Charge density waves induced superconductivity in LaSrCuO with intra-layer interactions

Dinesh Varshney1, G S Patel1 & R K Singh2

1School of Physics, Vigyan Bhawan, Devi Ahilya University, Khandwa Road Campus, Indore 452 017
2M P Bhoj (Open) University, Shivaji Nagar, Bhopal, MP 462 016

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An effective two-dimensional dynamic interaction is developed which incorporates screening of holes by charge density fluctuations and by optical phonons to discuss the nature of the pairing mechanism leading to superconductivity in layered La-based cuprates. The authors treat the La-Sr-CuO system as an ionic solid containing layers of holes as carriers and a model dielectric function is set up which fulfills the appropriate sum rules on the electronic and ionic polarizabilities. Estimate of the Coulomb pseudo-potential ($\mu^*=0.22$), describing the screening effects on superconductivity is due to reduced carrier density and large value of the optical dielectric constant as well as the effective mass of holes. The hole-phonon coupling constant ($\lambda$) is evaluated as 1.5, which infers strong strength of coupling. Following strong coupling theory, the superconducting transition temperature of optimally doped La-Sr-CuO is estimated as 42 K. The isotope exponent, coherence length and magnetic penetration depth are also estimated. The implications of the model and its analysis are discussed.

[Keywords: Charge density, Superconductivity, LaSrCuO]

1 Introduction

Superconducting materials were once thought to be limited to transition temperature ($T_c$) below 25 K, but superconductivity in cuprates1 at 35 K in La-Ba-CuO, 90 K in Y-Ba-CuO, 110 K in Bi-Sr-Ca-CuO, 125 K in Tl-Ba-Ca-CuO, and 136 K in Hg-Ba-Ca-CuO (Hg-1223) was achieved, at ambient pressure. It was further enhanced to 150 K, by applying pressure. A change in copper valence of La$_2$CuO$_3$ is known to be caused by the substitution of alkaline earths (Ba, Sr, and Ca) or alkaline metals (Na and K) and with appropriate doping, the compound becomes superconducting with $T_c = 20-40$ K.

La$_{2-x}$Sr$_x$CuO$_{4+y}$ has a complicated crystal structure among the high-$T_c$ cuprates, but more important is a single CuO$_2$ layer sandwiched in between other metal oxide layers in a unit cell. In tetragonal La-Sr-CuO, apical oxygen atoms sit above and below the Cu atoms yielding a nearly octahedral coordination. Therefore, one would expect this system to be relevant from the theoretical as well as from the experimental point of view. Furthermore, in La-based system, the superconductivity is induced by substituting La$^{3+}$ with Sr$^{2+}$ and CuO$_2$ planes contains holes as carriers for the formation of superconducting pairs.

The Sr doping in La$_{2-x}$Sr$_x$CuO$_{4+y}$ influences the transition temperature ($T_c$) of the superconducting state and much interest has been focussed on this issue2. The parabolic dependence of $T_c$ on the number of carriers (holes) per CuO$_2$ reveals that, superconducting transport takes place in the CuO$_2$ plane, in the wide range of $x$ ($0.06 \leq x \leq 0.30$), $T_c$ has its maximum at 38 K for $x = 0.15$, and it drops when the Sr content ($x$) shifts to either side of the optimal value. In view of the parabolic dependence of $T_c$ on $x$, it seems important, as a first step to understand the nature of attractive pairing mechanism and the physical parameters that influenced the superconducting state in doped La-Sr-CuO, at $x = 0.15$. The increase in $T_c$ with the increase in the number of CuO$_2$ layers is also a significant problem that is not discussed in this work.

An overwhelming amount of theoretical effort is being devoted to the question of the electron-pairing mechanism that is responsible for the high-$T_c$ phenomenon. Since the first discovery of cuprate superconductors, it has been felt that, most
probably, new mechanisms must be required to explain the high-$T_c$, although the electron-phonon interaction cannot be ruled out as a contributing mechanism. Optical phonons are important in an ionic lattice, such as, La-Sr-CuO and various groups have employed Raman$^8$ and IR spectroscopy$^9$ to probe frequencies of the optic phonon modes at the zone centre. The features in the phonon spectrum indicate the strong electron-phonon interaction between the relevant phonons and the electronic states. Therefore, electron-phonon interactions are expected to play an important role in the electron-electron pairing mechanism in cuprates.

Possible mediated agencies, i.e., phonons$^5$, polaron$^6$, bipolarons$^6$, excitons$^6$, plasmons$^6$, spin fluctuations$^{10}$, magnons$^{11}$, and resonating bond state$^2$. It is frequently argued that, when the superconducting gap parameter has a $d$-wave symmetry$^{13}$, it implies a Coulomb mechanism, while the strong electron coupling to low-frequency phonons causes an $s$-wave or extended $s$-wave symmetry of gap parameter. Indeed, the magnetic models favour $d$-wave pairing. This argument is based on a long experience with the phonon-mediated mechanism, since the work of BCS, Gorkov, Nambu & Eliashberg$^4$. For cuprates, with a magnetic ion, several groups suggested that, the spin-density waves induce superconductivity and would provide the $d$-wave pairing. Although, this idea is down-played due to two reasons: as the spin fluctuation mechanism yield low transition temperature, it is generally believed that, on increasing the doping from $x=0$, anti-ferromagnetic state disappears, even at $T=0$, before superconductivity sets in.Spin density waves are relevant in over and under-doped cuprates and are not meaningful at the optimized, doped La-Sr-CuO, as we deal with.

In the high-$T_c$ cuprates, screened electron-ion potential is highly anisotropic, with predominant forward scattering. As a consequence, this favours the superconductivity with $d$-wave symmetry of gap parameter, even for the phonon-mediated interaction process. The point is, whatever the mechanism, it must be sufficiently strong in order to account for the observed short coherence length$^6$ with respect to conventional materials. Plasmon-mediated electron-electron interaction potential has been studied by various authors$^8$ among potential non-phonon mechanisms of high-$T_c$ superconductors. However, the kind of plasmon modes proposed to give rise to an attractive interaction potential varies from one calculation to the other. On the other hand, the discovery of the existence of an isotopic effect coefficient (even if it is small), is dependent on the doping concentration$^{16}$ suggested that, standard electron-phonon combined with the plasmonic mechanisms are realistic ones for the explanation of the occurrence of high-$T_c$.

