Adaptive tracking control scheme for an autonomous underwater vehicle subject to a union of boundaries

Zool Hilmi Ismail\(^1\) & Mohd Bazli Mohd Mokhar\(^2\)

\(^1\)Robotics Research Group,
Centre for Artificial Intelligence and Robotics (CAIRO), Universiti Teknologi Malaysia,
Jalan Semarak, 54100, Kuala Lumpur, Malaysia.
\(^2\)Faculty of Electrical Engineering, Universiti Teknologi Malaysia, 81310, Skudai, Malaysia.

[E-mail: \(^1\)zool@fke.utm.my, \(^2\)bazli.mokhar@gmail.com]

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Present study presents a novel region boundary-based tracking control for an Autonomous Underwater Vehicle (AUV). The control objective is to track a moving target formed by the union of all boundaries. In this case, multiplicative potential energy function is used to unite all the boundaries and various shapes can be created using this function. It is interesting to note that the AUV will be placed at a specific position on the dynamic region boundaries. A Lyapunov-like function is presented for stability analysis of the AUV. Simulation studies will be performed to illustrate the effectiveness of the proposed controller.

[Keywords: Multiplicative potential energy function, dynamic region boundary-based control; autonomous underwater vehicle]

Introduction

The current research studies of an Autonomous Underwater Vehicle (AUV) have focused on the development and improvement of regulation and tracking control schemes. Breakthrough from these findings can be deployed for numerous applications including industrial and military applications. Point stabilization is regarded as a problem of commanding an AUV to a final target point with a desired orientation. Set-point control is one of the most important topics in underwater vehicle navigation. However, when the desired target of the AUV will be a point, it consumes a lot of energy due to propeller usually being active to counter the wave until the AUV arrived to its set-point. Hence, an adaptive region tracking control is proposed by\(^1\). From their work, the target is defined as a moving region, thus it would require less energy to perform docking or tracking. This is due to the controller only activated when the AUV outside of the region rather than always pushing the AUV into the point target in conventional method. The proposed technique not only applicable for an AUV but also can be utilized for a swarm of robots as reported by\(^2\). While the robots are moving in a specific region, the formation can be rotated and/or scaled depending on their mission. On the other hand,\(^3\) proposed a multiplicative energy function together with region control for a swarm of robots. Within this control law, various shapes such as star shape and N-shape can be formed based on the union of all the regions. In addition, a research work by\(^4\) combined the multiplicative potential energy function with transformation mapping that enables the swarm robot to change their shape and size while moving. Recently, the multiplicative potential energy function and adaptive boundary-based region was presented to control a single AUV\(^5\). The vehicle has been able to navigate and locate itself to a desired position on a moving target using an edge-based segmentation technique. Note that the respective moving target is created via the union of region boundaries.

In this paper, a dynamic boundary-based region tracking control approach and multiplicative potential energy function are proposed for an AUV. The desired target is formed using the union of region boundaries and the AUV will move in a specific direction alongside the moving region boundary as to monitor the pipeline thoroughly from any degree. The region can be scaled up or down according to the state or size of the pipeline as depicted in Fig. 1.
Fig. 1—An illustration of dynamic boundary-based region control technique for tracking a pipeline

Materials and Methods

Modeling of an AUV

Kinematic Model

The relationship between inertial and body-fixed vehicle velocity can be described using the Jacobian matrix \( J \) in the following form

\[
\begin{bmatrix}
\dot{\eta}_1 \\
\dot{\eta}_2
\end{bmatrix} =
\begin{bmatrix}
J_1(\eta_2) & 0_{3\times3} \\
0_{3\times3} & J_2(\eta_2)
\end{bmatrix}
\begin{bmatrix}
v_1 \\
v_2
\end{bmatrix} \Rightarrow \dot{\eta} = f(\eta_2)v \quad ... (1)
\]

Where \( \eta_1 = [x, y, z]^T \in \mathbb{R}^3 \) and \( \eta_2 = [\phi, \theta, \psi]^T \in \mathbb{R}^3 \) denote the position and the orientation of the vehicle, respectively, expressed in the inertial-fixed frame. \( J_1 \) and \( J_2 \) are the transformation matrices expressed in terms of the Euler angles. Linear and angular velocity vectors, \( v_1 = [u, v, w]^T \in \mathbb{R}^3 \) and \( v_2 = [p, q, r]^T \in \mathbb{R}^3 \), respectively, are described in terms of the body-fixed frame.

