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Charge imbalance in a non-equilibrium superconductor

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A general formalism for charge imbalance in a non-equilibrium superconductor is obtained using ξ-integrated Green's function, which is applicable at arbitrary temperature, even when the width of quasi-particle states is appreciable due to pair breaking processes and when strong coupling effects are significant.

[Keywords: Superconductor, Green's function, Quasi-particle states]

1 Introduction

One of the important aspects of non-equilibrium superconductivity is the concept of branch imbalance in electron-like and hole-like branches of the excitation spectrum[1]. Since branch imbalance corresponds to net charge in the excitation spectrum, interpretation in terms of charge imbalance is imperative to explain the experimental observations[2]. Pethick & Smith[3] have discussed many aspects of charge imbalance using quasi-particle distribution function for excitation spectrum and its coupling with condensate. This approach is physically appealing, but is limited in its validity, since, it is applicable only if the width of quasi-particle states is small, and pair-breaking effects may be treated as perturbation. To cope with strong pair-breaking, one must resort to a more general approach, and the particular one the author employs is, that based on the quasi-particle Green's function. This is due to Eilenberger[4], Eliashberg[5], Larkin & Ovchinnkov[6], Schimd & Schon[7] and Mattoo[8].

In this paper, the author first reviews briefly, the quasi-particle distribution function approach, for calculating charge imbalance in non-equilibrium superconductor in Sec 2. Then, a general formalism for charge imbalance is obtained in Sec 3, using ξ-integrated Green's function and the results compares with those obtained, using other techniques.

2 Quasi-particle Approach

The quasi-particle formalism is valid provided that, spatial variations are slow on the scale of temperature-dependent coherence length $\xi(T)\sim\hbar v_F/\Delta$. Also, temporal variations are slow on a scale of $\hbar/\Delta$. Here, v_F is the Fermi velocity and Δ the energy gap. Under these circumstances, the internal structure of quasi-particles respond, essentially, instantaneously to the local gap and the state of normal component may be specified in terms of a scalar quasi-particle distribution function. Likewise, the gap responds, essentially, instantaneously to the local quasi-particle distribution, and is therefore given by the BCS equation. The time development of the superconducting component is given by the usual super-fluid acceleration equation, and the development of the quasi-particle distribution function is governed by the quasi-particle Boltzmann equation. The resulting Boltzmann equation closely resembles that for normal metals and super-fluid correlations enter through changes in the quasi-particle dispersion relation and through coherence factors that occur in transitional probabilities.

In the BCS limit, the total charge-density (measured in units of the electronic charge) may be written as:

$$n=\sum_{p\sigma}\left\{u_p^2 f_p+v_p^2(1-f_{-p})\right\} \quad \ldots(1)$$

Here, the summation is over states, normalized to unit volume; f_p the distribution function for Bogoliubov quasi-particles, characterized by momentum p and spin σ; u_p and v_p are the coherence factors defined by:

$$u_p^2=\frac{1}{2}\left\{1+\frac{\xi_p}{E_p}\right\} \quad \ldots(2)$$

$$\text{and } v_p^2 = \frac{1}{2}\left\{1 - \frac{\xi_p}{E_p}\right\} \qquad \ldots(3)$$

The quantity $\xi_p = \varepsilon_p - \mu$ is the normal state quasi-particle energy, ε_p, measured with respect to the chemical potential, μ, and $E_p = \sqrt{\xi_p^2 + \Delta^2}$ is the energy of a quasi-particle in the superconductor, with Δ being the gap parameter.

In Pethick & Smith's approach to charge imbalance[3], one writes variations of n as:

$$\delta n = \delta Q_n + \delta Q_s \qquad \ldots(4)$$

where:

$$\delta Q_n = \sum_{p\sigma} q_p \, \delta f_p \qquad \ldots(5a)$$

is identified as the normal charge density, and:

$$\delta Q_s = \sum_{p\sigma} (1 - 2 f_p) \, \delta v \qquad \ldots(5b)$$

is defined as the charge, associated with condensate.

One can immediately see from Eq. (5) that, addition of a quasi-particle adds a charge to the system. This effective charge depends on quasi-particle energy and is given by:

$$q_p = u_p^2 - v_p^2 = \frac{\xi_p}{E_p} \qquad \ldots(6)$$

The microscopic approach is more closely related to somewhat different decomposition of Eq. (1). As per Kadin *et al.*[9]:

$$n = Q^* + \sum_{p\sigma} v_p^2 \qquad \ldots(7)$$

where:

$$Q^* = \sum_{p\sigma} (u_p^2 - v_p^2) f_p \qquad \ldots(8)$$

An important point is that, u^2 and v^2 in Eq. (8) are not necessarily equal to their values in equilibrium. The quantity Q^* is, therefore, a measure of the amount by which, quasi-particle distribution is asymmetric about the actual Fermi surface, not the Fermi surface in equilibrium. From Eq. (8), one finds for first-order variations:

$$\delta Q^* = \sum_{p\sigma} \left(q_p \delta f_p + f_p \delta f_p \right) \qquad \ldots(9)$$