The motions of the carriers are highly correlated owing to their strong Coulomb repulsions, giving rise to low-lying plasmon modes (by low energy, i.e. $<2$ eV). The charge fluctuation mechanism is emphasized in cuprate superconductors by their strong anisotropy and by the presence of a layer-stacking sequence. Electron-energy-loss-spectroscopy$^{17}$ and optical reflectivity measurements$^{18}$ have been used to probe the electronic structure of La-Sr-CuO superconductor. The reflectivity spectrum reveals a metallic plasma reflection due to free-carriers and the plasma frequency of about 1 eV. This implies that, charge fluctuations are large in the hole-doped cuprates and may contribute to the pairing mechanism.

In trying to understand the mechanism leading to an increased pair binding in the hole-doped cuprates, it is quite natural to incorporate electron-phonon and electron-plasmon interactions. In the present work, a pairing mechanism for La-Sr-CuO is evaluated, which arises from the coupling of the carriers to both the optical phonons and the plasmon excitations. Specifically, the authors shall aim at assessing, whether this screened mechanism can explain the $T_c$ (~40 K) value. The gas of hole carriers lies in the layered sequence of copper oxide planes and their Coulomb repulsion are screened by both charge fluctuations and optical phonons. It is worth mentioning that a strong coupling of the electrons with the oxygen vibrations is shown in the electron-tunnelling spectra$^{19}$. This evidence relating to the coupled phonon-electron excitations associated with the oxygen in La-Sr-CuO together with Raman$^5$, IR spectroscopy$^1$, Electron-energy-loss-spectroscopy$^{17}$ and reflectance studies$^{18}$ of La$_{1-x}$Sr$_x$CuO$_2$ have provided the motivation for the present work.

Tachiki & Takahashi$^{20}$ have attributed superconductivity in La- and Y-based cuprates to a pairing interaction, mediated by charge transfer
oscillations associated with LO phonons and stressed that, the charge fluctuations are large, which lead to a negative dielectric constant in the cuprates, and to the work of Falter et al. 21 on cuprates, relating to mode-mixing between phonons and low energy plasmons, whenever the dispersion of the electronic bands in the c direction is sufficiently small. One may also refer to a work by Leggett 22 who discussed the superconductivity in some cuprates by the interaction of carriers in the conducting CuO₂ planes with one another and with static lattice. It was shown that, in an n-layered structure, there is an optical plasmon mode and one acoustic plasmon mode for symmetry of two CuO₂ planes, and in general n-1 acoustic modes. This study may also have potential usefulness in relation to materials containing (CuO₂)ₙ stacks.

The plan of the paper is as follows: In section 2, an effective interaction potential for La-Sr-CuO cuprates is developed by studying the collective excitations of ions and electrons in a single conducting CuO₂ layer as a limit of a periodic stack of such layers which are well separated from each other by insulating metal oxygen layers. The random phase approximation is adopted for the polarizabilities and leads to a model dielectric function, which obeys the appropriate sum rules. The zeros of the dielectric function yield two modes, namely, a low-energy 2D plasmon and a mixed optical phonon-plasmon mode. The static limit of the effective potential is then used to calculate the effective hole-hole coupling strength and hence the transition temperature Tₐ. The isotope effect, the energy-gap ratio, the in-plane coherence length and magnetic penetration depth are also estimated. The details of the numerical analysis and its results are discussed in Section 3.

The main findings include: (i) the optical phonons combined with collective charge fluctuations describes an attractive pairing mechanism and leads to predict a Tₐ ~ 40 K; (ii) in comparison with the BCS results, a null isotope effect and an enhanced energy gap reflect a strong coupling regime; (iii) the in-plane coherence length is smaller and the magnetic penetration depth is larger relative to conventional superconductors. Therefore, it appears that, there is a transition from conventional phonon-mediated to a screened phonon mechanism in cuprates. A summary of the main conclusions are presented in Section 4.

2 Effective Interaction Potential

The layered structure of Sr doped La₂Cu₃O₈ superconductor is composed of two-dimensional (2-D) sheets of CuO₂ layers and metal oxide layers. Such layers as, La₃₋ₓSrₓO₃, LaₓSrₓO₃, CuO₂, La₃₋ₓSrₓO₃, LaₓSrₓO and CuO₂ can conveniently describe the stacking layer sequence. The x-y plane containing the a and b axes is taken to lie in the CuO₂ layer, with the c axis lying along the z direction. Neighboring CuO₂ planes are at a distance d equal to one-half of the lattice parameter along the c axis.

Each basic structural unit in the stoichiometric material is insulating. Substitution of the trivalent La by divalent Sr introduces free carriers as holes in each CuO₂ layer, which is necessary to change La₂Cu₃O₈ into a superconducting compound. Further, the contraction and expansion of Cu and O ions within the octahedral CuO₆ cluster will lead to in-plane (breathing) and out-of-plane (buckling) phonon modes. The effective interaction between each pair of holes as carriers is, in this model, determined by screening of the Coulomb repulsion via collective excitations in the layered fluid of holes (plasmons) and in the underlying ionic lattice (optical phonons). The plasmon mechanism is as follows.

The Fourier transformation in the x-y plane of the bare Coulomb potential between a pair of carriers in the layered system is:

\[
V(q, z - z') = \frac{2\pi e^2}{q\varepsilon_m} \exp[-q|z - z'|]
\]

with \(q\) (\(q_x, q_y\)) as the wave vector in the x-y plane and \(\varepsilon_m\) the high-frequency dielectric constant accounting for core-electron polarization. The main focus is in the case, where \(z - z' = jd\) with \(j\) an integer.

The effective dynamic interaction between the charge carriers with singlet spin is obtained by a simple diagrammatic approach in terms of the polarizability function \(\Pi(q, \omega)\) for the carriers in each layer. The result is:

\[
V(q, j; \omega) = V(q, j) + \sum_j V_j(q, j' \Pi(q, \omega) V(q, j'; \omega)
\]

However, since all CuO₂ planes are identical, the polarizability function can be assumed to be
independent of \( j \). Summation in Eq. (2) is then performed via Fourier transform along the \( z \)-direction, namely, by setting:

\[
V(q, j; \omega) = \frac{d}{2\pi} \int \frac{V_S(q, q_z; \omega) \exp(-i q_z d)}{d q_z} ... (3)
\]

Here, the wave vector \( q_z \) is along the perpendicular to the \( x-y \) plane. Eq. (2) then yields:

\[
V_S(q, q_z; \omega) = \frac{2\pi e^2}{q \varepsilon(q, q_z; \omega)} S(q, q_z) ... (4)
\]

with

\[
S(q, q_z) = \tanh(qd)[1 + \text{sech}(qd) \cos(qd)]^{-1} ... (5)
\]

being a static form factor for the carriers. Fetter\textsuperscript{23} originally derived the above expression for a layered electron gas within a hydro-dynamical approach. This result is of interest for a cuprate superconductor insofar as it can be usefully modelled by a stacking sequence of conducting layers. The dielectric function in Eq. (4) is given by:

\[
\varepsilon(q, q_z; \omega) = \varepsilon_\infty + P(q, \omega) S(q, q_z) ... (6)
\]

with

\[
P(q, \omega) = -\frac{2\pi e^2}{q} \Pi(q, \omega) ... (7)
\]

in terms of the polarizability function \( \Pi(q, \omega) \) for a single band of holes. The long-wavelength form of this function will be used while estimating the effective coupling strength in the subsequent section.

