Effect of doping on stimulated Brillouin scattering in piezoelectric magnetized III-V semiconductors

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Received 15 October 2006; accepted 17 April 2007

Using the hydrodynamic model of a semiconductor-plasma and following the coupled mode approach, an analytical investigation has been made for stimulated Brillouin scattering (SBS) resulting from the nonlinear interaction of an intense pump beam with acoustic perturbations internally generated due to piezoelectric and electrostrictive property of the crystal. The analysis deals with the qualitative behaviour of threshold pump electric field for the onset of SBS and resulting gain coefficient with respect to doping concentration (through plasma frequency) in piezoelectric III-V semiconductors subjected to transverse magnetostatic field. The numerical estimates are made for \( n \)-type InSb crystal at 77 K duly shined by pulsed 10.6 \( \mu \)m CO\(_2\) laser. The threshold value of pump field required for the onset of SBS is found to lower in the presence of magnetostatic field. At resonance, the Brillouin gain is two hundred and 10\(^4\) times in the presence of piezoelectricity and magnetostatic field, respectively, than in their absence. Moreover, the SBS gain coefficient increases with increasing in scattering angle and results in maximum value for the backscattered mode. The backward Brillouin gain is found to be 10\(^4\) times larger than the forward gain. The analysis also suggests the possibility of observing phase conjugation reflectivity as high as \(~10^6\) in weakly piezoelectric doped III-V semiconductors subjected to a magnetic field.

Keywords: Stimulated Brillouin scattering, Doping, Piezoelectricity, Magnetic field, III-V semiconductors
IPC Code: G02F1/35

1 Introduction

Stimulated Brillouin scattering (SBS) discovered by Chiao \textit{et al.}\(^1\), is a third-order nonlinear optical phenomenon caused by third-order optical phase-coherent interaction of an intense coherent pump beam with acoustic waves in an active medium demonstrating a large amplification of the scattered wave at Stokes shifted frequency. The study of SBS is an interesting field of research due to its manifold technological applications in the area of modern optics such as amplifier isolation, pre-pulse suppression, pulse squeezing, laser induced fusion, real time holography and optical phase conjugation\(^2\,\,\,3\). Among the various nonlinear optical effects responsible for the occurrence of optical phase conjugation (OPC), SBS is regarded to be of definite advantage because of its high conversion efficiency\(^8\). Il’ichev\(^9\) has reported that the OPC-SBS conversion efficiency can be varied as a function of parameters of the scattering medium. The Brillouin nonlinearity can also produce efficient four-wave mixing and yield reflection coefficients\(^10\) approaching 10\(^6\).

SBS has been mostly studied in media such as liquids and plasma\(^11\), in which electrostrictive phenomena are being taken as the origin of this process. A competition between forward, side, and backward Brillouin scattering of the finite size laser beam has been discussed in a two dimensional plasma of planar geometry\(^12\). The review work on the early research activities on SBS in solids was made by Starunov and Fabelinskii\(^13\) and by Fabelinskii\(^14\), respectively. Aghamkar and Sen\(^15\) studied SBS in doped semiconductors which exhibit centre of symmetry. However, in weakly piezoelectric semiconductors, a low effective mass allows one to work at drift velocities many times than the sound velocity, which leads to considerable reduction of effects of inhomogeneity. It is suggested\(^16\) that in piezoelectric semiconducting crystals, magnitude of free carrier absorption is quite small and it may be neglected. The effect of piezoelectricity on degenerate four-wave mixing has been investigated\(^17\) and found that the piezoelectricity considerably enhances the magnitude of third-order optical susceptibility. Apart
from the effect of crystal properties on optical nonlinearity, externally applied electric and magnetic fields play crucial role in enhancing optical nonlinearity of the medium. Recently, an external magnetic field has been used to generate surface-field terahertz (THz) emission\textsuperscript{18} and its enhancement in III-V semiconductors\textsuperscript{19}.

The effect of free carrier concentration on SBS in III-V semiconductors to (i) minimize the threshold field for the onset of SBS process and (ii) enhance the effective Brillouin gain and the phase conjugation reflectivity well above the threshold has been studied in the present paper.

The weakly piezoelectric \textit{n}-type III-V semiconducting crystal such as \textit{n}-InSb, \textit{n}-GaAs, etc is considered as Brillouin sample which is subjected to a pump wave. Further, the pump photon energy $\hbar \omega_p$ is taken well below the band-gap energy $\hbar \omega_e$ of the sample. This choice allows the optical properties of the sample to be influenced considerably by free carriers and to remain unaffected by the photo-induced interband transition mechanisms. The third order nonlinear optical susceptibility $\chi^{(3)}$ responsible for the occurrence of SBS process has been obtained by following the coupled-mode approach. Numerical estimates have been made for an \textit{n}-type InSb crystal at 77 K duly irradiated by few nano second pulsed 10.6 $\mu$m CO$_2$ laser.

