Signature of void probability scaling in high energy hadronic interactions

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The rapidity gap probability in the p-AgBr interactions at 200 GeV/c and π-AgBr interactions at 200 and 350 GeV/c have been studied in detail. The data indicate scaling behaviour of the void probability in the central rapidity domain, which hints the validity of the linked-pair approximation for the N-particle cumulant correlation function.

[Keywords: Hadronic interactions, Void probability, Rapidity gap probability]

1 Introduction

Over the past few years, some physicists have brought into focus their interesting observation that, the frequency distribution of galaxy counts exhibits some common features with the multiplicity distribution of charged hadrons produced at high energy collisions available in contemporary accelerators. It is a well-known fact that, both galaxy counts and multiplicity distribution have been successfully described by the negative binomial distribution as well as, log normal distribution. It has further been observed that, the galaxy counts in space and rapidity distribution of produced hadrons have scaled invariant self-similar pattern, which seems to be an intrinsic feature of both phenomena. All these facts and closed correspondences between two fields have provoked the authors to study further, the similarity between the multi-hadron distribution and galaxy counts. The real question is, where new physics is implied by this phenomenon. Experimental results have inspired the physicists to think that, there is connection between these two fields, sometimes called the inner space-outer space connection1.

The investigational work of Carruthers & Sarcevic2 on inner-outter space phenomenology developed the idea that, the higher order cumulant correlation functions may be built out of two particle cumulant correlation functions which leads to confirm the structural similarity of the multi-hadron and galaxy N-body correlation3,4. They2 have also proposed that, the higher order cumulant function composed of two-particle cumulant correlation is defined by:

\[ C_2(\eta_1, \eta_2) = \rho_1(\eta_1) \rho_2(\eta_2) - \rho_1(\eta_1) \rho_1(\eta_2) \] (1)

where \( \rho_1 \) and \( \rho_2 \) are the inclusive single and two-particle densities.

According to the linking scheme, for N-particle system, the higher order, \( C_N \) which can be composed in a similar way, is found to be proportional to the product of (N-1) two particle reduced cumulants summed over all permutations, defined as:

\[ C_N(\eta_1, \eta_2, ..., \eta_N) = A_N \sum_{\text{Perm}} \prod_{i=1}^{N-1} C_i(\eta_i, \eta_j) \] (2)

where \( A_N \) is the dimensionless coefficient to be so-called, correlation amplitude of order N. Peebles et al.5,6 while investigating in the field of two-dimensional galaxy catalogs, invented the above scheme and have proved this to be successful in describing galaxy-galaxy correlation. Mandelbort also independently proposed the same structure for galaxy correlation functions on the basis of theoretical approach. Numerous hierarchical models in the astrophysical literature exist with this linking scheme for galaxy correlation functions. In multi-particle phenomenology, this ansatz for the N-particle cumulant correlation functions acquired the name-linked pair approximation.

Hegyi’s recent investigation7 which is based on the rapidity gap probability \( P_a(\Delta \eta) \), that measures
the chance of finding no particle in the rapidity interval $\Delta \eta$, used UA5 data in $p\bar{p}$ collisions\textsuperscript{9,10}, establishes that a scaling behaviour predicted by hierarchical models with linked pair approximation in the rapidity gap probability has a close correspondence with the scaling of a void probability in galaxy clustering studies\textsuperscript{9,10}. The similarities in the two scaling behaviours give us an indication of the presence of a common pattern in the dynamics underlying multi-particle production and galaxy clustering.

2 Theory

In this paper, a similar analysis is presented for the first time of hadron-nucleus collisions. Specifically, the data sets of $p$-AgBr interactions at 200 GeV/c and the $\pi$-AgBr interactions at 200 and 350 GeV/c, respectively, have been used here to study this analysis. The main aim in this paper is to test the validity of the scaling of the void probability and the linked-pair approximation to higher-order cumulant correlation, based on the pseudo-rapidity gap probability. Hegyi\textsuperscript{9} is the first to start with the probability generating functions $Q(\lambda)$, for $P_n(\Delta\eta)$, defined as:

$$Q(\lambda) = \sum_{n=0}^{\infty} (1-\lambda)^n P_n(\Delta\eta)$$ \hspace{1cm} (3)

where $P_n(\Delta\eta)$ is the probability of detecting exactly $n$ particles in $\Delta\eta$. Mueller\textsuperscript{11} and White\textsuperscript{10} expressed $Q(\lambda)$ in terms of integrals of the $N$-particle correlation function, as given below:

$$Q(\lambda) = \exp \left( -\sum_{n=1}^{\infty} \frac{(-\Delta\eta)^n}{n!} \Delta\eta \cdots \Delta\eta \right)$$ \hspace{1cm} (4)

where $\bar{n} = \frac{\Delta\eta}{N!} \Delta\eta$ is the average number of particles in $\Delta\eta$ and $N$-fold integral of $C_N$ over an $N$-cube of side length $\Delta\eta$ defined the $N$th reduced factorial cumulant moment $\bar{K}_N$ for bin $\Delta\eta$. Now, the probability of detecting no particle in $\Delta\eta$ is related to the generating function through:

$$P_e(\Delta\eta) = Q(\lambda = 1)$$ \hspace{1cm} (5)

which can be acted\textsuperscript{13} as a generating function for $P_o(\Delta\eta)$.

$$P_o(\Delta\eta) = \left( \frac{\bar{n}}{n!} \right) \left( \frac{\Delta\eta}{\bar{K}_N} \right)^n P_n(\Delta\eta)$$ \hspace{1cm} (6)