For a layered crystal, in a situation when Brillouin zone is open in the \( q_z \) direction, the integration in \( q_z \), for any reasonable values of the hole density, is over the full Brillouin zone, i.e., \(-\pi/d \leq q_z \leq \pi/d \). While to that, the integration in \( q_z \) and \( q_y \) does not cover the full Brillouin zone in the reciprocal-layer plane. It is, thus, useful to use the cylindrical coordinates \( q_y \), \( \phi \), and \( q_\perp \) to take advantage of an additional orthogonality condition related to the integration over \( \phi \), as \( \phi = \tan^{-1}(q_y/q_x) \) and \( q_z = q_x \), with \( 0 \leq \phi \leq 2\pi \). In view of the large anisotropy\textsuperscript{24} of the mass of the holes as carriers in La-Sr-CuO, the main interest of the authors is in the effective dynamic interaction between pairs of carriers in the plane at \( z = 0 \). This is given by:

\[
V(q, \omega) = \frac{d}{2\pi} \int \frac{V_S(q, q_z; \omega) dq_z}{D(q, \omega)} ... (8)
\]

Using Eqs (4)-(7), one can obtain:

\[
V(q, \omega) = \frac{2\pi e^2}{q \varepsilon_\infty} \frac{D(q, \omega) \sinh(qd)}{|D(q, \omega)| [D^2(q, \omega) - 1]^{1/2}} ... (9)
\]

with

\[
D(q, \omega) = \cosh(qd) + \sinh(qd) P(q, \omega) / \varepsilon_\infty ... (10)
\]

An effective attractive interaction will result in ranges of \( q \) and \( \omega \) where \( D(q, \omega) \) is negative, as evident from Eq. (9). In fact, the long-wavelength limit \( (q \to 0) \) is especially relevant in case of screened Coulomb interactions, and for small \( qd \) values, the result in Eq. (9) simplifies to:

\[
V(q, \omega) = \frac{2\pi e^2 d}{\varepsilon_\infty} \frac{D(q, \omega)}{|D(q, \omega)| [D^2(q, \omega) - 1]^{1/2}} ... (11)
\]

and

\[
D(q, \omega) = 1 + qxP(q, \omega) / \varepsilon_\infty ... (12)
\]

In what follows the above effective dynamic interaction is, in between a pair of carriers with spin-singlet in the consecutive CuO\textsubscript{2} layers separated by a distance \( d \) i.e., the interlayer interactions. In the La-Sr-CuO superconductors, the dominant conduction process is relevant to the two-dimensional CuO\textsubscript{2}O\textsubscript{2}o(a,b) layers. At low temperatures, these CuO\textsubscript{2} layers exhibit a certain in-plane anisotropy, i.e. formation of an oblique lattice, where the distance between neighbouring Cu\textsubscript{2} and O(a) atoms (apical O atoms (O\textsubscript{b})) is slightly longer than that between Cu\textsubscript{2} and O(b) atoms (O atoms in the plane O\textsubscript{b}). In such an anisotropic situation, the Coulomb potential energies at O(a) and O(b) sites would be radically different from one another, together with the anisotropy of the intra-layer transfer integral.

When the temperature exceeds a certain critical value, the oblique lattice changes into a square one. The high-\( T_c \) superconductivity is not observed in symmetric structure. From such evidence, there is a possibility that, the structural in-plane anisotropy and intra-layer interactions play a significant role in
enhancing the high-$T_c$ state. Note that, since there exists a good preliminary knowledge of the structure, it is possible to examine the layered interactions in determining the effective interaction and the resulting $T_c$. The approach that is meaningful consists of an infinite number of periodically stacked layers with carriers inside the layers separated by a distance $d$.

As one examines the available experimental data, one restricts to the case where carriers are constrained to move within the planes and no interlayer motion is allowed. In such a case, for small wave-vectors with in-plane and perpendicular component $q$ and $q_z (\neq 0)$, the charge density wave dispersion is linear in $|q|$, with dispersion proportional to $q^2$. A finite probability for tunnelling between neighbouring layers shifts the plasma frequency $\omega_0 (0, q_z)$ to a finite value. It is important to note that, in the limit $q_z \gg 1/d$, the interlayer interaction does not play an important role and it can be seen that, one is dealing with a two-dimensional dispersion relation $\omega \approx q$.

One must continue to examine and explore the possible layered interactions, and a reliable calculation of $T_c$ with interlayer interactions is quite delicate, and the authors would endeavour to explore this task in a later publication. It is in this spirit that the authors begin with the intra-layer interactions and a search for finding the superconducting transition temperature. Thus, the extreme anisotropy limit of a system, constituted by a periodic stack of isolated conducting layers, is achieved by setting $d \to \infty$ in Eq. (9), yielding:

$$V(q, \omega) = \frac{2 \pi e^2}{q \epsilon(q, \omega)} \quad \text{...(13)}$$

with

$$\epsilon(q, \omega) = \epsilon_\omega + P(q, \omega) \quad \text{...(14)}$$

For an attractive interaction, $\epsilon(q, \omega)$ must be negative. Of course, the long-wavelength collective excitations propagating in the $x$-$y$ plane in the gas of carriers will have the character of a 3D plasmon in the case of finite $d$ described by Eq. (11) and that of a 2D plasmon in the extreme anisotropy limit given by Eq. (13). The authors shall proceed to include the effect of optical vibrations of the ionic lattice in the latter limit.

