Theoretical analysis of linear array antenna of stacked patches

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A linear four-element stacked array antenna is analyzed. It is observed that stacked element array antenna shows marked improvement in the bandwidth and gain over the single-element array antenna. The bandwidth of the stacked antenna array improves to 14.8% over the 4% bandwidth of the single-element array antenna, whereas the radiated power is enhanced by 0.82 dB as compared to the single-element array antenna. It is further observed that the resonance frequency, gain and directivity are highly dependent on the thickness and permittivity of the upper substrate.

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1 Introduction

Microstrip antennas are well suited for arrays due to their low weight, low profile with conformability and low manufacturing cost. However the major drawbacks of microstrip antennas are low gain and narrow bandwidth. A number of papers have appeared in the literature on bandwidth enhancement of microstrip antennas, such as electrically thick substrate, patch loaded with slot, an integrated reactive loading with reduced cross-polarization radiation, patch with an air gap between substrate and ground, gap coupled parasitic patches, stacked patch antennas and impedance matching network. The bandwidth of the basic form of microstrip antenna can be enhanced by using parasitic elements to create dual or multiple closely spaced resonances, either in another layer (stacked geometry) or in the same layer (coplanar geometry), which is significantly larger than that of a conventional single-layer microstrip antenna (Fig. 1). Therefore, the bandwidth of the array antenna can also be increased by using stacked microstrip patch antenna.

In the present endeavour, an attempt has been made to enhance the bandwidth of the microstrip array antenna by using stacked antenna as an element of the array antenna. The proposed model for stacked antenna is based on the circuit theory concept, which is used to calculate the input impedance, bandwidth and radiation pattern of single element as well as for array antenna. The details of the entire analysis are given in the following section.

2 Theoretical analysis

2.1 Two-layers stacked patch

Due to the presence of the parasitic element in the stacked configuration [Fig. 2(a)] there are two resonances associated with the two constitutive resonators. One resonance is associated with the resonator formed by the lower patch and the ground plane and the second resonance is associated with the resonator formed by the parasitic patch and lower patch. The first resonator is considered as a microstrip patch with dielectric cover (superstrate). Due to the presence of superstrate the effective dielectric constant is changed and the resonance frequency will decrease with increase in thickness and the dielectric constant of the superstrate. The effective dielectric constant of this structure is given as:

\[ \varepsilon_{eff} = \varepsilon_{r} q_{2} + \varepsilon_{r} \frac{(1-q_{1})^{2}}{\varepsilon_{r} (1-q_{1}-q_{2}) + q_{2}} \]  \( \text{(1)} \)

Fig. 1—Single-layer rectangular microstrip antenna geometry
Fig. 2—(a) Side view of the two layers stacked rectangular microstrip antenna, (b) equivalent circuit of the first resonator and (c) equivalent circuit of the second resonator

where, $q_1$ are the effective filling fractions and are given as:

$$q_1 = 1 - \frac{h_1}{2w_{le}} \ln \left( \frac{\pi}{h_1} \frac{w_{le}}{w_{le} - 1} \right)$$

$$q_2 = 1 - q_1 - q_3$$

where the effective width

$$w_{le} = w_i + \frac{2h_1}{\pi} \ln \left\{ 17.08 \left( \frac{w_i}{2h_i} + 0.92 \right) \right\}$$

and

$$g = \frac{2h_1}{\pi} \tan^{-1} \left( \frac{\pi}{\frac{w_{le}}{2h_i} - \frac{1}{2}} \right)$$

Using this effective dielectric constant the microstrip with superstrate can be represented as a single patch with a semi-infinite superstrate with relative permittivity equal to unity and a single relative dielectric constant equal to $\varepsilon'_{eff}$, which is given as

$$\varepsilon'_{eff} = \frac{2\varepsilon_{eff} - 1 + p}{1 + p}$$

where $p = (1 + 10h_i/w_{le})^{-1/2}$

The new resultant resonance frequency of the first resonator is given as

$$f'_{r1} = \frac{c}{2\left(l_i + 2\Delta l' \right)\sqrt{\varepsilon_{eff}}}$$

where fringing length

$$\Delta l' = 0.412h_i \frac{(\varepsilon'_{le} + 0.3)(w_i/h_i + 0.264)}{(\varepsilon'_{le} - 0.258)(w_i/h_i + 0.8)}$$

effective dielectric constant

$$\varepsilon'_{le} = \frac{\varepsilon'_{le} + 1}{2} + \frac{\varepsilon'_{le} - 1}{2} \left( 1 + 12h_i/w_i \right)^{-1/2}$$