2 Theory

The theoretical formulations of complex effective optical Brillouin susceptibility and consequent gain coefficient for the Stokes component of scattered electromagnetic wave in weakly piezoelectric doped III-V semiconductors subjected to a magnetostatic field have been studied. SBS occurs due to parametric coupling among three waves which in the present scheme are: (i) the input pump beam of field strength $E_0(x,t) = E_0 \exp[\ii (k_0 x - \omega_0 t)]$, (ii) the idler acoustic phonon modes $u(x,t) = u_0 \exp[\ii (k_x x - \omega_0 t)]$ and (iii) the scattered Stokes component of pump electromagnetic field of wave strength $E_s(x,t) = E_s \exp[\ii (k_x x - \omega_0 t)]$. We employ the well known hydrodynamic model (for which $k_j < \ll 1$, $k_x$ and $l$ being the acoustic wave vector and mean free path of an electron, respectively) of homogeneous one-component (electron) plasma subjected to an electromagnetic pump field under thermal equilibrium. The time varying pump field exerts force on piezoelectric crystal which results in piezoelectric and electrostrictive strain and is, thus, capable of driving acoustic waves in the medium. The induced acoustic wave modulates the optical dielectric constant and thus, can cause an exchange of energy between electromagnetic waves whose frequencies differ by an amount equal to acoustic wave frequency. Let the deviation of a point $x$ from its equilibrium position be $u(x,t)$, so that the one dimensional strain is $\|u/\|x$. The force along $x$-direction acting on a unit volume of the crystal due to piezoelectricity and electrostriction can be expressed as $\beta \|E \|x^*$ and $\gamma \|E \|x^*$, respectively. $\beta$ and $\gamma$ are the piezoelectric and electrostriction coefficients of the medium.

The equation of motion for $u(x,t)$ of lattice vibrations due to process of piezoelectricity and electrostriction in the presence of magnetostatic field may be given as:

$$\rho \frac{\dd^2 u}{\dd t^2} = \frac{\beta}{\|x^2} \|E \|x^* + \frac{\gamma}{2\|x^2} (\|E \|x^*)^2 - 2\Gamma_u \frac{\dd u}{\dd t} \ldots (1)$$

where $\|E = \|E_0 + \|E_0 + \|B_0$, $\|v_0$ being the oscillatory fluid velocity of an electron of effective mass $m$ and charge $e$ at pump frequency $\omega_0$. $\|B_0$ is an external magnetostatic field which is applied perpendicular to the direction of the pump wave. $\rho$, $C$ and $\Gamma_u$ being the mass density, elastic constant and phenomenological damping parameter of the medium, respectively. In this paper the asterisk denotes the conjugate of a complex entity. The other basic equations of present formulation are:

$$\frac{\dd v_0}{\dd t} + \|v_0 = \frac{e}{m} \|E_0 + \|v_0 + \|B_0 \ldots (2)$$

$$\frac{\dd v_1}{\dd t} + \|v_1 + \|v_0 = \frac{e}{m} (\|E_1 + \|v_1 + \|B_0 \ldots (3)$$

$$\frac{\dd n_1}{\dd t} + n_0 \|v_1 + n_1 \|v_0 + \|v_0 \|n_0 = 0 \ldots (4)$$

$$\frac{\dd E_1}{\dd x} = \frac{n_1 e}{\varepsilon} - \frac{\beta}{\varepsilon} \|E \|x^* + \frac{\gamma}{\varepsilon} (\|E \|x^*)^2 \ldots (6)$$
Eqs (2) and (3) are the zeroth- and first-order electron moment transfer equations under the influence of magnetostatic field, \( v_1 \) being the perturbed oscillatory fluid velocity of an electron. \( v \) is the electron collision frequency. Eq. (4) is the continuity equation where \( n_0 \) and \( n_1 \) are the equilibrium and perturbed carrier densities, respectively. Eq. (5) describes electrostrictive polarization in the presence of an external magnetic field. The induced space charge field \( E_1 \) due to nonlinear interaction is determined by Poisson’s Eq. (6) where \( \varepsilon \) is dielectric function of the medium and varies as \( \exp[i(k_a x - \omega_0 t)] \). As we have considered the resonant sideband frequencies \( (\omega_0 \pm p \omega_a) \), where \( p \) is an integer) by assuming the interaction path as sufficiently long so that the higher order components (for \( p \leq 2 \)) are neglected under off resonant conditions and only the first order Stokes component (with \( p = 1 \)) has been considered.