The screening in the La-Sr-CuO system has been evaluated by treating the system as a layer of gas of holes as carriers inside the CuO$_2$ layers, the holes being introduced by the substitution of La$^3+$ with Sr$^{2+}$ in the parent La$_2$CuO$_4$ compound. In addition, the contraction and expansion of the Cu-C network associated with the motions of the copper and oxygen ions will contribute to the screening mechanism (buckling modes). The effective dynamic interaction $V_i (q, q_i; \omega)$ between the carriers is still given by Eq. (4). However, the dielectric function of the oxide appearing in this equation is given by the sum of contributions from the core electrons, from the charge fluctuations in the gas of carriers and from the ionic motions. The dielectric function in Eq. (13) is then replaced by:

$$\epsilon(q, \omega) = \epsilon_\omega + P(q, \omega) + P_i(q, \omega) \quad \text{...(15)}$$

$P_i(q, \omega)$ being the polarizability associated with the ionic motions and $P(q, \omega)$ given by Eq. (7).

A realistic calculation of $\Pi(q, \omega)$ in Eq. (7) for a single 2D CuO$_2$ layer depends on several aspects of the behaviour of charge carriers. To calculate $\Pi(q, \omega)$ one requires the detailed knowledge of eigenstates and eigenvalues for the carriers in the $x$-$y$ plane. A rigorous numerical calculation of $\Pi(q, \omega)$ can be made using the existing information on eigenstates and eigenvalues. However, it is impossible to obtain analytical result for the effective dynamic interaction with the use of existing calculations of eigenstates and eigenvalues.

Marginal-Fermi-liquid and nested-Fermi-liquid approaches have been proposed for calculations of appropriate $P(q, \omega)$ in a strongly correlated 2D system such as cuprates. However, a random phase approximation (RPA) form of the carrier polarizability is widely used to describe the plasmon excitation and can be helpful to display the coupling effects in a simple manner. In particular, in the long-wavelength limit ($q \to 0$) for a single-band model of holes as carriers with density $n$ and effective mass $m^*$, this approximation yields:

$$P(q, \omega) = \frac{\Omega_p^2}{(q^2 v_F^2 / 2) - \omega^2} \quad \text{...(16)}$$

Here, $\Omega_p = (2 \pi e^2 q/m^*)^{1/2}$ is the 2D plasma frequency and $v_F$ the Fermi velocity. It is quite straightforward that, RPA includes long-range
interactions with the short-range correlations. Considering the regime \( q \to 0 \), only the long wave character of \( V \) in Eq. (13) is important and from Eq. (14), one can recover the thermodynamic relation \( (q = q_0 \to 0) \), \( \varepsilon(\omega) = \varepsilon_\infty + i \sigma(\omega)/\omega \) \( \sigma(\omega) \) is the complex planar conductivity, which includes correctly the correlation effects. However, the present analysis is restricted to small \( q \), i.e., \( q d < 1 \) and \( q d < 1 \), with \( d \) as the interlayer distance. While to that for finite but small \( q \), the analysis is also valid for \( q d \to 1 \) the corrections to the RPA becomes substantial and also short-range properties of the interaction potential becomes dominant. One may comment that, the microscopic models as Hubbard model and the \( t-J \) model\(^{28}\) for strongly correlated electron systems do not incorporate the long range Coulomb interaction, which are of importance for the collective excitations that are dealt with.

The ionic polarizability in Eq. (15) is expressed as:

\[
P_l(q, \omega) = \frac{\varepsilon_\infty \omega^2 - \omega_{LO}^2}{\omega_{TO}^2 - \omega^2} \quad \ldots(17)
\]

where \( \omega_{TO} \) and \( \omega_{LO} \) are the frequencies of the longitudinal and transverse optical vibrations of the system. The single CuO\(_2\) layer La-Sr-CuO system contains seven atoms in a unit cell and their motions will be described by several phonon modes; however, for the sake of simplicity a single optical vibration without dispersion is considered here.

The model dielectric function is thus written as:

\[
\varepsilon(q, \omega) = \varepsilon_\infty + \frac{D_1^2}{\omega_{LO}^2 - \omega^2} + \frac{D_2^2}{\omega_{TO}^2 - \omega^2} \quad \ldots(18)
\]

where \( D_1^2 = \Omega_{\omega}^2 \), \( D_2^2 = \omega_{LO}^2 - \omega_{TO}^2 \), \( A_1^2 = q^2 v_F^2 / 2 \) and \( A_2^2 = \omega_{\infty}^2 \), respectively.

The inverse dielectric function for a two-coupled-oscillator follows:

\[
\varepsilon^{-1}(q, \omega) = 1 + \sum_{i=1}^{2} \frac{f_i}{\omega^2 - \Omega_i^2} \quad \ldots(19)
\]

Here, \( \Omega_i \) are the frequencies of the two modes, to be obtained from the solution of the coupled-mode equation which follows by setting \( \varepsilon(q, \Omega) = 0 \), at the resonance frequencies:

\[
\left( \Omega_i^2 - A_i^2 \right) \left( \Omega_i^2 - A_i^2 \right) = \Omega_i^2 (A_1^2 - \Omega_i^2) + D_1^2 (A_2^2 - \Omega_i^2) = 0 \quad \ldots(20)
\]

with \( D_i^2 = D_i^2 / \varepsilon_\infty \). The oscillator strengths in Eq. (11) are defined as:

\[
f_1 = \frac{\left( \Omega_i^2 - A_i^2 \right) \left( \Omega_i^2 - A_i^2 \right)}{\Omega_i^2 - \Omega_i^2} \quad \ldots(21)
\]

and

\[
f_2 = \frac{\left( \Omega_i^2 - A_i^2 \right) \left( \Omega_i^2 - A_i^2 \right)}{\Omega_i^2 - \Omega_i^2} \quad \ldots(21a)
\]

The following sum rules are valid:

\[
\sum_{i=1}^{2} \Omega_i^2 (q) = D_1^2 + D_2^2 + A_1^2 + A_2^2 \quad \ldots(22)
\]

\[
\sum_{i=1}^{2} f_i(q) = D_1^2 + D_2^2 \quad \ldots(23)
\]

and

\[
\sum_{i=1}^{2} f_i(0) / \Omega_i^2 (0) = 1 \quad \ldots(24)
\]

with \( f_i (0) / \Omega_i^2 (0) \) being the limiting value of \( f_i(q) / \Omega_i^2 (q) \) at long-wavelengths. This implies that, \( \varepsilon^{-1} (q \to 0, 0) \to 0 \) in the static limit. The solution of Eq. (20) is:

\[
2 \Omega_1^2 = \left[ \Omega_{\omega}^2 + A_1^2 + \omega_{LO}^2 \right]
\]

\[
\pm \left[ \left( \Omega_{\omega}^2 + A_1^2 + \omega_{LO}^2 \right) - 4 \left( A_1^2 \omega_{\infty}^2 + \omega_{LO}^2 \right) \right]^{1/2} \quad \ldots(25)
\]

