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Formulation of muon range 0-100 TeV and transmission through lead

R Darijani[1]*, A Negarestani[2], M Reza Rezaie[2], J Fatami[3] & A Akhond[1]

[1]Department of Physics, Payame Noor University Tehran, Iran

[2]Department of Physics, Kerman Graduate University of Technology, Kerman, Iran

[3]Department of Physics, Shahid Bahonar University, Kerman, Iran

*E-mail: Reza.Darijani@yahoo.com

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Muons can penetrate long distances into matter and are less susceptible to radiate effects as compared to electrons. Over a broad energy range, the dominant energy loss is due to ionization mechanism. This makes the shielding of muons and the knowledge of their range considerably important at high-energy accelerators and in other high energy applications. In the present paper, the muon range $R(E)$ from 0-1 GeV in lead is obtained by using muon range data which were generated by SRIM 2012.03 code. For muon energy versus traveled distance E(X), an analytical formula is extracted by a fitting procedure by the aid of Origin 8.0 and Find Graph softwares. For extracting the values of R(E) in the range 0-100TeV and E(X) for less than 100 TeV using the current method faced a limitation. This limitation is due to maximum SRIM energy input card which is 1GeV, so the SRIM range data are used for less than 1 GeV and the GROOM range data for above 1 GeV. In the present paper, the muon energy is calculated using this technique at energy less than 100 TeV for arbitrary traveled distance.

Keywords: Muon range, Travelled distance, SRIM, GROOM, Transmission, Lead

1 Introduction

The study of the passage of fast-moving charged particles through matter has been important since the early days of nuclear physics[1]. Muons are leptons that have a rest mass of 105.7 MeV and a mean lifetime of 2.2×10^{-6}s. These particles do not participate in the strong interaction. Muons can penetrate long distances into the matter and are less susceptible to radiate effects as compared to electrons[2]. Individual electrons never show a well defined range: at low energy, multiple scattering causes them to diffuse through matter, and at high energies, the initial electron is concealed by a complex shower of secondary particles. From this point of view, muons are more satisfactory particles to consider because multiple scattering is not too serious at low energies and the effect of nuclear collisions is small at intermediate energies, and, Bremsstrahlung and direct pair production do occur at extremly high energies which increase the energy loss above that due to ionization. These make the shielding of muons and the knowledge of their range considerably important at high energy accelerators and in other applications such as muon propagation through thick layers of matter and electric field gradient calculations for muon in simple metal[3-5]. So the muon range as a function of its energy $R(E)$ and muon energy versus travelled distance $E(X)$ in a medium are needed for these applications. $R(E)$ and $E(X)$ formula for energies greater than 10 GeV have been introduced[6,7]. The present study is concerned with a quantitative evaluation of the muons range over the energy range 0-1 TeV. Mixing the SRIM range data for muon energies less than 1 GeV with the Groom range data[8] for energies in the range 1GeV-100 TeV, the muon range equations in the range 0-100 TeV and energy versus traveled distance for less than 100 TeV in any medium can be obtained. Also using $E(X)$ and $R(E)$ equations allow the following problems to be quantified: (a) The muon energy with arbitrary initial energy less than 100 TeV after any traveled distance in a medium and (b) The traveled distance for any energy loss.

2 Theory and Discussion

Overview of previous studies for calculating charged particle energy versus traveled distance shows that the stopping power for a charged particle when passing through a medium is defined as Eq. (1):

$$f(E)=-\frac{\mathrm{d}E}{\mathrm{d}x} \qquad \dots (1)$$

where dx is the traveled distance interval by a particle and dE is the particle energy loss interval. To define $g(E)=[1/f(E)]$, Eq.(1) gives Eq.(2):

$$\mathrm{d}x = -g(E)\mathrm{d}E \qquad \dots (2)$$

If the charged particle energy decreases to E from an initial value E_0, the traveled distance is given by Eq. (3), and the range obtains from Eq. (4).

$$x = -\int_{E_0}^{E} g(E)d(E) \qquad \dots (3)$$

$$R = -\int_{E_0}^{0} g(E)d(E) \qquad \dots (4)$$

For solving Eq. (3) several methods have been proposed. Comparing them, it seems that for low and medium energy (0-10 GeV) the method mentioned in reference[10] is the best, but the method in reference[11] is more suitable for higher energies.