The phase matching conditions which are to be satisfied in the present case, are as follows:

\[
h\omega_0 = h\omega_a + h\omega_s \quad \ldots (8a)
\]

\[
hk_0 = hk_s + hk_a \quad \ldots (8b)
\]

where the frequencies \( \{\omega_0, \omega_a, \omega_s\} \) and wave vectors \( \{k_0, k_a, k_s\} \) correspond to the incident laser, scattered Stokes and acoustic waves, respectively. The scattering geometry is shown in Fig. 1, where \( \theta \) is the scattering angle. The relation between three wave vectors is given by \( k_s = \sqrt{k_0^2 + k_a^2 - 2k_0k_a \cos \theta} \). In principle, SBS can occur in any direction provided that energy and momentum conservation given in Eq. (8) is satisfied.

We may obtain the following coupled wave equations under rotating-wave approximation (RWA):

\[
\frac{\partial^2 n_0}{\partial t^2} + \nabla \nabla n_0 + \Omega^2 n_0 \Delta_i - \frac{i\beta\gamma k_a^2 \Omega^2 n_0 \Delta_i}{2e\Delta_a} \frac{\partial}{\partial \theta} E_0 E^* \quad \ldots (9a)
\]

\[
\frac{\partial^2 n_1}{\partial t^2} + \nabla \nabla n_1 + \Omega^2 n_1 \Delta_i - \frac{i\beta\gamma k_a^2 \Omega^2 n_0 \Delta_i}{2e\Delta_a} \frac{\partial}{\partial \theta} E_0 E^* \quad \ldots (9b)
\]
Thus, the slow and fast components of the density perturbations are coupled to each other via the pump electric field.

For a medium whose Brillouin effect is dominated by piezoelectricity and electrostriction, such a complex absorption-attenuation coefficient for the Stokes component of the signal is obtained by the solution of the coupled wave equations, from Eqs (9a) and (9b) as:

\[ n_{s}^{*} = \frac{c\delta_{0}\delta_{1}\Omega_{s}^{2}k_{s}^{2}}{2\eta\rho\Delta_{s}} \left[ \beta\gamma(m/e) + \gamma^{2}k_{a}\right] E_{0}|E_{s}|^{2} \]

\[ \chi_{B_{,cd}}(\omega) = - \frac{\gamma^{2}k_{a}^{2}}{2\rho\epsilon_{0}\epsilon_{r}^{2}} \left[ \frac{\Delta_{s}{\omega_{s}}}{\Delta_{s} + \omega_{s}^{2}} \right] \] (10)

where \( \Delta_{sp}^{2} = \Omega_{s}^{2}\delta_{1} - \omega_{s}^{2} - i\omega_{s} \), \( \Delta = \eta\epsilon_{0}\Delta_{p}^{2}\Delta_{s}^{2} \), and

\[ \Delta_{sp}^{2} = \Omega_{s}^{2}\delta_{0} - \omega_{s}^{2} - \frac{2i\gamma\epsilon_{0}}{\rho} \] (with \( c \) and \( \eta = \sqrt{\epsilon_{r}} \)

being the absolute velocity of light and background refractive index of crystal medium).

The Stokes component of the nonlinear current density due to finite nonlinear polarization of the medium has been deduced neglecting the transition dipole moment and represented as:

\[ J_{NL}(\omega) = n_{s}^{*}e\nu_{0} \]

The time integral of Eq. (11) yields the expression for nonlinear induced polarization due to the perturbed carrier density as:

\[ P_{cd}(\omega) = \int J_{NL}(\omega)dt \]

\[ \chi_{B_{,cd}}(\omega) = - \frac{ce^{2}\delta_{0}\delta_{1}\Omega_{s}^{2}k_{s}^{2}}{2\eta\epsilon_{0}\epsilon_{r}^{2}m^{2}\Delta_{a}^{2}(\Delta_{s} + \omega_{s}^{2})\Delta_{sp}^{2}} \]

\[ \times \beta\gamma(m/e)\Delta_{p}^{2} - 2\gamma^{2}k_{a}\Delta_{p}^{2} \] (12)