with \( \Omega_{\omega} = \Omega_{\omega} / \varepsilon_\infty \). The above values of the eigenfrequencies arise from mode-mixing between the optical phonons and the 2D plasmons. Further simplification is achieved by treating the second term under the square root as small. This yields:

\[
\Omega_1^2 = A_1^2 + \omega_1^2 \quad \ldots(26)
\]

for the upper mode in the adiabatic approximation, which is the dispersion relation of a 2D plasmon. The lower mode frequency is:

\[
\Omega_2^2 = \frac{\omega_0^2 \Omega_{\omega}^2 + A_1^2 \omega_{\infty}^2}{A_1^2 + \omega_{\infty}^2 + \Omega_{\omega}^2} = \omega_0^2 \quad \ldots(27)
\]

again in the adiabatic approximation.
Finally, the dielectric function can be rewritten in terms of the exact solution of Eq. (25) for the coupled modes as:

$$\epsilon(q, \omega) = \epsilon_m \left( \frac{\omega^2 - \Omega^2}{\omega^2 - \omega_{\text{TO}}^2} \right) \left( \frac{\omega^2 - \omega_{\text{LO}}^2}{\omega^2 - \omega_{\text{LO}}^2} \right)$$

... (28)

and thus, the effective interaction potential takes the form:

$$V(q, \omega) = \frac{2 \pi \omega^2}{\epsilon_m} \left[ 1 + \frac{\Omega^2 - \omega_{\text{TO}}^2}{\omega^2 - \omega_{\text{TO}}^2} \left( \frac{\Omega^2 + A^2_{\text{LO}} - \omega_{\text{LO}}^2}{\omega^2 - \omega_{\text{LO}}^2} \right) \right. \left. \right]$$

... (29)

The above form of the effective potential essentially describes the coupling strength for the scattering of a fermion from the state $|k\rangle$ to the state $|k'\rangle$ in the long-wavelength limit. Having discussed the effective interaction potential, the authors now evaluate the effective hole-hole coupling strength in the superconducting state of single CuO$_2$ layer cuprate in the next Section.

### 3 Effective Coupling Strength

#### 3.1 Hole-hole screening parameter

The evaluation of the superconducting transition temperature $T_c$ requires information about the hole-hole and the hole-phonon coupling strengths. While calculating $T_c$, the mass enhancement parameter $\lambda$ due to hole-phonon coupling is usually treated first with utmost care and the effective Coulomb interaction between carriers is, subsequently, introduced in terms of the pseudo-potential $\mu^*$ in dealing with a high-$T_c$ superconductor. The latter parameter within the two-dimensional model presented in the previous Section is evaluated.

The re-normalised Coulomb repulsive parameter is defined as:

$$\mu^* = \mu \left[ 1 + \mu \ln \left( \frac{\epsilon_m}{\omega_{\text{TO}}^2} \right) \right]$$

... (30)

where the cut-off frequency has been set equal to the screened-phonon frequency $\omega_{\text{TO}}$ and the choice justified in the following Section. The Coulomb strength parameter $\mu$ in Eq. (30) is $\mu = N(\epsilon_F)U$, where $N(\epsilon_F) = n^*A/(2\pi\hbar^2)$ is the 2D density of states at the Fermi energy, $A$ the area of the unit cell and $U$ the static screened interaction $V(q_\parallel, q_\parallel) = V(q_\parallel, q_\parallel, \omega = 0)$ averaged over the 2D Fermi circle. The result for bare Coulomb screening parameter is

$$\mu = \frac{1}{2\epsilon_m a_k^2} \ln \left[ 1 + \frac{\epsilon_m a_k^2}{\omega_{\text{TO}}^2} \right]$$

... (31)

with $a_k = \hbar^2/(m^*e^2)$ the Bohr radius. Here, $\epsilon_m = \epsilon_e \omega_{\text{TO}}^2/\omega_{\text{LO}}^2$ is the low-frequency dielectric constant of the lattice, including the contribution from the optic-phonon modes. Substitution of this value of $\mu$ into Eq. (30) yields the re-normalised Coulomb parameter. The hole-phonon coupling strength shall be evaluated in the next sub-section.

#### 3.2 Hole-phonon coupling strength

Superconducting tunnelling data yields the momentum dependent electron-phonon spectra weight $\alpha F(\omega)$ from a strong coupling inversion procedure for the estimation of the coupling strength $\lambda$. Here, the cylindrical coordinates are used as $\phi = \tan^{-1} (q_y/q_x)$ and $\theta = \tan^{-1} (q_x/q_y)$. McMillan$^{50}$ noted that, the first moment $\alpha F(\omega)$ is related to the density of states at Fermi level $N(\epsilon_F)$ and the mean square electron-ion matrix $(\langle P \rangle)$. The electron-phonon coupling strength $\lambda$ is:

$$\lambda = 2 \int_0^{\epsilon_F} \frac{\alpha^2 F(\omega)}{\omega}$$

... (32)

$$= \frac{N(\epsilon_F)}{\omega_{\text{TO}}} \langle I^2 \rangle$$

... (33)

with the earlier choice of screened-phonon frequency $\omega_{\text{TO}}$ as the cut-off frequency. The above is appropriate for the present system, in view of the enhanced density of states at the Fermi level, relative to a conventional superconductor and of the large mass difference between Cu and O. In application to La-doped cuprates the ionic mass $M$ should be replaced by the reduced mass of the Cu-O pair. The quantity $\langle I^2 \rangle$ in Eq. (32) is:

$$\langle I^2 \rangle = \frac{\Omega}{(2\pi)^3} \int |d^3k|^2 \int d^3k' \frac{\delta(E_i - \epsilon_F)\delta(E_{i'} - \epsilon_F)}{\delta(E_i - E_{i'})\delta(E_{i'} - E_{i'''})}$$

... (34)
\[
< I^2 > = \frac{\left[ N(e_F) \right]^{2N_F} \langle q^2 dq/2k_F^2 \rangle V(q) \right|^2}{\left[ N(e_F) \right]^{2N_F} \langle dq/2k_F^2 \rangle} \quad \text{(35)}
\]

\( \Omega \) being the volume of unit cell. It is seen from the delta functions entering the integrals in Eq. (33) that, the scattering wave vector \( q \) has a maximum value equal to \( 2k_F \). The screened phonon mediated electron-electron interaction is either from the on-site interaction or from the nearest-neighbour interaction. The on-site interaction \( V_{\text{on}} \) is:

\[
V_{\text{on}} = \sum_{q_i, q_s} V(q_i, q_s) \quad \text{(36)}
\]

and the nearest-neighbour interaction \( V_{\text{nn}} \) follows:

\[
V_{\text{nn}} = \sum_{q_i, q_s} V(q_i, q_s) \cos q_i \quad \text{(37)}
\]