Dynamic Model

The investigation of dynamic equation of motion for an underwater vehicle has been previously reported\(^6\). The underwater vehicle dynamic equation can be expressed in closed form as

\[
M\ddot{\eta} + C(\eta)v + D(v)\dot{\eta} + g(\eta) = \tau \quad ... (2)
\]

where \( \eta \in \mathbb{R}^6 \) is the velocity state vector with respect to the body-fixed frame, \( M \) is the inertia matrix including the added mass term, \( C(\eta) \) represents the matrix of the Coriolis and centripetal forces including the added mass term, \( D(\eta) \) denotes the hydrodynamic damping and lift force, \( g(\eta) \) is the restoring force and \( \tau \) is the vector of generalized forces acting on the vehicle. The dynamic equation in (2) preserves the following properties:\(^8\):

1. The inertia matrix \( M \) is symmetric and positive definite such that \( M = M^T > 0 \).
2. \( C(\eta) \) is the skew-symmetric matrix such that \( C(\eta) = -C^T(\eta) \).
3. The hydrodynamic damping matrix \( D(\eta) \) is positive definite, i.e.: \( D(\eta) = D^T(\eta) > 0 \).
4. The dynamic model as described in (2) is linear in a set of dynamic parameters \( \theta_d \in \mathbb{R}^{n_p} \) and can be written as

\[
M\ddot{\eta} + C(v)\dot{\eta} + D(\eta)\dot{\eta} + g(\eta) = Y_d(\eta, v, \dot{\eta}) \quad ... (3)
\]

where \( Y_d(\eta, v, \dot{\eta}) \in \mathbb{R}^{6 \times n_p} \) is a known regression matrix; \( n_p \) is the total number of physical parameters. It is assumed that if the arguments of \( Y_d(\cdot) \) are bounded then \( Y_d(\cdot) \) is bounded.

Tracking Control Using Multiplicative Potential Energy Function

In region boundary-based control, the desired moving target is specified by at least two sub-regions intersecting at the same point. Inner sub-region acts a repulsive region while the outer sub-region acts as an attractive region. Regulation control concept that has been presented\(^7\) is extended for coordination control of multiple AUVs\(^8\). A new proposed tracking control for an AUV subject to the union of boundaries is formulated as follows:

First, a dynamic region of specific shape is defined and this can be viewed as a global objective of the proposed control law. Global objective functions for outer sub-region and inner sub-region of AUV are defined by the following inequalities:

\[
f_{out}(\delta\eta) \leq 0 \quad ... (4)
\]

where \( \delta\eta = \eta - \eta_d \in \mathbb{R}^6 \) is the continuous first partial derivatives; \( \eta_d \) is the time-varying reference point inside the region. The following inequality function can be used for the inner sub-region

\[
f_{in}(\delta\eta) \geq 0 \quad ... (5)
\]

where the primary and secondary sub-regions share the same reference point, \( \eta_d \). Note that, (4) and (5) are defined arbitrarily close to each other, such that

\[
f_{out}(\delta\eta) \approx f_{in}(\delta\eta) \quad ... (6)
\]
The corresponding potential energy function for the desired sub-region describes in (4) can be specified as:

\[ P_b(\delta \eta) = \frac{k_p}{2} \left[ \max\left(0, f_{OUT}(\delta \eta)\right) \right]^2 \leq \begin{cases} 
0, & f_{OUT}(\delta \eta) \leq 0 \\
\frac{k_p}{2} f_{OUT}^2(\delta \eta), & f_{OUT}(\delta \eta) > 0 
\end{cases} \quad \ldots (7) \]

where \( k_p \) is positive scalar for an AUV. Similarly, the potential energy function for the inner sub-regions in (5) can defined as follows

\[ P_s(\delta \eta) = \frac{k_s}{2} \left[ \max\left(0, f_{IN}(\delta \eta)\right) \right]^2 \leq \begin{cases} 
0, & f_{IN}(\delta \eta) \geq 0 \\
\frac{k_s}{2} f_{IN}^2(\delta \eta), & f_{IN}(\delta \eta) < 0 
\end{cases} \quad \ldots (8) \]

where \( k_s \) is positive scalar. Differentiating (7) and (8) with respect to \( \delta \eta \) gives

\[ \left( \frac{\partial P_b(\delta \eta)}{\partial \eta} \right)^T = k_p \max\left(0, f_{OUT}(\delta \eta)\right) \left( \frac{\partial f_{OUT}(\delta \eta)}{\partial \eta} \right)^T \quad \ldots (9) \]

\[ \left( \frac{\partial P_s(\delta \eta)}{\partial \eta} \right)^T = k_s \max\left(0, f_{IN}(\delta \eta)\right) \left( \frac{\partial f_{IN}(\delta \eta)}{\partial \eta} \right)^T \quad \ldots (10) \]