Here, the differentiation is carried out with the correlation functions, held fixed. This important feature of hole probability was stressed by White\textsuperscript{12} and discussed by Balian & Schuefler\textsuperscript{13}. The cumulant expression of the logarithm of $P_n(\Delta\eta)$ is:

$$\ln P_n(\Delta\eta) = \sum_{n=1}^{\infty} \frac{(-\bar{n})^n}{n!} \frac{\Delta\eta}{K_N}$$ \hspace{1cm} (7)

where $P_n(\Delta\eta)$ is symmetrically dependent on the entire hierarchy of correlation functions. If the correlation functions satisfy the linked-pair ansatz, the reduced factorial moments take the form\textsuperscript{14}:

$$\bar{K}_N = A_N \bar{K}_2^{N-1}$$ \hspace{1cm} (8)

The scale rapidity gap probability is defined as:

$$\chi = -\ln \frac{P_n(\Delta\eta)}{\bar{n}}$$ \hspace{1cm} (9)

If $A_N$ linking coefficient becomes independent of collision energy and rapidity bin size\textsuperscript{9,13}, $\chi$ depends only on the single momentum combination $nK_2$, so that we can write:

$$\chi = \sum_{n=1}^{\infty} \frac{1}{N!} A_N (-\bar{n}K_2)^{N-1} = \chi(\bar{n}K_2)$$ \hspace{1cm} (10)

Thus, $\chi$ gives the measure of no particle probability by normalizing out the contribution from uncorrelated particle emission\textsuperscript{16,17}. So, $\chi<1$ deals only with the clustering properties of secondaries involving correlation of all orders. This scaling feature was derived by White\textsuperscript{12} and the recently available observational data for scaled void probability data exhibit the hierarchical scaling. Two hierarchical models shall be considered to compare $\chi_e$ in galaxy clustering studies in which, the reduced factorial cumulants $\bar{K}_N$ satisfy Eq. (8). Negative binomial model is one of them with linking coefficients $A_N = (N-1)!$, where the scaled rapidity gap probability is given by:

$$\chi = \ln \frac{1 + \bar{n}K_2}{n\bar{K}_2}$$ \hspace{1cm} (11)

The other minimal model known as the hierarchical Poisson model where $A_N=1$ for all $N$, is defined as:

$$\chi = [1 - \exp (-\bar{n}K_2)]/n\bar{K}_2$$ \hspace{1cm} (12)

Since $A_N\rightarrow A_1$, are somewhat smaller in strength than predicted, by the negative binomial model\textsuperscript{13,16}.

Eqs (11) and (12) are expected to provide the upper and lower bounds for $\chi_e$. 
Fig. 1 — Experimentally obtained values for the scaled rapidity gap probability $\chi^{\text{th}}$ for $\pi$-AgBr interactions at 350 GeV/c. Data are compared with negative binomial model shown as dashed line and with the minimal model as shown by the solid line.

Fig. 2 — Experimentally obtained values for the scaled rapidity gap probability $\chi^{\text{th}}$ for $\pi$-AgBr interactions at 200 GeV/c. Data are compared with negative binomial model shown as dashed line and with the minimal model as shown by the solid line.

Fig. 3 — Experimentally obtained values for the scaled rapidity gap probability $\chi^{\text{th}}$ for $p$-AgBr interactions at 200 GeV/c. Data are compared with negative binomial model shown as dashed line and with the minimal model as shown by the solid line.

3 Results and Discussion

The charged particle multiplicity distributions are obtained from the hadron-nucleus interaction data of $p$-AgBr at 200, and $\pi$-AgBr at 200 and 350 GeV/c incident energies. The stack of G5 nuclear emulsion plates were exposed horizontally to a $\pi$ beam at CERN with 350 GeV/c and to proton and $\pi$ beam at Fermilab with 200 GeV/c incident energies. Details of the experimental set-up is described earlier. The sample of data is made through two sub-groups characterized by the number of heavy prongs ($N_h$). The grouping is usually done to distinguish between the interactions with light CNO ($2 \leq N_h < 8$) and heavy AgBr ($N_h \geq 8$) emulsion nuclei. Elastic and coherently produced events are excluded from the analysis. Pseudo-rapidity ($\eta$) at all shower particles were determined from the measured space angle ($\theta$) with reference to the beam by the relation:

$$n = -\ln \tan \theta/2$$

Rapidity gap probability $P_{r}(\Delta\eta)$ have been found. Initially, a bin centre ($\eta_r$) at $O$ is taken and found the rapidity gap probability for a particular
pseudo-rapidity size. Then, the centre had been shifted in steps of 0.5 from 0 to 6 for each of the window sizes \( \Delta \eta = 0.5, 1, 1.5, 2, 2.5, 3 \) and \( P_T(\Delta \eta) \) calculated for each case, separately. Scaled rapidity gap probability \( (\chi^m) \) and single momentum combination \( (n K) \) have been calculated for each case, separately.

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From the figures it is seen that, although not all experimental points fall inside the region, bounded by the dashed and solid lines in all cases, i.e. \( p-AgBr \) interactions at 200 GeV/c and \( \pi-AgBr \) interactions at 200 and 350 GeV/c, if errors of the experimental points are considered (not shown in the figure for clarity) more points may be considered to be within the two predicted, bounded lines. Thus, these analyses may possibly indicate the existence of void probability scaling in hadron-nucleus interactions. One should note more analyses with other data as increased statistics is essential for arriving at a confident consideration.

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References