The equivalent circuit of the first resonator based on the modal expansion cavity model is shown in Fig. 2(b) and the values of the circuit parameters are defined as:

$$C_i = \frac{\varepsilon_e \varepsilon_{le} l_i w_i}{(2h_i)}$$

$$L_i' = \frac{1}{\left( 2\pi f'_{r1} \right)^2 C_i}$$

and

$$R_i' = \frac{Q_i}{\left( 2\pi f'_{r1} C_i \right)}$$

The equivalent circuit of the second resonator based on the modal expansion cavity model is shown in Fig. 2(c) and the values of the circuit parameters are given as

$$C_2 = \frac{\varepsilon_e \varepsilon_{le} l_2 w_2}{(2h_2)}$$

$$L_2 = \frac{1}{\left( 2\pi f_{r2} \right)^2 C_2}$$

and

$$R_2 = \frac{Q_i}{\left( 2\pi f_{r2} C_2 \right)}$$
where resonance frequency

\[ f_r = \frac{1}{2 \left( L_1 + 2 \Delta L_2 \right) / \varepsilon_{\text{eq}}} \]  \hspace{1cm} (5)

effective dielectric constant

\[ \varepsilon_{\text{eq}} = \frac{\varepsilon_{\text{r1}} + 1}{2} + \frac{\varepsilon_{\text{r2}} - 1}{2} \left( 1 + \frac{1.12 h_2}{w_2} \right)^{3/2} \]

fringing length

\[ \Delta l_2 = 0.412 h_2 \left( \frac{\varepsilon_{\text{r1}} + 0.3}{\varepsilon_{\text{r2}} - 0.258} \right) \left( \frac{w_2}{h_2} + 0.8 \right) \]

The coupling factor \( C_p \) between the two resonators may be given as

\[ C_p = \frac{1}{\sqrt{Q_i Q_f}} \]  \hspace{1cm} (6)

where \( Q_i \) and \( Q_f \) are the total quality factor\(^{9}\) of the first and second resonator, respectively. Considering both inductive and capacitive coupling, the resulting equivalent circuit of the stacked antenna can be represented as in Fig. 3 and the values of mutual inductance \( L_m \) and mutual capacitance \( C_m \) are defined as\(^{9}\)

\[ L_m = \frac{C_i^p \left( L_1 + L_2 \right) + \sqrt{C_i^p \left( L_1 + L_2 \right)^2 + 4 C_p^2 \left( 1 - C_p^2 \right) L_1 L_2}}{2 \left( 1 - C_p^2 \right)} \]

\[ C_m = \frac{\left( C_i + C_2 \right) + \sqrt{\left( C_i + C_2 \right)^2 - C_i C_2 \left( 1 - 1/C_p^2 \right)}}{2} \]  \hspace{1cm} (7)

Now the input impedance of the stacked antenna (Fig. 3) is given by

\[ Z_{in} = \frac{\omega^2 L_m^2 + j \omega R^2 L_1 \left( 1 - \omega^2 L C \right)}{\omega^2 \left( \omega^2 L_m^2 L C^2 - 2 R^2 L C + L^2 \right) + R^2} \]  \hspace{1cm} (8)

where

\[ R = \frac{R_1 R_2}{R_1^2 + R_2^2} \]

\[ L = \frac{L_1^2 L_2}{L_1^2 + L_2^2} + L_m \]

and

\[ C = \frac{C_i + C_m}{C_i + C_2 + C_m} \]

The E-plane (x-z plane) radiation field of the rectangular microstrip patch antenna (Fig. 1), can be given as\(^{10}\)

\[ E(\Phi) = j \frac{k_0 W_0}{\pi r} e^{-j \Phi} \left\{ \sin \left( \left( k_0 h_1 \cos \Phi \right)/2 \right) \frac{\cos \left( \left( k_0 l_0 \cos \Phi \right)/2 \right)}{\cos \left( \left( k_0 l_0 \sin \Phi \right)/2 \right)} \right\} \]

\[ -90^\circ \leq \Phi \leq 90^\circ \]  \hspace{1cm} (9)

The far field E-plane radiation of the stacked antenna was derived with the following assumptions:

(1) The induced slot voltage of the parasitic patch is \( C_i \) times the slot voltage of the driven patch.