Now, let (9) and (10) be represented as the primary region error \( \tilde{e}_b \) and secondary region error \( \tilde{e}_s \) respectively in the following form

\[ \tilde{e}_b = \max\left(0, f_{OUT}(\delta \eta)\right) \left( \frac{\partial f_{OUT}(\delta \eta)}{\partial \eta} \right)^T \quad \ldots (11) \]

\[ \tilde{e}_s = \max\left(0, f_{IN}(\delta \eta)\right) \left( \frac{\partial f_{IN}(\delta \eta)}{\partial \eta} \right)^T \quad \ldots (12) \]

Next, an edge-based segmentation approach is utilized to ensure an AUV is placed at a desired position in the target. Potential energy function for a segmented boundary can defined as follows:

\[ P_{edge}(\delta \eta_1) = \frac{k_e}{2} \left[ \max\left(0, f_{edge}(\delta \eta_1)\right) \right]^2 \leq \begin{cases} 
0, & f_{edge}(\delta \eta_1) \geq 0 \\
\frac{k_e}{2} f_{edge}^2(\delta \eta_1), & f_{edge}(\delta \eta_1) < 0 
\end{cases} \quad \ldots (13) \]

where \( k_e \) is a positive constant. Differentiating (13) with respect to \( \delta \eta_1 \) gives

\[ \left( \frac{\partial P_{edge}(\delta \eta_1)}{\partial \eta_1} \right)^T = k_e \max\left(0, f_{edge}(\delta \eta_1)\right) \left( \frac{\partial f_{edge}(\delta \eta_1)}{\partial \eta_1} \right)^T \quad \ldots (14) \]

which leads to

\[ \tilde{e}_e = \max\left(0, f_{edge}(\delta \eta_1)\right) \left( \frac{\partial f_{edge}(\delta \eta_1)}{\partial \eta_1} \right)^T \quad \ldots (15) \]

To implement the multiplicative potential energy, let \( P_i \) be the potential energy function associated with region boundary \( RB_l, l = 1, 2, 3, \ldots, L \)

\[ RB_1: P_1 = P_{p1}(\delta \eta) + P_{s1}(\delta \eta) + P_{edge1}(\delta \eta_1) \]

\[ RB_2: P_2 = P_{p2}(\delta \eta) + P_{s2}(\delta \eta) + P_{edge2}(\delta \eta_1) \quad \ldots (16) \]

\[ \vdots \]

\[ RB_l: P_l = P_{pl}(\delta \eta) + P_{sl}(\delta \eta) + P_{edgel}(\delta \eta_1) \]

where \( L \) is the number of desired boundaries. A multiplication method is adopted in this paper, thus the total potential energy \( P_T \) associated with the desired boundary in (16) is defined by

\[ P_T(\delta \eta) = \prod_{l=1}^{L} P_l(\delta \eta) \quad \ldots (17) \]

where \( P_l \) is defined in (16). The desired boundary produced from this multiplicative of the potential energy is the union of all the boundaries \( RB_l \) that is \( RB = RB_1 \cup RB_2 \cup \ldots \cup RB_L \). Note that \( P_T \) has a minimum value of zero when \( \eta \) is within any of the desired boundaries. Equation (17) expresses that the potential energy is at the minimum value (zero) at the desired target. This potential function will ensure the AUV move toward the overall region produced by union of all the boundaries \( RB_1, RB_2, \ldots, RB_L \). This function is useful when the AUV need to adapt the moving boundary, depending on the situation and environment such as avoiding obstacle on its path or capturing images at some different angles.