The electrostrictive effect gives rise to an additional sound wave, which scatters the incident pump wave at a frequency corresponding to the material excitation of the medium. It is this electrostrictive polarization that couples the scattered wave and the sound wave in the presence of an intense pump wave. This results in transfer of energy to the Stokes signal wave, which is subsequently amplified at the expense of the pump and the acoustic waves. However, we neglect the pump depletion in the present analysis. The complex Brillouin susceptibility at Stokes frequency due to process of electrostriction in a magnetic field is obtained as:

\[ \chi_{B_{,cd}}(\omega) = \chi_{B_{,cd}}(\omega) + \chi_{B_{,cd}}(\omega) \]

\[ \chi_{B_{,eff}}(\omega) = \chi_{B_{,cd}}(\omega) + \chi_{B_{,cd}}(\omega) \]

In order to obtain the effective Brillouin gain coefficient \( g_{B}(\omega) \), we employ the relation:

\[ g_{B}(\omega) = - \frac{k_{a}}{2\epsilon_{0}} \chi_{B_{,eff}}(\omega) \left| E_{0} \right|^{2} \]

\[ \chi_{B_{,eff}}(\omega) = \chi_{B_{,eff}}(\omega) + \chi_{B_{,eff}}(\omega) \]

where

\[ \chi_{B_{,eff}}(\omega) = \chi_{B_{,eff}}(\omega) + \chi_{B_{,eff}}(\omega) \]

The threshold value of pump field \( E_{0, th} \) required for the onset of SBS is obtained by setting \( \chi_{B_{,imag}} = 0 \).

3 Results and Discussion

The present theoretical formulations are analyzed to study the dependence of threshold and gain coefficient for the onset of SBS on free carrier concentration in transverse magnetostatic field. For this purpose, we have chosen III-V semiconducting crystal, viz., InSb as the Brillouin active medium. In order to make the estimation compatible with requirements like off-resonant laser excitation, we consider the irradiation of InSb at 77 K by 10.6 \( \mu \)m CO_{2} laser. The material constants are as follows:

\[ \omega_{0} = 1.78 \times 10^{14} \text{s}^{-1}, \quad m = 0.014 \quad m_{0}, \quad m_{0} \quad \text{the rest mass of an electron,} \quad \epsilon_{r} = 15.8, \quad \rho = 5.8 \times 10^{3} \text{kgm}^{-3}, \]

\[ \beta = 0.054 \text{cm}^{-2}, \quad \gamma = 5 \times 10^{-10} \text{MKS Units,} \quad v = 3 \times 10^{11} \text{s}^{-1}, \quad \omega_{p} = 2 \times 10^{11} \text{s}^{-1}, \quad \Gamma_{a} = 2 \times 10^{10} \text{s}^{-1}, \]

\[ \eta = 4, \quad v_{u} = 4 \times 10^{5} \text{m s}^{-1}. \]
3.1 Threshold characteristics

In order to study the effect of doping on threshold condition of SBS, in Fig. 2, we have plotted $E_{0,th}$ as a function of plasma frequency $\Omega_p$ in the presence and absence of an external magnetostatic field $B_0$. Curves “a” and “b” represent the features of $E_{0,th}$ for $B_0 = 0$ and $B_0 = 10^4$ T, respectively. In the absence of magnetostatic field, $E_{0,th}$ has a gradual linear increase with plasma frequency. However, in the presence of magnetostatic field, one observed a critical dependence of $E_{0,th}$ on plasma frequency when $\Omega_p$ is almost in resonance with signal frequency, $\omega_s$, and

$$E_{0,th} = \frac{\omega_s}{\omega_p} \frac{\omega_p^{1/2}}{\omega_s^{1/2}} - \frac{\omega_s^{1/2}}{\omega_p^{1/2}}$$

becomes minimum. With further increase in doping level, the departure from resonance causes a rapid increase in $E_{0,th}$ saturating at a considerable large value. This figure leads one to infer that considerable reduction in $E_{0,th}$ (from $10^6$ to $5 \times 10^4 \text{Vm}^{-1}$) can be achieved by proper tuning of plasma frequency ($\approx 4.9 \times 10^{12}$ to $5.8 \times 10^2 \text{s}^{-1}$).

3.2 Brillouin gain

We now concentrate our attention to the quantitative analysis of gain constant $g_B$ associated with the SBS process as given by Eq. (16). One has to consider excitation field well above the threshold value, i.e. $E_0 \gg E_{0,th}$. In our further analysis, we have taken $E_0 = 8 \times 10^6 \text{Vm}^{-1}$.

