \nu(z) \), which is given by

\[
\nu(z) = \int_0^z \frac{q_0}{m} [E + \nu \times (B + B_0)] \, dz \n\]

where \( E \) is wave-electric field and \( B_0 \) is geomagnetic field. The triggered wave magnetic field is given by

\[
B_\perp(z) = \frac{\mu_0}{2} \int_0^z J(z) \times \hat{n} \, dz \n\]

where \( \hat{n} \) is unit vector in \(-z\) direction and \( \mu_0 \) permeability of the free space.

At any time \( t \), the total field is vector sum of applied field and triggered field, which causes phase bunching of new electrons flowing into the interaction region. Thus, a self-sustained feedback loop may be established when the energetic particle distribution function \( f(\nu, \alpha) \) is such that the generated wave field exceeds the field required to generate it. The wave field is obtained by solving coupled equations [Eqs (2)-(4)].

The characteristic time for electron phase bunching known as “bunching time” is one-quarter of the trapping period of an electron in the wave-field, which is obtained as

\[
T_b = \left[ \frac{\pi^2 m}{4 q k \nu_\perp B} \right]^{1/2} \n\]

where \( m \) is mass of an electron. Helliwell and Inan have discussed that saturation occurs when resonance time becomes equal to bunching time. For a dipolar geomagnetic field resonant interaction length is given by

\[
S = 2 \left[ \frac{R_e^2}{4.5 f_{He} (f_\perp)} \left( \frac{f_{He}}{f_\perp} \right)^{5/3} \left( \frac{\nu_\perp}{c} \right)^{3/2} \right]^{1/3} \n\]

\[
= 5.85 \times 10^4 \left( \frac{1 - \lambda}{\nu_\perp} \right) \times \frac{1 - \lambda}{\nu_\perp} \text{ km} \n\]

where all the frequencies are expressed in Hz. Here \( \lambda = \nu_\perp / \nu_\perp \), where \( \nu_\perp \) is the equatorial electron gyrofrequency at the Earth’s surface (873.6 kHz) and \( f_{He} \) the equatorial electron gyrofrequency along the path of propagation (source region). Also \( R_e = 6370 \) km is radius of the Earth.

The transverse current, which is directly proportional to the intensity of triggered emissions, depends on the magnitude and phase of transverse velocities of energetic electrons and their number density. Assuming complete bunching of the resonant electrons and isotropic distribution of pitch angles, the transverse current density per unit pitch angle is written as

\[
J_\perp = J_{\perp \max} \cos^2 \alpha \sin^2 \alpha \frac{A}{m^2 \text{ radian}} \n\]

where \( J_{\perp \max} \) is the maximum current density per unit pitch angle. This expression was derived assuming electron spectrum proportional to \( \nu^{-5} \) (Frank). It can easily be seen that the function \( \cos^2 \alpha \sin^2 \alpha \) peaks at \( \alpha = 30^\circ \). Thus, for the maximum contribution to the current and hence to the triggered wave-field, we
assume $\alpha = 30^\circ$ and hence $v_\perp = 0.577 \, v_p$, where $v_\parallel = v_R$ (resonance velocity) in the interaction region. Here, it should be noted that usually the loss cone pitch angle is less than 10° and hence its neglect will not seriously affect the computed results.

The power radiated by the phase bunching electrons depends on the length of the radiating region, transverse current density, transverse electric field, and phase angle $\theta$ between transverse current and wave electric field, and is given by

$$P = \int_0^{2\pi} J_\perp E \cos \theta \, ds \quad \text{W/m}^2 \quad \text{...(8)}$$

where $E$ is intensity of wave electric field in V/m and $ds$ the elementary length within the interaction region. The maximum power radiated from the interaction region can be written as

$$P_{\text{max}} = \frac{16\pi^2}{\varepsilon_0} J_{\perp\text{max}}^2 \quad \text{...(9)}$$

where $Z = 377/\varepsilon_0^{1/2}$ $\Omega$ is the characteristic impedance of the medium and $\varepsilon_0$ is its relative dielectric constant. Using Maxwell’s equations, limiting value of the wave magnetic field $B_0$ is obtained as

$$B_0 = 5.8 \times 10^{-13} \frac{f_{\text{max}}^4}{f_p^2} \lambda^{-2} \cot \alpha \quad \text{Weber/m}^2 \quad \text{...(10)}$$

In the above equation frequencies are expressed in Hz. The wave magnetic field increases with decreasing wave frequency, plasma frequency and electron gyrofrequency in the equatorial region. The number of energetic electrons taking part in the generation of triggered emission is evaluated from the definition of transverse current $J_\perp = n \, q \, v_\perp$.