(2) The distance between two patches is very small compared to far field point, hence the radiations from the two patches are in same phase.

(3) The radiation pattern depends on the fringing length, hence the fringing length for stacked antenna is replaced by sum of the fringing length of the two patches. Hence the radiated far field of the stacked antenna in E-plane \((-90^\circ \leq \Phi \leq 90^\circ\)\) can be written as

\[ E(\Phi) = j \frac{k_0 W_0}{\pi r} e^{-j \Phi} \left\{ \sin \left( \left( k_0 h_1 \cos \Phi \right)/2 \right) \frac{\cos \left( \left( k_0 l_0 \cos \Phi \right)/2 \right)}{\cos \left( \left( k_0 l_0 \sin \Phi \right)/2 \right)} \right\} \]

\[ + j \frac{k_0 W_2}{\pi r} e^{-j \Phi} \left\{ \sin \left( \left( k_0 h_2 \cos \Phi \right)/2 \right) \frac{\cos \left( \left( k_0 l_2 \cos \Phi \right)/2 \right)}{\cos \left( \left( k_0 l_2 \sin \Phi \right)/2 \right)} \right\} \]

\[ -90^\circ \leq \Phi \leq 90^\circ \]  \hspace{1cm} (10)
Here the first term indicates the radiated field due to lower patch and the second term due to upper patch.

2.2 Four-element antenna array

Let us consider a corporate feed for 4-element linear antenna array as shown in Fig. 4, in which all elements are placed in y-z plane along y-axis with inter-element spacing $d = 0.8 \lambda_0$. Angle $\phi$ is measured from the broad side direction, i.e. from the x-axis. The bandwidth of a corporate-fed microstrip array antenna is limited by two factors: the bandwidth of the patch element and the impedance matching circuit of the power dividing transmission line.

Using the theory of small reflections\(^{12}\) taking only first-order reflections into account, the reflection coefficient of the array can be given as

$$\Gamma = \Gamma_1 + \Gamma_2 e^{j2\theta_1} + \Gamma_3 e^{j(2\theta_1 + 2\theta_2)} + \Gamma_4 e^{-j(2\theta_1 + 2\theta_2)} \ldots (11)$$

where $\theta_1$ and $\theta_2$ are the electrical lengths of quarter-wave transformer ($T_1$ and $T_2$) and distance between two transformers respectively and given by:

$$\theta_1 = \frac{\lambda_g}{4} \quad \text{and} \quad \theta_2 = \frac{\lambda_g}{2}$$

where $\lambda_g$ is the guide wavelength.

The VSWR of the antenna array is given as

$$VSWR = \frac{1 + |\Gamma|}{1 - |\Gamma|} \ldots (12)$$

The E-plane radiated field of the antenna array is given as\(^{10}\)

$$E_T(\Phi) = E(\Phi) \frac{\sin(n\pi d \sin(\Phi))}{n \sin(\pi d \sin(\Phi))} \ldots (13)$$

where $E(\Phi)$ is given by Eqs (9) and (10) for single-element array and stacked element array, respectively.

3 Design specifications and calculations

The centre design frequency of both, single-layer rectangular microstrip antenna and two-layer stacked rectangular microstrip antennas is 5 GHz. Other specifications are presented in Table 1.

The calculated values of various parameters of single\(^8\) and stacked rectangular microstrip antenna and for $1 \times 4$ linear arrays are presented in Table 2.