The pairing interaction is of \( s \)-wave type for \( V_{\text{on}} \) while to that \( V_{\text{nn}} \) represents the \( d \)-wave type pairing interaction. While considering the ordinary unit cell, i.e., \( V_{\text{on}} \) is attractive and stronger. Hence phonons leads to \( s \)-wave superconductivity. On the other hand, the effective interaction \( V_{\text{nn}} \) yields \( d \)-wave symmetry with nodes due to momentum dependence. The buckling phonon modes generate an attractive interaction between nearest-neighbour sites, thereby increasing the electron-phonon coupling in the \( d \)-channel. As evident from Eq. (36), the term \( \cos q_i \) gives more weight to small momenta transfer. The net result with \( V_{\text{on}} \) is:

\[
< I^2 > = \frac{1}{2k_F} \left[ \frac{2 \pi e^2}{\lambda} \right]^2
\]

\[
\frac{2e_0 k_F + 4a_0^{-1}}{e_0^2(e_0 + a_0^{-1}k_F^{-2})} - \frac{4}{a_0 e_0^3} \ln(1 + e_0 a_0 k_F^{-2}) \quad \text{(38)}
\]

Here, \( e_0 = \varepsilon_0 \omega_0^2 \) is used in the low-frequency dielectric constant of the lattice.

Finally, the electron-phonon coupling strength within the 2D model takes the following form:

\[
\lambda = \frac{\pi e^2}{a_0 k_F A \omega_{\text{ro}}^2}
\]

\[
\frac{2e_0 k_F + 4a_0^{-1}}{e_0^2(e_0 + a_0^{-1}k_F^{-2})} - \frac{4}{a_0 e_0^3} \ln(1 + e_0 a_0 k_F^{-2}) \quad \text{(39)}
\]

The values of the in-layer Coulomb repulsive parameter and of the hole-phonon coupling strength are used in the next section for a quantitative comparison between theory and experiment.

4 Results and Discussion

While calculating the superconducting state parameters of hole doped La_{1-x}Sr_xCuO_4 superconductors, realistic values of some physical parameters are derived from the experimental data. For a layered stacking sequence well separated by an average spacing, \( d_s (= c/2) \), the effective mass of holes as carriers along the conducting CuO_2 plane is obtained from the electronic specific heat coefficient, \( \gamma \), following the relation, \( m^* = 3h^2\gamma/d_s^2 \). Taking inter-planar distance from the lattice parameter \( d_s = 6.6 \text{ Å} \) and \( \gamma = 4.5 \text{ mJ/mol/K}^2 \) from the specific heat measurement \( m^* \approx 3m_e \) is obtained.

The behaviour of the multi-layer systems critically depends only on the planar electron density and the spacings between the layers, and not on the form of single particle distribution function, the layer thickness, or the number of layers. For a stack of 2D conducting planes well separated by an average spacing, \( d_s \), the condition for optimised pairing infers the 2D electronic charge carrier density and follows \( n_d d_s^2 = 1 \) to obtain \( n_e \equiv 2.3 \times 10^{14} \text{ cm}^2 \). The other parameters of the holes as carriers are the Fermi velocity \( v_F \equiv 1.46 \times 10^6 \text{ cm/s} \) and the Fermi energy \( \varepsilon_F \equiv 0.18 \text{ eV} \).

Turning to the parameters related to the ions, a reasonable value of the background dielectric constant \( \varepsilon_0 \) in hole-doped cuprates is \( \varepsilon_0 \equiv 4.5 \). The optic-mode frequencies, on the other hand, can be estimated in an ionic model using a value of \( Ze = -2e \). Such a model yields \( \omega_{\text{ro}}^2 = (\kappa + \eta)/M \quad \text{(40)} \)

and

\( \omega_{\text{ro}}^2 = (\kappa - \eta)/M \quad \text{(41)} \)

where \( M \) is the reduced mass of the Cu-O pair and is 12.77 amu, while \( \kappa \) and \( \eta \) are suitable force constants defined for an inverse-power \( r^{-\alpha} \) overlap repulsion as:

\( \kappa = (Z e)^2 \frac{s-1}{r_0^3} \quad \text{(42)} \)
and

\[ \eta = \frac{8\pi (Z_c)^2}{3 \Omega} \]  

(43)

The authors use \( s = 12 \) [Ref. 33] and \( n_0 = 4.82 \text{Å} \) [Ref. 31], to find \( \kappa = 9.05 \times 10^4 \text{g cm}^{-2} \text{s}^{-2} \) and \( \eta = 4.02 \times 10^4 \text{g cm}^{-2} \text{s}^{-2} \), and hence \( \omega_{\text{LO}} = 416.3 \text{ cm}^{-1} (51.65 \text{ meV}) \) and \( \omega_{\text{TO}} = 257 \text{ cm}^{-1} (32 \text{ meV}) \). The estimated value of the static dielectric constant of the lattice then is, \( \epsilon_0 \approx 11.8 \). The calculated values of the LO/TO frequency are consistent with the measured values of the out-of-plane optical phonon (\( A_{1g} \)) from the Raman spectra of La\(_{1.33}\)Sr\(_{0.15}\)CuO\(_4\) superconductors.

With the above values for the input parameters, the upper and lower mode frequencies \( \hbar \omega \) and \( \hbar \Omega \), are then estimated. The 2D-plasmon energy from Eq. (26) is estimated, as 1.07 eV at \( 2k_F \) and the frequency of the optical phonon dressed with electronic excitations is about 36 meV. Reflectivity spectraw and electron-energy loss spectroscopy\(^7\) of La\(_{1.33}\)Sr\(_{0.15}\)CuO\(_4\) reveals a charge carrier plasmon typically of 1 eV. The effective hole-hole dynamic interaction corresponding to the above values of the system parameters is illustrated in Fig. 1 as a function of frequency in a semi-logarithmic plot. The first pole at \( \omega = \Omega \) arises from the screened optical phonon and a second lies at \( \omega = \Omega \) in correspondence with the 2D plasmon energy. The effective potential is attractive in the ranges \( \omega < \Omega \) and \( \lambda_c < \omega < \Omega \), while it is repulsive in the regimes \( \Omega < \omega < \lambda_c \) and \( \omega > \Omega \).