The analysis of whistler accompanying triggered emissions shows that they have propagated along $L$-value lying between 1.9 and 2.4. Therefore, we consider that these emissions may have been generated in the equatorial region corresponding to these $L$-values. The computations have been made for parameters corresponding to $L$-values 1.9, 3, 4 and 5 for the normal and disturbed conditions of the magnetosphere. The $L$-values $\leq 3$ is chosen to explain the observations reported in the present paper, where, as $L \geq 4$ has been chosen to represent the values often reported from mid/high latitudes. Bulk of the observed data corresponds to the latter $L$-values. The results are shown in Fig. 4(a)-(d). It is seen that for a given geomagnetic field line the length of interaction region, wave magnetic field and resonant current decreases as wave frequency increases during the normal and disturbed magnetosphere. The interaction length increases as $L$-value increases at a fixed frequency. The value of resonant current and wave magnetic field decreases as $L$-value increases. For a fixed $L$-value, the interaction length, wave-magnetic field and resonant current increase during the disturbed conditions of the magnetosphere. However, the number of energetic electrons participating in the resonance process could not show any appreciable change during the normal and disturbed conditions (Table 2). Further, the density of energetic electrons decreases with the increase in $L$-value and is in accordance with the measured values. For example, for wave frequency 5 kHz and $L = 1.9$, under the normal and disturbed conditions the interaction length varies from 1063 to 1328 km, wave magnetic field from $1.06 \times 10^{-11}$ to $2.6 \times 10^{-11}$ Weber/m$^2$, resonant current from $2.34 \times 10^{-11}$ to $4.56 \times 10^{-11}$ A/m$^2$. These values are in reasonably good agreement with those reported by others.

The risers shown in Fig. 1(a), (d) could be explained by considering the interaction region to start from the equator and move in the northern hemisphere along the geomagnetic field line. The positive drift provides positive slope, which may result into a riser. Oscillating tones could be triggered when the interaction region oscillates near the equatorial region along the field line [Fig. 1(c), (d)]. Hellinwell has argued that the peak in electron velocity spectrum could cause a periodic movement of the interaction region between points on the opposite sides of the equator and this movement of interaction region may give rise to oscillating tone. If we assume that the input particle flux increases as the wave frequency falls, then we expect that at some frequency, drift velocity will become positive and the process repeats itself. This is a simple schematic explanation and the exact mechanism of the generation of oscillating tones in the magnetosphere is not well understood. Sonwalker and Inan have suggested two possibilities to explain whistler triggered hiss: (a) triggered emissions along with the first reflected component of the whistler from the magnetosphere may undergo multiple reflections in the magnetosphere and finally may result in appearance of whistler triggered hiss, (b) the whistler
Fig. 4—Variation of interaction parameters with wave frequency at $L = 1.9$, $3.0$, $4.0$ and $5.0$ for the normal and disturbed conditions of the magnetosphere ($N$ for normal condition, $D$ for disturbed condition) [(a) resonance interaction length, (b) wave magnetic field, (c) transverse resonant current, and (d) number of energetic electron participating in the triggering process]

<table>
<thead>
<tr>
<th>Frequency kHz</th>
<th>L = 1.9</th>
<th>L = 3.0</th>
<th>L = 4.0</th>
<th>L = 5.0</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N</td>
<td>D</td>
<td>N</td>
<td>D</td>
</tr>
<tr>
<td>1.0</td>
<td>0.3885</td>
<td>0.3884</td>
<td>0.1541</td>
<td>0.1539</td>
</tr>
<tr>
<td>2.0</td>
<td>0.3915</td>
<td>0.3914</td>
<td>0.1592</td>
<td>0.1590</td>
</tr>
<tr>
<td>3.0</td>
<td>0.3945</td>
<td>0.3944</td>
<td>0.1646</td>
<td>0.1644</td>
</tr>
<tr>
<td>4.0</td>
<td>0.3975</td>
<td>0.3974</td>
<td>0.1704</td>
<td>0.1702</td>
</tr>
<tr>
<td>5.0</td>
<td>0.4006</td>
<td>0.4005</td>
<td>0.1767</td>
<td>0.1765</td>
</tr>
<tr>
<td>6.0</td>
<td>0.4037</td>
<td>0.4036</td>
<td>0.1834</td>
<td>0.1832</td>
</tr>
</tbody>
</table>
triggered emissions lead only to an initial weak hiss, which undergoes further amplification, so as to reach the observed hiss intensity.

4 Conclusions
Whistler-triggered emissions were observed at low latitude ground station Varanasi during the main phase of the magnetic storm ($K_p$ index varied from 4" to 4" and $D_s$ value changed from +22γ to -49γ at 1200 hrs UT). Based on analysis it is suggested that these emissions may have been triggered in the equatorial region of $L$-value lying between 1.9 and 2.4 by the non-linear Doppler-shifted cyclotron resonance interaction. The required parallel resonant energy has been computed for the normal and disturbed conditions of the magnetosphere, which is found to decrease with increase in $L$-value and emitted wave frequencies. Using the theory of the second order resonance interaction, wave magnetic field, interaction length, transverse resonant current and number of electrons participating in the resonance interaction process for different location of the magnetosphere as a function of wave frequency under the normal and disturbed conditions of the magnetosphere have been estimated, which were found to be in good agreement with those reported in the literature.

Acknowledgements
One of the authors (DS) is thankful to the Head, I & OT Division, IITM, for his encouragement and help throughout the preparation of this manuscript. The work is partly supported by the Department of Science and Technology, New Delhi under SERC project (SS and RPS). Thanks are also due to V Gopalakrishnan for his help in computation work.

References
27 Hayakawa M, Okada T & Tanaka Y, Morphological characteristics and the polarization of plasmaspheric ELF hiss observed at Moshiri \((L = 1.6)\), *J Geophys Res (USA)*, 90 (1985) 5133.