4 Discussion of results

The designed parameters of rectangular microstrip patch antenna at 5 GHz are shown in Table 2. By using the same patch in stacked configuration the effect of upper substrate thickness ($h_2$) and permittivity ($\varepsilon_r$) were studied and the results are shown in Figs 5 and 6. It is observed [Fig. 5(a)] that the stacking of the patch with parasitic element of same dimensions reduces the input resistance from 162 ohm to 80 ohm and the resonance frequency reduces from 5 GHz to 4.9 GHz. It is further observed that on increasing the thickness of upper substrate ($h_2$), the resonance frequency of the stacked antenna decreases with small increase in resistance at resonance. From Fig. 5(b) it is found that the gain and directivity of the stacked antenna are greater than that of single patch antenna and increase with thickness of

<table>
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<th>Table 1—Patch antenna specifications</th>
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<tr>
<td><strong>Substrate material</strong></td>
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<tr>
<td><strong>Dielectric constant</strong></td>
</tr>
<tr>
<td><strong>Loss tangent (tan$$\delta$$)</strong></td>
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<tr>
<td><strong>Substrate thickness</strong></td>
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<tr>
<td><strong>The inter-element spacing</strong> of the array</td>
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<table>
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<tr>
<th>Table 2—Calculated values of single and stacked microstrip antenna at 5 GHz</th>
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<tbody>
<tr>
<td><strong>Single-layer</strong></td>
</tr>
<tr>
<td>$w_1$ = 2.37 cm</td>
</tr>
<tr>
<td>$l_1$ = 1.96 cm</td>
</tr>
<tr>
<td>$h_1$ = 1.2 mm</td>
</tr>
<tr>
<td>$\varepsilon_r$ = 2.2</td>
</tr>
<tr>
<td>$Z_0$ = 162$\Omega$</td>
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<tr>
<td>Beam width = 91.2°</td>
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<tr>
<td>Characteristic impedance of</td>
</tr>
<tr>
<td>$T_1$ = 130$\Omega$ &amp; $T_2$ = 70$\Omega$</td>
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<tr>
<td>Bandwidth = 4%</td>
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the upper substrate due to the fact that as coupling factor $C_p$ increases, the total quality factor of second resonator decreases. Similar observations were also made by Lee and Lee\textsuperscript{13}. From Fig. 6(a), it is observed that as the permittivity of the upper substrate increases, the resonance frequency decreases with small increase in the resistance at resonance. It is also observed that as permittivity increases, the resonance curve becomes sharper. Figure 6(b) shows that the gain decreases with increasing permittivity of the upper substrate. Thus the thickness and permittivity of upper substrate play an important role in controlling the resonance behaviour of stacked antenna. From the above observations it is clear that for wide bandwidth, $h_2$ should be high and $\varepsilon_r$ must be low. Similar results were also observed by Mitchell et al\textsuperscript{14}. An increase in the thickness $h_2$ reduces the resonance frequency. Therefore to work at the same centre frequency, the patch dimensions must be reduced, which may reduce the radiated power, i.e. antenna efficiency. Hence a compromise has to be made between bandwidth and radiated power. Thus in order to have design frequency of 5 GHz for stacked antenna, an optimization\textsuperscript{5} process was adopted (resonance frequency is controlled by $l_1$, $l_2$ and resonance peak resistance by the ratio $w_1/w_2$ and $l_1/l_2$) and the optimized parameters for stacked antenna are given in the Table 2. A similar result was reported by Lee and Chen\textsuperscript{15} for stacked antenna having similar specification, in which they got 15% bandwidth. These parameters were used for developing the array antenna for further investigations. The input impedance and radiation patterns of these designed antennas are presented in Fig. 7. The array [Fig.8 (a)] of the stacked antenna shows an improvement in the bandwidth to 14.8% as compared to 4% of single-element array.

It is further observed [Fig.8 (b)] that the radiation power of array antenna with stacked antenna is enhanced by 0.82 dB over the array antenna with
single element, while the side lobe level remains almost at the same level. Thus the array with stacked antenna improves the bandwidth of operation as well as gain over the array antenna with single element.

5 Conclusion
It may be concluded that both resonance frequency and gain are highly dependent on the thickness \( h_2 \) and permittivity \( e_{r_2} \) of the upper substrate. Further the bandwidth of the antenna improves when the relative permittivity of upper substrate approaches to unity.

References