3 Muon Range Calculations

3.1 Overview on previous works in high energy

The mean stopping power for high-energy muons in a material[11] can be described by Eq. (5):

$$f(E) = \frac{1}{g(E)} = -\mathrm{d}E/\mathrm{d}x = a\,(\mathrm{E}) + b\,(\mathrm{E})E$$

$$b = b_{\mathrm{brems}} + b_{\mathrm{pair}} + b_{\mathrm{nucl}} \qquad \dots (5)$$

where E is the total energy, $a(E)$ is the electronic stopping power, and $b(E)$ is due to radiative processes, including bremsstrahlung, pair production and photonuclear interactions. The quantities $a(E)$ and $b(E)$ are slowly varying functions of E at high energies where radiative contributions are important. As the energy increases, the contribution of ionization decreases. Of the three remaining processes, bremsstrahlung dominates up to about 1000 GeV and then pair production dominates. The term $b(\mathrm{E})E$ is less than 1% of $a(E)$ for $E \leq 100$ GeV for most materials. For example, $a(E)$ is ≈0.002 GeVcm^2/g in iron, and $b(E)$ has values of about 1, 3, 5.5, 7.5, 8 and 8.4×10^{-6}cm^2/g for 1, 10, 100, 1000, 10000 and 100000 GeV muon energy, respectively[2]. The data for $a(E)$ and $b(E)$ have been presented for most materials by Lawrence Berkeley National Laboratory[8]. For a and b (essentially) constant, the continuous-slowing-down-approximation (CSDA) range is obtained from Eq. (4) as Eq. (6).

$$R(E) = (1/b)\ln(1 + E/E_\mathrm{c}) \qquad \dots (6)$$

where $E_\mathrm{c} = a/b$ is the muon critical energy, which is the energy at which electronic and radiative losses are equal. The critical energies for muons incident on water, air, concrete, standard rock and lead are 1.03, 1.28, 1.11, 0.70, 0.693 and 134 TeV, respectively[2]. In previous works, the muon range as a function of energy below 10 Gev was not characterized exactly[11,12]. The main goal in the present paper is to obtain this function exactly. For this purpose, the SRIM 2012.03 software[13], which is a popular and widely used software[14], was used. This software, in addition to the stopping power data, can also produce the range data. By fitting a function on these data, the muon range is obtained as a function of its energy as Eq. (7).

$$R(E) = a_1E^{0.5}a_2E^{1.5} + \sum_{i=1}^{5} a_{i+2}E^i \quad \text{0-1 GeV} \qquad \dots (7)$$

In Eq. (7) and for 0-1 GeV, the $a_1E^{0.5}$ and $a_2E^{1.5}$ components describe the muon range at very low energy, the a_3E^2 and a_4E^3 components describe the muon range at low and medium energies, and for high energies the a_4E^3, a_5E^4 and a_6E^5 components are introduced. With these parameters, the $R(E)$ equation is fitted to the SRIM data. Mixing the SRIM and Groom range data[8], the muon range equations for 1 GeV to 100 TeV are obtained as Eqs (8 and 9).

$$R(E) = \frac{\sum_{i=0}^{5} b_iE^i}{\sum_{i=0}^{3} c_iE^i} \quad \text{1 GeV-1 TeV} \qquad \dots (8)$$

$$R(E) = \frac{\sum_{i=0}^{4} d_iE^i}{\sum_{i=0}^{2} e_iE^i} \quad \text{1-100 TeV} \qquad \dots (9)$$

3.2 Muon energy versus traveled distance calculations E(x)

Muon energy versus traveled distance (METD) equations in a media are needed for many applications in shielding, muon propagation through matter, muography and cosmology. Previously, the METD equation was introduced for muons in biosphere after traversing a thickness x of rock (or ice or water) by Beatty *et al*[8]. as Eq. (10).

$$E = E_0\exp(-bx) - E_c[\exp(-bx) - 1] \qquad \dots (10)$$

where E is the muon energy after traveling distance x in rock, E_0 is the initial energy, E_c and b have been introduced in Eqs (5 and 6). In the present work for calculating the muon traveled distance versus energy, Eq. (3) is used which is as follows:

$$X = -\int_{E_0}^{0} g(E)d(E) - \int_{0}^{E} g(E)d(E) = R_0 - R \qquad \ldots(11)$$

where x is the traveling distance, $R_0=R(E)$ and $R=R(E)$ are the range of muons with energy E_0 and E, respectively.