Partial differentiating the total potential energy function described by (17) with respect to \( \delta \eta_i \) leads to

\[ \left( \frac{\partial P_T(\delta \eta)}{\partial \eta} \right)^T = \left( k_{p1} \tilde{e}_{p1} + k_{s1} \tilde{e}_{s1} + k_{e1} \tilde{e}_{e1} \right) \prod_{i=1}^{L} P_i(\delta \eta) \]

\[ + \left( k_{p2} \tilde{e}_{p2} + k_{s2} \tilde{e}_{s2} + k_{e2} \tilde{e}_{e2} \right) \prod_{i=2}^{L} P_i(\delta \eta) + \ldots \]

\[ + \left( k_{pl} \tilde{e}_{pl} + k_{sl} \tilde{e}_{sl} + k_{el} \tilde{e}_{el} \right) \prod_{i=L}^{L} P_i(\delta \eta) \prod_{i=1}^{L} P_i(\delta \eta) = \Delta \xi \quad \ldots (18) \]

where the product rule is used to obtain the derivatives of products of two or more functions.
When the AUV are outside the desired boundary, the control force $\Delta \xi$ described by (18) is activated to attract the AUV toward the desired boundary. When the AUV is inside the desired boundary, then the control force is zero or $\Delta \xi = 0$. Next, a vector $v_r$ that is useful is defined

$$v_r = f^{-1}(\dot{\eta}_d - \delta \eta) - \alpha f^{-1} \Delta \xi$$ ...(19)

where $f^{-1}$ is the inverse of the Jacobian matrix, and $\alpha$ is a positive constant. The error term $\Delta \xi$ is given in (18). Based on the structure of (18) and (19) and the subsequent stability analysis, a filtered tracking error vector for an underwater vehicle is defined as

$$\hat{\xi} = v - f^{-1} \dot{\eta}_d + \alpha f^{-1} \Delta \xi$$ ...(20)

where $f^{-1}$ is the inverse of the Jacobian matrix, and $\alpha$ is a positive constant.

$\tau_v = C(v)\dot{r} + D(v)\dot{r} + Y_d(\cdot)\theta_d = \tau$ ...(21)

Where

$$Y_d(\cdot)\theta_d = M\ddot{r} + C(v)\dot{r} + D(v)\dot{r} + g(\eta)$$ ...(22)

and the derivative of $v_r$ in (22) is given as

$$\dot{v}_r = f^{-1} \ddot{\eta}_d + f^{-1} \dot{\dot{\eta}}_d - \alpha f^{-1} \Delta \xi - \alpha f^{-1} \dot{\Delta} \xi$$ ...(23)

Based on the error system development and the subsequent stability analysis, the proposed adaptive control law for AUV is

$$\tau = -f^T(\eta)\Delta \xi - K_v \dot{r} + Y_d(\cdot)\dot{\theta}_d$$ ...(24)

where $K_v$ is positive constant matrix. The estimated parameters $\dot{\theta}_d$ are updated using the following update law

$$\dot{\theta}_d = -L_d Y_d^T(\cdot)\dot{r}$$ ...(25)

where $L_d$ is a symmetric positive definite. Substituting (24) into (21) produces a closed-loop dynamic for $r(t)$ as follows

$$M\ddot{r} = -C(v)\dot{r} + D(v)\dot{r} - K_v \dot{r} - f^T(\eta)\Delta \xi + Y_d(\cdot)\dot{\theta}_d$$ ...(26)

where $\dot{\theta}_d = \dot{\theta}_d - \theta_d$ denotes the parameter estimation error. Next, the following non-negative function is introduced to analyze the stability of the proposed control law

$$V = \frac{1}{2} r^T M \ddot{r} + \frac{1}{2} \dot{\theta}_d L_d^{-1} \dot{\theta}_d + P(\delta \eta)$$ ...(27)

Differentiating $V$ with respect to time and using the update law (25) yields

$$\dot{V} = r^T M \ddot{r} - \dot{\theta}_d L_d^{-1} \dot{\theta}_d + \frac{1}{2} \dot{\theta}_d L_d^{-1} \dot{\theta}_d + \frac{1}{2} P(\delta \eta) + \cdots$$

Utilizing equation (19), (20), a closed-loop dynamic (26) and cancelling the common terms leads to

$$\dot{V} = -r^T D(v)\dot{r} - r^T K_v \dot{r} - \alpha^2 \Delta \xi^T \Delta \xi \leq 0$$ ...(29)

where Property 3 is used. Now, a new theorem can be stated as follows:

**Theorem:** Given a closed-loop of AUV in (26), the proposed adaptive control law (24) and the update parameter laws (25) guarantees the convergence of $\Delta \xi \to 0$ and $r \to 0$, as $t \to \infty$.