Finally, \( 2k_F \) scattering of the charge carriers on the 2D Fermi circle is evaluated. It is found that, \( \mu = 0.36 \) from Eq. (31) and \( \mu^* = 0.22 \) from Eq. (30). The choice of the screened optical phonon as the cut-off frequency is because, retardation effects due to very different energy scales for electrons and phonons reduce the Coulomb interactions. In other words, the large reduction of \( \mu^* \) relative to \( \mu \) is because, the ratio \( \epsilon_0 \hbar \omega_{\text{TO}} \) is much larger than unity. It has, therefore, been asserted that phonon energy scale is relevant for retardation effects. The only thing one must be sure of is that, if a boson is a candidate for superconductivity, one must expect \( \mu^* < \mu = 0.5 \) and of course \( \lambda - \mu > 0 \).

In usual metals \( \mu^* \) is taken in the range 0.10-0.13. The estimated value of \( \mu^* \) is large as compared to conventional metals and is due to sufficient screening despite low carrier concentration. The reason to have obtained the result on \( \mu^* \), contrary to the usual metals, is attributed to increased effective mass of the carriers as well as the large value of high frequency dielectric constant \( \epsilon_0 \). In a true sense, the holes move in a correlated way, which quenches the Coulomb interaction in the La-Sr-CuO superconductor and leads to an attractive interaction via exchange of screened phonons. The dimensionless Coulomb pseudo-potential is re-normalised in cuprates. On the other hand, the cut-off frequency would be instead at plasmon energy, the ratio \( \epsilon_0 \hbar \Omega \), smaller than unity. In this situation, Migdal theorem\(^4\) is violated and vertex corrections to electron-phonon interactions are important and hence the use of pseudopotential approximation is incorrect for which retardation effect is ineffective. The electron-phonon coupling strength is estimated from Eq. (24) as 1.5, that infers the strong coupling theory.

To evaluate the transition temperature \( T_c \) for La-Sr-CuO one can proceed as follows. Under the situation when electron-phonon coupling strength is larger than unity, the strong coupling theory\(^5\) applies, yielding:

\[ T_c = 0.25 \omega_{\text{TO}} \left\{ \exp \left( \frac{2}{\lambda_{\text{eff}}} \right) - 1 \right\}^{-1/2} \]  

(44)

where

\[ \lambda_{\text{eff}} = \left( \lambda - \mu^* \right) \left[ 2 \mu^* + \lambda \mu^* i(\lambda) \right]^{-1} \]  

(45)

and

\[ i(\lambda) = 1.5 \exp (-0.28 \lambda) \]  

(46)

Fig. 2 shows the variation of function \( i(\lambda) \) with \( \lambda \). It is found that, \( \lambda_{\text{eff}} = 0.72 \) and \( T_c = 23.8 \text{ K} \), lower with the measured values \( T_c = 38-40 \text{ K} \) in La\(_{1.33}\)Sr\(_{0.15}\)CuO\(_4\). In Fig. 3, the result for \( T_c \) as a function of \( \lambda_{\text{eff}} \) for \( \omega_{\text{TO}} = 369.8 \text{ K} \) is shown. The analytic expression for \( T_c \) from Eq. (44) clearly demonstrates that, \( T_c \) is strongly influenced by the effective coupling strength, which in principle depends on Coulomb repulsive parameter as well as electron phonon coupling strength.

For a set of parameters \( \omega_{\text{TO}} = 369.8 \text{ K}, \lambda = 1.5 \) and \( i(\lambda) = 0.98 \), the variation of \( T_{c} \) with \( \mu^* \) is plotted in Fig. 4. When the Coulomb interactions are not present (that would be considered un-physical
for strongly correlated electron system here, in the present investigation) i.e., a T_{c}' of 55 K is estimated. The enhanced value of $\mu^*$ gives a reduced $T_{c}'$. The net result is that, the increased electron-electron repulsion reduces electron-phonon attraction and points to suppressed $T_{c}'$.

The transition temperature $T_c$, after inclusion of plasmon mechanism, is\(^{36}\):

$$T_c = T_{c}'^{ph}[\Omega_p/T_{c}'^{ph}]$$

...(47)

with

$$\eta = \lambda_{pl}/[\lambda + \lambda_{pl}]$$

...(48)

$\lambda_{pl}$ being the hole-plasmon coupling strength. Taking $\lambda_{pl} = 0.1$, one obtains $T_c = 35$ K and thus, $T_c$ enhances with the increase in $\lambda_{pl}$ value (see Fig. 5). Value of $T_c$ is calculated to be equal to 42 K for $\lambda_{pl} = 0.15$, in agreement with the earlier reported data.\(^{3}\)

A steep increase in $T_c$ values is thus predicted by incorporating the additional coupling of low energy plasmons although weakly coupled with a pair of carriers, over the value obtained by the electron-phonon mechanism. One can in principle explain the enhanced $T_c$ values in cuprates by starting with a strong coupling phonon mechanism and later on adding a correction term with weakly coupled plasmons. However, the essence of collective charge excitations will be further verified to see whether, by this way, one can enhance $T_c$ and whether they play a special role in the pairing mechanism of high-$T_c$ cuprates.

The isotope effect of superconducting transition temperature is described in terms of the isotope effect exponent defined by $T_{c} - M^{\alpha}$, where $M$ is the ionic mass. For a conventional mono-atomic BCS superconductor $\alpha$ is 0.5, when the Coulomb interactions as well the anharmonicity of phonons are not present. Assuming a mass dependence through the frequency pre-factor, the isotope effect exponent can be derived as:

$$\alpha = \lambda^{-1} \sum_{\nu} \lambda_{\nu} \alpha_{\nu}$$

...(49)

with

$$\lambda = \sum_{\nu} \lambda_{\nu}$$

...(50)

The $\lambda_{\nu}$ are the coupled mode strengths and $\alpha_{\nu}$ are defined as:

$$\alpha_{\nu} = \frac{\partial \ln \Omega_{\nu}}{\partial \ln M}$$

...(51)

Here, $\nu = \pm$ and $\Omega_{\nu}$ are the frequencies of the coupled phonon-plasmon modes as defined earlier in Eq. (25) and the result for $\alpha = 0.5 - \chi_3$, with:

$$\chi_3 = \frac{(\Omega_{\nu}^2 + A_{\nu}^2)}{2\Omega_{\nu}^2}$$

$$\pm \left[ \Omega_{\nu}^2 - (\Omega_{\nu}^2 + A_{\nu}^2 + \omega_{\nu}^2)\Omega_{\nu}^2 + A_{\nu}^2 \right] \left[ \Omega_{\nu}^2 + A_{\nu}^2 + \omega_{\nu}^2 \right] - 4(\Omega_{\nu}^2 + A_{\nu}^2 + \omega_{\nu}^2)$$

...(52)

In the adiabatic limit [$\Omega_{\nu} \gg \omega_{\nu}$, $\omega_{\nu}$], Eq. (52) leads to $\chi_3 = 0.5$ and $\chi_3 = 0.0$ thus, $\alpha = 0.0$ and $\alpha = 0.5$. There are two points to be made here. Firstly, the severely reduced value of $\alpha (\alpha = 0.15)$ as obtained by Crawford et al.\(^{16}\) is predicted in a simple manner and secondly the normal isotope exponent for conventional superconductors is recovered.