Using Eqs (8 and 9), the traveled distance equations are obtained as Eqs (12-14).

$$X(E,E_0) = R(E_0) - R(E)$$
$$X = a_1(E_0^{0.5} - E^{0.5}) + a_2(E_0^{1.5} - E^{1.5}) + \sum_{i=1}^{5} a_{i+2}(E_0^i - E^i) \quad \text{for 0-1 GeV} \qquad \ldots(12)$$

$$X = \frac{\sum_{i=0}^{5} b_i E_0^i}{\sum_{i=0}^{3} c_i E_0^i} - \frac{\sum_{i=0}^{5} b_i E^i}{\sum_{i=0}^{3} c_i E^i} \quad \text{for 1 GeV-1 TeV} \qquad \ldots(13)$$

$$X = \frac{\sum_{i=0}^{5} d_i E_0^i}{\sum_{i=0}^{3} e_i E_0^i} - \frac{\sum_{i=0}^{5} d_i E^i}{\sum_{i=0}^{3} e_i E^i} \quad \text{for 1-100 TeV} \qquad \ldots(14)$$

The equation of muon energy versus traveled distance $E(x,E_0)$ is the inversion of the $X(E,E_0)$ equation. For inversion of $X(E,E_0)$ by Origin 8, it is sufficient to exchange the E and X axes and fitting a function on data.

Using the range in Eqs (7-9), the $E(x,E_0)$ equations are obtained as Eqs (15-19).

$$E = E_0 - \sum_{i=1}^{5} f_i X^i + f_6 \exp[f_7 x - R_0)^2] \qquad \ldots(15)$$

$$E = E_0 - \left(\sum_{i=1}^{7} g_i X^i\right) \text{ for E(100 TeV –100 GeV) or } x \text{ (0-315770.9 mm)} \qquad \ldots(16)$$

$$E = \left(\sum_{i=1}^{7} h_i X^i\right) \text{ for } E \text{ (100 GeV -1 GeV) or } x \text{ (315770.9-354698.9 mm)} \qquad \ldots(17)$$

$$E=h_8+h_9\exp(-h_{10}x) \text{ for } E\text{(1 GeV-100 MeV) or } x\text{(354698.9-355346.3 mm)} \qquad \ldots(18)$$

$$E=h_{11}+h_{12}\exp[-h_{13}(x-h_{14}) \text{ or } E\text{(100 MeV –0) or } x \text{ (355346.3-355393.8 mm)} \qquad \ldots(19)$$

For plotting of the Eqs (16-19), first g_i and h_i coefficients have been calculated. Second by using add function graph option in Origin 8.0 software the equations were defined and plotted.These equations for muon initial energy less than 100 TeV are very useful because these can be calculated: (I) The range of muon. (II) The muon energy for arbitrary traveled distance. (III) The muon traveled distance for arbitrary energy loss.

For case II, the procedure is as follows:

$x_0=R_0-R_1$, $R_0=R(E_0=10^8)$ and
$R_1=R(E_1$: initial energy) … (20)

In Eq. (20), the form of R_1 equation is according to the energy interval as presented in Table1.
E(u) is the muon energy after distance traveling x with initial energy E_1. Where $U=x_0+x$

In case III, the procedure is described by Eq.(21):

$X=R(E_1)-R(E_2)$, $R(E_1)=R(E_1$: initial energy) and
$R(E_2)=R(E_2$: arbitrary energy) … (21)

Eq. (21) gives muon traveled distance for energy loss from E_1 to E_2.

4 Results

4.1 Calculating the energy of muon as a function of traveled distance in lead

As an example, this method used for calculating the muon energy as a function of traveled distance in lead. Firstly, the muon range data from 0-1GeV are generated by the SRIM2008 software. And an analytical $R(E)$ formula is extracted by a fitting procedure by the aid of Origin 8.0 and Findgraph. Then the muon range has been plotted as shown in Fig. 1 based on the expression given in Table1.

The agreement between the fit and the data is found to be very good. Using Table 1 equation, the muon range can be calculated in every energy between 0-1 GeV. Secondly muon range data that generated by SRIM2008 (0-1 GeV) are mixed with Groom data (1 GeV-100 TeV). These mixed data are fitted by the aid of Origin 8.0 and Findgraph softwares. These equations have been presented in Table 2 and have been plotted in Fig. 2(a and b).