**Proof:** See previous work by Ismail et al.\textsuperscript{5} for proof.

Remark: The proposed dynamic control concept can be extended to the case of rotating and scaling region boundary. In this case, continuous first partial derivatives in (4) needs to be exploited such that $\delta \eta = RS(\eta - \eta_d) \in \mathbb{R}^6; R(t)$ is rotational matrix given as:

$$R = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$$ ...(30)

where $\theta(t)$ is an angle of rotation. Generally, the rotation matrix $R$ need not be specified for orientation function. Meanwhile $S(t)$ is a time-varying and nonsingular scaling factor that is defined as:
where $S_1$ is the scaling matrix of $\eta_1$ and $S_2$ is the scaling matrix of $\eta_2$. The scaling of the orientation of AUV is not required in general, so $S_2$ can be set as an identity matrix. Thus, the scaling matrix $S_1$ is given by

$$S_1 = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & s_z \end{bmatrix}$$

where $s_x(t), s_y(t)$ and $s_z(t)$ are scaling factors.

**Results and Discussion**

**Simulation Results**

In this section, two simulation studies are carried out to assess the efficacy of the proposed dynamic region boundary-based control law for an underwater vehicle. The vehicle is required to track a pipeline in this simulation. An ODIN with full 6-DOF is chosen as autonomous underwater vehicle model for numerical simulation.

In simulation, the following inequality functions are defined for a boundary of a spherical region

$$f_{out_1} = (x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 \leq \kappa_x^2 \quad \ldots(33)$$

$$f_{in_1} = (x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 \geq \kappa_x^2 \quad \ldots(34)$$

while the subsequent inequality functions are defined for a boundary of an ellipsoid region

$$f_{out_2} = (x - x_0)^2 + \frac{(y - y_0)^2}{4} + (z - z_0)^2 \geq \frac{\kappa_x^2}{4} \quad \ldots(35)$$

$$f_{in_2} = (x - x_0)^2 + \frac{(y - y_0)^2}{4} + (z - z_0)^2 \leq \frac{\kappa_x^2}{4} \quad \ldots(36)$$

Equations (33) and (34) represent the outer sub-regions and (35) and (36) represent the inner sub-regions. $\kappa_{\eta} \equiv \kappa_{\eta}$ is a time-varying tolerance vector. An underwater vehicle is required to track a straight-line trajectory with green and magenta (cross-section lines) trajectory is the horizontal basis position initialized at $[1.45 - 0.6]^T$ m. The green and magenta designate spherical and ellipsoid regions, respectively. The solid blue lines represent the position of an AUV at various time instances.

In the first simulation, the vehicle is initialized to the position $\eta_1(0) = [1.35 - 1 - 1]^T$ m and it is required to converges into the bottom of a moving scalable spherical region, while in the second simulation, the vehicle is initialized to the position $\eta_2(0) = [1.35 - 1]^T$ m and it is required to converges the top of a moving scalable ellipsoid region. The scaling matrix presented in (32) is used in both simulations. The orientation is kept constant with the allowable errors, denoted by $\kappa_{\phi}, \kappa_{\theta}$ and $\kappa_{\varphi}$, are set to 0.1 rad, and the initial values are $\eta_2(0) = [0 0 0]^T$ rad. To ensure the vehicle moves towards its desired position on a moving boundary, the following inequality function is defined

$$f_{edge}(\delta \eta_1) = \begin{cases} (x - x_0) + R_x - \kappa_x \geq 0, \\
(y - y_0) + R_y - \kappa_y \geq 0, \\
(z - z_0) + R_z - \kappa_z \geq 0, \end{cases} \quad (37)$$

where the vector $[R_x \ R_y \ R_z]^T$ is the circumradius for an AUV and $[\kappa_x \ \kappa_y \ \kappa_z]^T$ is a tolerance vector. The control gains are set to the following:

$$k_p = 218; k_s = k_e = 156.4; \alpha = 1;$$

$$K_v = diag([2.08 \ 2.08 \ 2.08 \ 1.1 \ 1.1 \ 1.1])$$

As can be seen from Fig. 2 to Fig. 5, an AUV initially converges into the desired position on a boundary line at various time instances.
Conclusion

In this paper, a new dynamic region boundary-based method has been proposed for an autonomous underwater vehicle. This control technique enables an AUV to perform a specific underwater tracking task. It has been shown that AUV is able to track a desired moving boundary produced by the union of two or more boundaries. Moreover, an AUV navigates into a specific location on the boundary lines or surfaces while the target itself is moving. The Lyapunov-like function is used to analyze the stability of the controller. Simulation results have been presented to demonstrate the performance of the proposed tracking controller.

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