If one considers only variation of q_p due to variations of μ, one may write:

$$\delta q_p = \frac{\partial q_p}{\partial \mu} \, \delta\mu = -\frac{\partial q_p}{\partial \xi_p} \delta\mu \qquad \ldots(10)$$

Performing a partial summation in the second term of Eq. (9), one then finds:

$$\delta Q^* = \sum_{p\sigma} \left(q_p \delta f_p^{l.e} \right) \qquad \ldots(11)$$

where:

$$\delta f_p^{l.e} = \delta f_p - \frac{\partial f^0}{\partial E_p} \delta E_p \qquad \ldots(12)$$

is the deviation of the distribution function from a Fermi function f^0, evaluated at the actual quasi-particle energy. The shift in the quasi-particle energy δE_p due to the chemical potential shift is given by:

$$\delta E_p = \frac{\partial E_p}{\partial \mu} \delta\mu = -q_p \delta\mu \qquad \ldots(13)$$

The result, Eq. (11) is also obvious from the fact that, to the extent that particle hole symmetry applies:

$$Q^* = 0 \quad if \quad f_p = f^0(E_p) \qquad \ldots(14)$$

and therefore:

$$Q^* = \sum_{p\sigma} q_p \left\{ f_p - f^0(E_p) \right\} \qquad \ldots(15)$$

which for first-order variations is identical with Eq. (13).

Sometimes, it is convenient to work with another definition of the quasi-particle energy, which is negative below the Fermi surface and take:

$$E_p = \sqrt{\xi_p^2 + \Delta^2} \; Sgn \; \xi_p \qquad \ldots(16)$$

The distribution function $\tilde{f}_p$ of these quasi-particles is defined by:

$$\tilde{f}_p = 1 - f_{-p} \qquad \ldots(17)$$

below the Fermi surface and:

$$\tilde{f}_p = f_p \qquad \ldots(18)$$

above the Fermi surface.

Therefore, the charge imbalance is given by:

$$\tilde{Q}^* = \sum_{p\sigma} | q_p | \delta f_p^{l.e} \quad \ldots(19)$$

3 Green Function Approach

Green's function formulation is capable of describing microscopic phenomena and allows one to take into account features of real metals, such as, the details of phonon spectrum, strong-coupling effects and the effects of magnetic and non-magnetic impurities. In the BCS limit, the Green's function, particularly, its Keldysh component is given by:

$$G^k(P,E) = \frac{\pi}{i}\left\{u_p^2 f_p \delta(E - E_p) + v_p^2(1 - f_{-p})\delta(E + E_p)\right\} \quad \ldots(20)$$

(Here the author does not consider the full matrix Green's function, but only its (1,1) component, which corresponds to the definition of the Green's function for the normal system). For simplicity, the author considers the pairing to be between states of momentum P and $-P$, which corresponds to working in a frame moving with super-fluid.

It is more convenient to work, not with the full Green's function but, rather with ξ-integrated function, g, for which transport equations have been derived by the author[8], directly applicable to strong-coupling superconductors, at arbitrary temperature. These give information only about deviations from equilibrium of those parts of G^k, which involves the quasi-particle distribution function, and other contributions to G^k must be determined by use of conservation laws. Thus one can have:

$$g^k(P,E) = \delta \int_{-\infty}^{\infty} d\xi \left\{u_p^2 f_p \delta(E - E_p) - v_p^2(1 - f_{-p})\delta(E + E_p)\right\}$$

$$= \int_{-\infty}^{\infty} d\xi \left\{u_p^2 \delta f_p^{l.e} \delta(E - E_p) - v_p^2 \delta f_{-p}^{l.e} \delta(E + E_p)\right\} \ldots(21)$$

The latter form follows, because, by virtue of the delta functions, one may write, u^2 and v^2 as a function of E alone $\left\{\frac{1}{2} 1 \pm (\sqrt{E^2 - \Delta^2} / E)\right\}$, and therefore, variations of u^2 and v^2 due to variations of ξ do not need to be taken into account. One may, therefore, write:

$$Q^* = 2N(0) \int_{-\infty}^{\infty} dE \int \frac{d\Omega_p^{\wedge}}{4\pi} g^k(P^{\wedge}, E) \quad \ldots(22)$$

where $N(0) = \sum_p \delta(\varepsilon_p - \mu)$ is the density of states at the Fermi surface for the normal metal for a single spin population and $\int_{-\infty}^{\infty} dE \int \frac{d\Omega_p^{\wedge}}{4\pi} g^k(P^{\wedge}, E)$ is the average value of g^k with $P^{\wedge}$ being the direction of momentum variable.