Table 1 — Range (mm) versus energy (MeV) and energy versus distance traveling (mm) for 1 GeV muon in lead

Muon range versus of energy in lead	Muon energy versus of traveling distance in lead
$R(E) = 5\times10^{-4}E^{0.5} + 9\times10^{-5}E^{1.5} + 0.27E + 2.57\times10^{-3}E^2 - 5.825\times10^{-6}E^3 + 5.8\times10^{-9}E^4 - 2.123\times10^{-12}E^5$	$E(X) = 1000 - 1.79x + 4\times10^{-3}x^2 - 1.66\times10^{-5}x^3 + 2.911\times10^{-8}x^4 - 1.781\times10^{-11}x^5 - 27\times exp(-2.4\times10^{-2}(x-694.8)^2$

4.2 Muon distance traveled data versus energy in interval 0-100 (TeV)

Using Eq. (11) and combined muon range data (Srim and Groom), the muon traveled distance data versus energy are extracted. These data have been plotted in Fig 3 and Fig. 4(a,b,c,d). Similarly, the method was used to obtain muon range equations, the muon traveled distance equations are extracted. These equations are presented in Table 3.

4.3 Two important applications for muon range and transmission in lead

Application 1: Energy of a muon with initial energy 400 GeV after traveling 2 m in lead is equal to 371.434 GeV. Because according to Eq. (23), the x_0 and U are as follows:

$R_0=R(E_0=10^8)=355394$ mm

$R_1=R(E_1$: initial energy=400 GeV)=90572 mm according the Eq. (9) and Table 3.

$x_0=R_0-R_1=264822$ mm

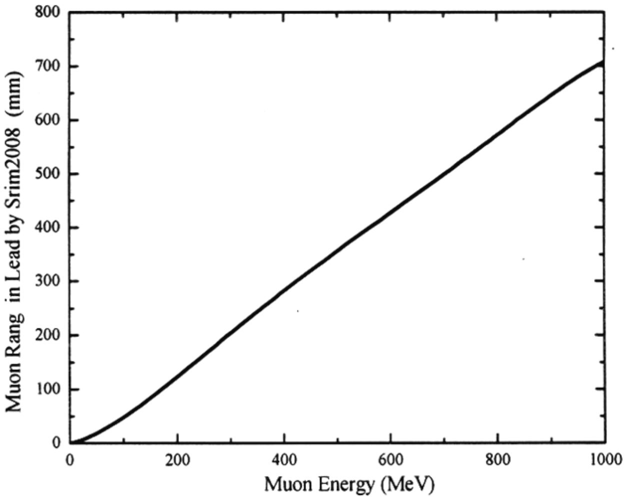

Fig. 1 — Muon range as a function of energy from 0 to 1Gev in lead

$U=x_0+x=264821.11+2000=266822$ mm

E(u) = 371GeV according the Eq.(19) and Table 3.

Application 2: Distance that muon travels until its energy decreases from 23 TeV to 152 GeV is equal to 231948 mm. Because according of Eq. (21), the value of x is as:

$R(E_1$: initial energy)= R(23 TeV)= 284818 mm according the Eq.(11) and Table 3

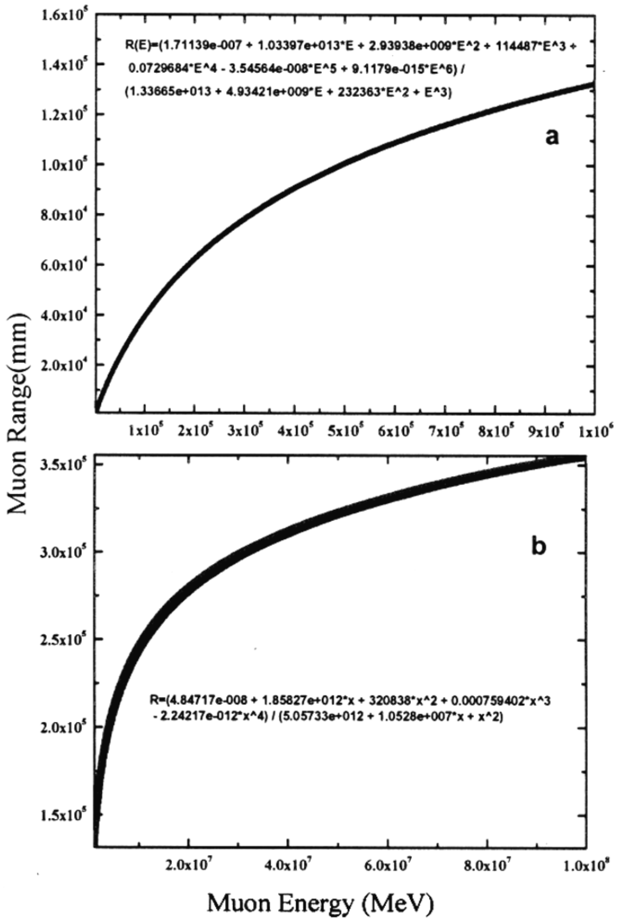

Fig. 2 — Muon range as a function of energy in lead (a) from 1 GeV to 100 GeV (b) from 100 GeV to 100 TeV