While in the situation of resonance coupling ($\Omega_{\nu} = \omega_{\nu}$), it is found that, $\alpha = \alpha = 0.25$, yielding an isotope exponent of 0.25, independent from the individual mode-coupling strengths. In principle, the isotope effect alone cannot be used to assess the role of the electron-phonon coupling in the pairing mechanism. It may be commented that, a reduced isotope-effect exponent usually implies an unconventional pairing mechanism, and in particular is a consequence of the screened phonon mechanism that have been treated here.

The other superconducting state parameters as in-plane coherence length are of great importance. The present calculations reveal a smaller Fermi velocity $v_F = 1.46 \times 10^7$ cm s \(^{-1}\) relative to the values in conventional metals (1.2-10^7 cm s \(^{-1}\)). A small value of $v_F$ along with a $T_c$ of 42 K leads to a coherence length of 42 Å using the precise definition $\xi = h v_F / \pi \Delta(0)$ with $2\Delta(0) = k_B T_c$. The value of $\beta$ critically depends on the strength of coupling. For La-Sr-CuO superconductor, it is found that, the electron-phonon coupling constant is about 1.5, which leads to a larger value of $\beta$ relative to BCS value of 3.52. Using $\beta = 4.1$ together with the $v_F$ value, it is found that, the coherence length is consistent with the value of 30-50 Å obtained from the magnetic measurements.\(^{35}\) It is interesting to comment that the coherence length which, even if
small, is much larger than inverse Fermi momentum.

Fig. 1 — Effective interaction potential \( V(q,\omega) \) at \( q = 2k_F \) as a function of \( \omega \) (eV) on a semi-logarithmic scale. The two poles lie at frequencies \( \Omega_+ (\approx 3188 \text{ meV}) \) and \( \Omega_- (\approx 11 \text{ eV}) \)

Fig. 2 — Function \( t(\lambda) \) is plotted with \( \lambda \)

Fig. 3 — \( T_e^{ph} (K) \) is plotted with \( \lambda_{eff} \) for \( \omega_{t(\lambda)} = 3188 \text{ meV} \)

Finally, the in-plane magnetic penetration depth \( \lambda_{d}(0) [\approx (m^*c^2/4\pi\hbar^2)] \) is estimated as 1565 Å, consistent with the values of 2000 Å by Kossler et al., from muon spin relaxation and of 2545 Å by Li et al., from penetration depth measurements in La_{1.8}Sr_{1.2}CuO_4 superconductors. Reduced value of the in-plane magnetic penetration depth is attributed to increased value of \( v_F \) value as deduced within the Fermi liquid approximation. For any realistic theory,
the band structure parameters are required that are quite complicated. These parameters, which are required to calculate various important physical quantities, vary from technique to technique. The small value of the in-plane coherence length and the large penetration depth over the conventional superconductors are another important features of hole-doped cuprates. While applying the present approach to single layer La-Sr-CuO, reasonable results have been obtained so far, but because of the complexity of layered interactions, much more work is required to analyse various important physical properties.

The model intra-layer dielectric function that the authors have evaluated accounts for screening of the Coulomb repulsions between the carriers by means of the RPA forms of the polarizabilities of the gas of holes as carriers and of the ionic lattice, satisfying the realistic sum rules in the long-wavelength limit. The screened phonon mechanism turns out to be quite effective in yielding a superconducting state in the strong coupling regime. The calculated values of the vibrational frequencies and of the plasma frequency at \(2k_F\) are in the range of values obtained from Raman and Electron Energy Loss spectroscopy.

Deduced value of renormalized Coulomb repulsion parameter \(\mu^*\) is large contrary to the normal metals, indicative of effective screening of holes despite the low carrier concentration in La-based cuprates. The large enhancement of the effective mass of the carriers from hole-optical phonon coupling, combined with an increased value of \(\mu^*\), and with the coupling of the carriers to 2-D plasmons, leads to a value of \(T_c\) in the measured range.

The symmetry of the order parameter and the role of inter-layer (intra-layer) interactions in most of the high-\(T_c\) cuprates is a significant problem and an attempt has been made to understand them. For screened phonon mode, a repulsive interaction may appear and the electron-ion matrix element with cos \(q\) term gives more weight to small momenta transfer for the nearest neighbour interactions. The electron-phonon coupling constant is large because of definitive sign dependence of momentum on the first moment of the density of states. In the present La_{1-x}Sr_xCuO_2 system at hand, with single CuO_2 layer in a unit cell, the authors have confined themselves only on the intra-layer interactions in view of the weak coupling of the conducting CuO_2 layer in the consecutive unit cell.

The present approach also evaluates the isotope-effect exponent and consistent with the experimental finding. In the adiabatic limit, the null isotope effect exponent is predicted and also the BCS value of conventional superconductors is recovered. The reduced isotope exponent arises from the screened phonon mechanism in place of a conventional phonon mechanism, as a consequence of a transition.
from the conventional phonon pairing mechanism to the screened optical-phonon mechanism with \( d \)-wave symmetry and is in accord with the reported value. The authors also found reduced in-plane coherence length and enhanced magnetic penetration depth value in La-based cuprate over the conventional superconductors. It is worth to point out that, the present in-layer screened phonon mechanism is sufficiently strong in order to account for the observed short coherence length with respect to conventional materials.

In conclusion, the authors have been able to relate the intra-layer coupling strengths to microscopic quantities relevant to the test material, and with reasonable values of these parameters one can obtain the parameters leading to a superconducting state. However, particular attention has not been paid to the interlayer interactions, which are also important for superconductivity in cuprates and this work will be reported subsequently. The existence of charge density waves along with phonons induced \( d \)-wave superconductivity in La-Sr-CuO within the intralayer pairing mechanism and the results obtained on microscopic quantities are in the observed range of high-\( T_c \) superconductors.

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