For quasi-particle having energy greater than Δ, it is convenient to work with the definition of quasi-particle energy given by Eq. (16), which is negative below the Fermi surface and positive above the Fermi surface. Thus, if one defines:

$$\delta f_p^{l.e} = 0 \text{ for } |E| < \Delta$$

and

$$\delta f_p^{l.e} = \delta f_p^{l.e}(P^{\wedge}, \pm) \text{ for } \xi_p = \pm\sqrt{E^2 - \Delta^2} \quad \ldots(23)$$

we find:

$$g^k(P^{\wedge} - E) = \frac{E}{\sqrt{E^2 - \Delta^2}}\left(\frac{1}{2}\left\{\delta f^{l.e}(P^{\wedge},+) + \delta f^{l.e}(P^{\wedge},-)\right.\right.$$
$$\left. + \frac{\sqrt{E^2 - \Delta^2}}{|E|}\left(\delta f^{l.e}(P^{\wedge},+) - \delta f^{l.e}(P^{\wedge},-)\right)\right\}\Theta(E)$$
$$-\left\{\frac{1}{2}\left(\delta f^{l.e}(-P^{\wedge},+) + \delta f^{l.e}(-P^{\wedge},-)\right)\right.$$
$$\left.\left.\frac{\sqrt{E^2 - \Delta^2}}{|E|}\left(\delta f^{l.e}(-P^{\wedge},+) - \delta f^{l.e}(-P^{\wedge},-)\right)\right\}\Theta(E)\right) \quad \ldots(24)$$

If g^k is isotropic, this reduces to:

$$g^k(P^{\wedge} - E) = \frac{E}{\sqrt{E^2 - \Delta^2}}\left(\frac{1}{2}\left\{\delta f^{l.e}(P^{\wedge},+) + \delta f^{l.e}(P^{\wedge},-)\right\}\right.$$
$$\left\{\Theta(E) - \Theta(-E)\right\}$$
$$\left. + \frac{1}{2}\frac{\sqrt{E^2 - \Delta^2}}{|E|}\left\{\delta f^{l.e}(P^{\wedge},+) - \delta f^{l.e}(P^{\wedge},-)\right\}\right\} \quad \ldots(25)$$

It is, therefore, seen that, the charge imbalance Q^* involves the part of g^k even in E, which is the part of $\delta f_p^{l.e}$, odd about the Fermi surface, and part of $\delta f_p^{l.c}$ even about the Fermi surface.

For a weak-coupling superconductor, where the quasi-particles have definite energy and momentum, Schmid & Schon[7] derived an expression for charge imbalance using generalized form of Eilenberger[4] equations introduced by Larkin & Ovchinnkov[6]:

$$Q^* = 2N(0)\int_{-\infty}^{\infty} \mathrm{d}E \int \frac{\mathrm{d}\Omega_p^{\wedge}}{4\pi} N_1(E) f^T \qquad \ldots(26)$$

It is to be noted that, Schmid & Schon's tranverse distribution function f^T is just the term in square brackets of Eq. (25) even in E:

$$f^T = \frac{1}{2}\frac{\sqrt{E^2-\Delta^2}}{|E|}\left\{\delta f^{l.e}(P^{\wedge},+) - \delta f^{l.e}(P^{\wedge},-)\right\} \quad \ldots(27)$$

Thus, for a weak-coupling case and in the BCS limit, where:

$$N_1(E) = \left\{\frac{|E|}{\sqrt{E^2-\Delta^2}}\right\}\Theta(|E|-\Delta) \qquad \ldots(28)$$

Eq. (22) is equivalent to Schmid & Schon's expression.

In the limit when the width of quasi-particle state is small, Eq. (27) corresponds to a distribution function for Bogoliubov quasi-particles times effective charge leading to an expression for charge imbalance given by Eq. (19).

4 Conclusion

In this paper, the author discusses expressions for the charge imbalance in the quasi-particle and ξ–integrated Green's function approach, respectively. The ξ–integrated Green's function formalism is a more general one, as it is applicable at arbitrary temperatures and when strong coupling effects are significant. For weak-coupling effects, the derived formalism is shown to be equivalent to that of the Schmid & Schon's expression. In the limit of small pair-breaking and for weak-coupling, the formalism reduces to that in quasi-particle approach.

References

1 Tinkam M, *Phys Rev B*, 6 (1972) 1747.

2 Clarke J, *Phys Rev Lett*, 28 (1972) 1363; Clarke J & Patternson J L, *J Low Temp Phys*, 15 (1974) 491.

3 Pethick C & Smith H, *Ann Phys*, 119 (1979) 113.

4 Elienberger G, *Z Phys*, 214 (1968) 195.

5 Eliashberg G M, *JETP*, 34 (1972) 668.

6 Larkin A I & Ovenhinnikov Yu N, *JETP*, 28 (1969) 2000; 46 (1977) 155.

7 Schmid A & Schon G, *Low Temp Phys*, 20 (1975) 207.

8 Mattoo B A, *Indian J Pure & Appl Phys*, 20 (2002) 402.

9 Kadin A M, Smith L N & Skopol W J, *Low Temp Phys*, 38 (1980) 497.