Table 2 — Muon range(mm) versus energy (MeV) for 100 TeV muon in lead

Energy interval	$R(E)=\frac{a(E)}{b(E)}$:Muon range equation in lead
10^8-10^6	$a(E)=4.847\times10^{-8}+1.86\times10^{12}E+3.208E^2+7.59\times10^{-4}E^3-2.242\times10^{-12}E^4$
	$b(E)=5.057\times10^{12}+1.053\times10^7E+E^2$
10^6-10^3	$a(E)=1.711\times10^{-7}+1.034\times10^{13}E+2.94\times10^9E^2+1.145E^3+7.3\times10^{-2}E^4-3.55\times10^{-8}E^5+9.12\times10^{-15}E^6$
	$b(E)=1.34\times10^{13}+4.93\times10^9E+2.32\times10^5E^2+E^3$
10^3-0	$a(E)=5\times10^{-4}E^{0.5}+9\times10^{-5}E^{1.5}+2.703\times10^{-1}E+2.57\times10^{-3}E^2-5.82\times10^{-6}E^3+5.8\times10^{-9}E^4-2.123\times10^{-12}E^5$
	$b(E)=1$

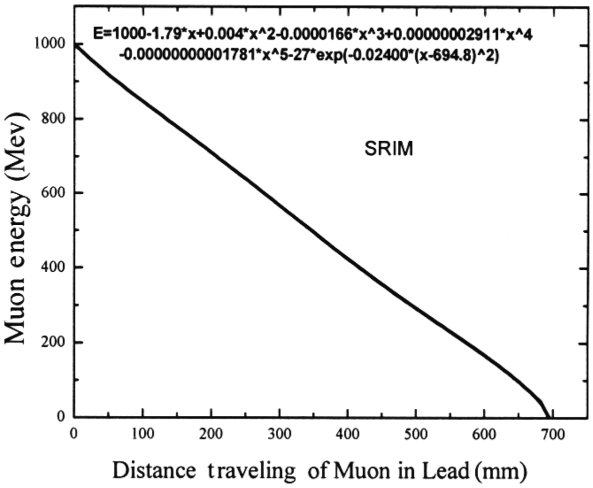

Fig.3 — Muon distance traveled data versus energy in interval 0-1 GeV

$R(E_2$: arbitrary energy$)= R(152$ GeV$)=52871$ mm according the Eq. (10) and Table 3

$X=R(E_1)-R(E_2)=284818-52871=231947$ mm

Table 3 — Energy (MeV) versus distance traveling (mm) for 100 TeV muon in lead

Energy interval	Distance traveling interval	E(x) : Muon Energy versus distance traveling
10^8-10^5	0-315770.9	$99996743 - 2.071X + 2.103 \times 10^{-2}X^2 - 1.344 \times 10^{-7}X^3 + 5.682 \times 10^{-13}X^4 - 1.54 \times 10^{-18}X^5 + 2.405 \times 10^{-24}X^6 - 1.633 \times 10^{-30}X^7$
10^5-10^3	315770.9- 354698.9	$-86960205 + 9.246X - 2.415 \times 10^{-3}X^2 - 3.226 \times 10^{-9}X^3 + 6.019 \times 10^{-15}X^4 + 9.473 \times 10^{-20}X^5 - 3.076 \times 10^{-24}X^6 + 2.722 \times 10^{-31}X^7$
10^3-10^2	354698.9- 355346.3	$-17125 + 1.603 \times 10^{16} exp\left(\frac{-x}{12894.06}\right)$
10^2-0	355346.3 - 355393.8	$187.22 - 2.13exp(\frac{x - 355402.67}{64.026625})$