Figure 3 exhibits the qualitative behaviour of effective backscattered ($\theta = 180^\circ$) Brillouin gain $g_B$ as a function of plasma frequency ($\Omega_p$) corresponding to the transverse magnetostatic field ($B_0 = 10^4$ T) with finite piezoelectricity ($\beta^1 0$). It has been observed that, initially $g_B$ is very small, increases sharply with plasma frequency to a peak value $\approx 36 \text{m}^{-1}$ at $\Omega_p \approx 5.3 \times 10^{12} \text{sec}^{-1}$ and decreases with further increase in $\Omega_p$. This typical behaviour of gain coefficient arises due to resonance condition. In Fig. 3, the inset plot ‘I’ shows the nature of dependence of $g_B$ on $\Omega_p$ in the absence of magnetostatic field ($B_0 = 0$ T) with finite piezoelectricity ($\beta^1 0$) in the same crystal. It can be noticed that the presence of magnetostatic field enhances $g_B$ by a factor of two near resonance. Inset plot ‘II’ in Fig. 3 represents the features of Brillouin gain at $B_0 = 10^4$ T but in absence of piezoelectricity ($\beta = 0$). One can notice that in presence of piezoelectricity the Brillouin gain is found to be 200 times than in its absence and leads to infer that piezoelectric doped n-type semiconductors in the presence of magnetostatic field are potential candidate materials to observe SBS with large Brillouin gain for proper selection of plasma frequency.

The nature of dependence of $g_B$ on scattering angle $\theta$ for plasma frequencies $\Omega_p \approx 1.2 \times 10^{13} \text{sec}^{-1}$ (the dotted line) and $\Omega_p \approx 5.3 \times 10^{12} \text{sec}^{-1}$ (the solid line) in the presence of magnetostatic field ($B_0 = 10^4$ T) is plotted in Fig. 4. It is found that for any particular value of $\Omega_p$, $g_B$ is quite small in the forward scattering...
direction (i.e. $\theta = 0^\circ$). However, Stokes mode grows exponentially between $\theta_0 < \theta < 50^\circ$ and then it increases gradually and becomes independent of scattering angle. A comparison between forward ($\theta = 0^\circ$) and backward ($\theta = 180^\circ$) Brillouin scatterings manifest that the backward Brillouin gain is nearly $10^4$ times the forward gain. It is also observed that the Brillouin gain is 100 times larger for $\Omega_p \approx 5.3 \times 10^{12}$ s$^{-1}$ than for $\Omega_p \approx 1.2 \times 10^{13}$ s$^{-1}$.

We now explore the possibility of the occurrence of optical phase conjugation induced by SBS, using the results of the above formulation of Brillouin growth and nonlinear susceptibility of the medium. The phase conjugate replica of the pump wave in the backward-scattering geometry experiences the largest spatial gain coefficient and is, therefore, favoured in the stimulation process. The optical phase conjugation for the threshold condition is obtained from the relation:

$$ R = \left( \frac{\Omega_p}{2} \chi_{B \omega} I_{exc} L \right)^2 $$

where $I_{exc} = \frac{1}{2} \varepsilon_0 \eta c |E_0|^2 \frac{\Omega}{\Omega_p}$ is the pump excitation intensity.

In Fig. 5, we have plotted the reflectivity coefficient $R$ against the plasma frequency $\Omega_p$ for $B_0 = 10$ T and $\theta = 180^\circ$ with finite piezoelectricity. It can be observed that, reflectivity increases sharply and one may achieve peak value of $R \approx 6 \times 10^5$ at $\Omega_p \approx 5.3 \times 10^{12}$ s$^{-1}$. Moreover, the order of third-order Brillouin susceptibility obtained from our theory, Eq. (15), ranges from $10^{-11}$ esu to $10^{-7}$ esu, which agrees well with theoretically reported values for semiconductors. Thus, it can be inferred that a backward scattered phase-conjugate wave can be resonantly enhanced by the operation of plasma frequency near the modified Stokes frequency.

4 Conclusions

The present analysis deals with an analytical investigation of stimulated Brillouin scattering in weakly piezoelectric doped III-V semiconductors subjected to a magnetostatic field. The externally applied magnetostatic field significantly lowers the threshold value of pump field required for the onset of SBS. The plasma frequency is found to augment the gain coefficient for the onset of SBS. At resonance, the Brillouin gain is enhanced by a factor as large as 200 if one incorporates the piezoelectric effect. The backward Brillouin gain is four-order larger than the forward Brillouin gain in piezoelectric low doped semiconductors in the presence of magnetostatic field. The analysis suggests the suitability of piezoelectric doped III-V semiconductors in the presence of magnetostatic field as the active materials for fabrication of SBS based phase conjugate mirrors.

Acknowledgement

Many fruitful discussions with Dr P Sen is thankfully acknowledged. One of the authors (P.A.),
acknowledges financial support from University Grants Commission, New Delhi and Council of Scientific and Industrial Research, New Delhi and Department of Science and Technology (DST), New Delhi.

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