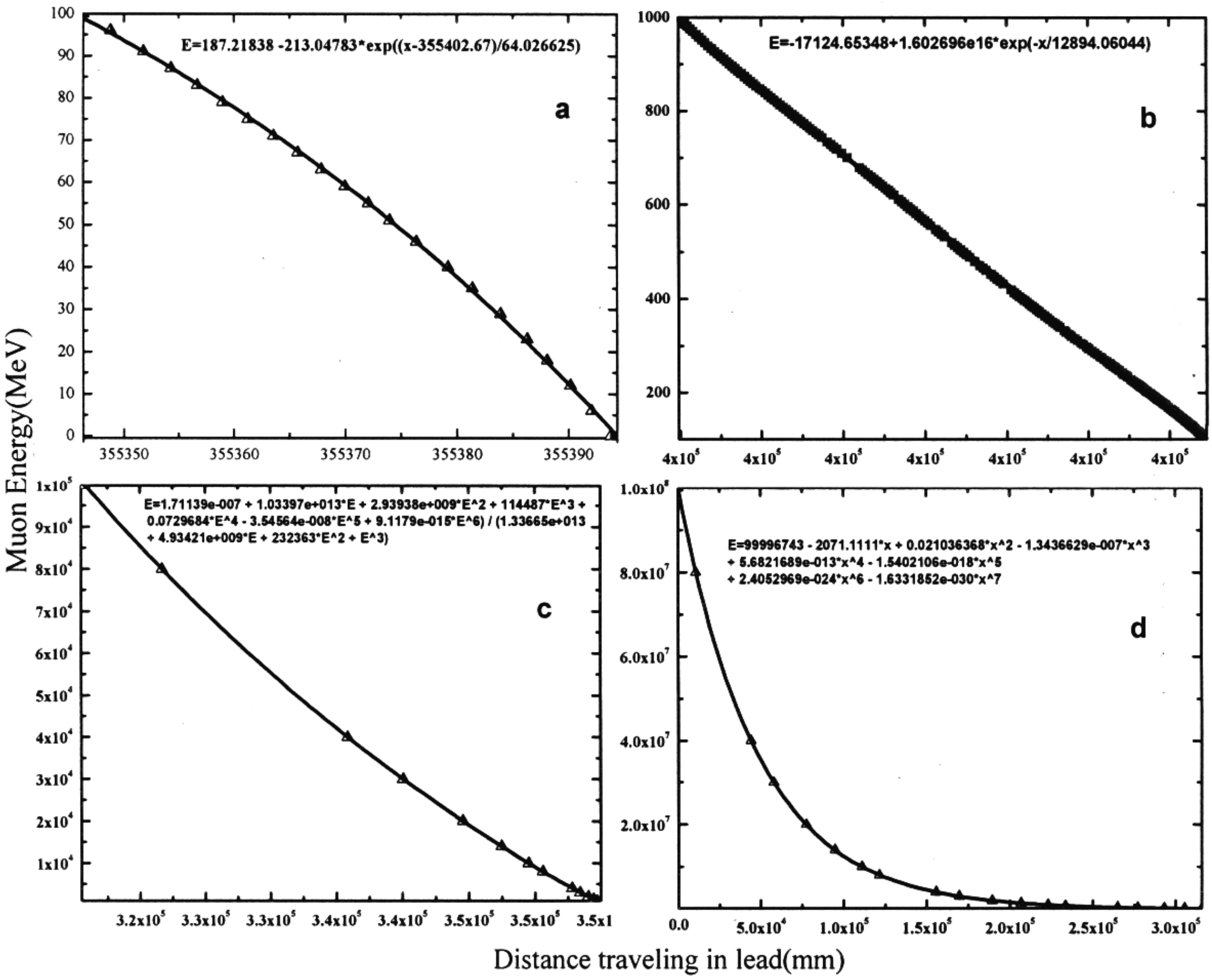

Fig. 4 — Muon distance traveled data versus energy from (a) 0-100 MeV, (b) 100 Mev-1 GeV, (c) 1 Gev-100 GeV and (d) 100 GeV-100 TeV

5 Conclusions

The method described is used for calculating the energy of muon as a function of traveled distance in lead. This method consists of four stops: first, it is used for calculating the muon range as a function of energy. For this purpose, the muon range data generated by SRIM2008 (0-1 GeV) are mixed with muon range data, were introduced by Groom (1 GeV-100 TeV).These combined data are fitted with the equations and are presented in Table 2, and shown in Fig. 2 (a,b). Secondly, using the equation $x=R_0-R$, the muon traveled distance data versus energy in interval 0-100 TeV are extracted. Thirdly, the $E(x)$ is obtained from inverse the $x(E)$. $E(x)$ functions in lead for energy less than 100TeV have been extracted. These equations are presented in Table 3 and plotted in Fig. 4(a,b,c,d). Finally, the muon energy after traveling distance x in medium for energy less than 100 TeV can be extracted by the following procedure introduced in paper with the help of the $U=x_0+x$ parameter.

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