Anomalous Hall constant of heavy fermion systems
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The large anomalous Hall constant of heavy fermion systems can be understood using Periodic Anderson model. A variational method in this work to study temperature variation of Hall constant \( R_H \) of cerium compounds has been used. The skew scattering plays a dominant role in deciding \( R_H \) in these compounds. The scattering phase shift is strongly magnetic field dependent and the Hall constant is expressible in terms of magnetic susceptibility.

[Keywords: Heavy fermions, Anomalous Hall constant, Skew scattering]

1 Introduction
An extraordinary Hall effect has been observed in various mixed-valence and heavy-fermion compounds, specially in the heavy-fermion Ce compounds\(^1\) CeAl\(_3\), CeCu\(_2\)Si\(_2\), CeCu\(_4\), and CeRu\(_4\)Si\(_2\). The Hall constant in these compounds is much larger than in normal metals, often positive and generally, drops rapidly, at low temperatures. The mechanism for this is attributed mainly, to intrinsic skew scattering arising from strong spin-orbit coupling for the Ce-ion. The intrinsic skew scattering increases rapidly from zero, as fluctuations about the low temperature coherent state develop characteristic temperature \( T_{co} \) and then decrease slowly at above temperatures.

A theoretical model of the skew scattering by Kondo impurities of cerium\(^2\) was first proposed by Fert in 1973. This model is valid only well above the Kondo temperature\(^3\) \( T_K \). Later on, Coleman, Anderson & Ramakrishnan\(^4,5\), in their work, discussed the skew scattering by cerium impurities in both the limits \( T \gg T_K \) and \( T \ll T_K \). The single impurity models of Fert\(^2\) and CAR (Refs 4, 5) are strictly valid only in the incoherent regime, where the heavy fermions can be thought of as, simply collection of independent resonant scattering centers. At present, there does not seem to be any specific theory of the skew scattering, which takes into account the coherent effects. However, it has been observed that, by extrapolating the single impurity results into the coherent regime, some of the experimental results, especially at low temperatures, may be accounted for\(^7\). Recently, the ground-state properties of Periodic Anderson Model (PAM) have been studied, using the variational techniques\(^x\). A variational technique to study the ground state and thermodynamic properties of PAM has been developed\(^{14-20}\) and used in this method here, to study the anomalous Hall effect of heavy fermion systems. The details of the variational method of the authors may be found in Refs (14, 15).

2 Formulation
The orbitally non-degenerate periodic Anderson model is described by the Hamiltonian,

\[
H = \sum_{k\sigma} \epsilon_k c_{k\sigma}^{\dagger} c_{k\sigma} + \epsilon_f \sum_{i\sigma} h_i^{\dagger} h_i \sigma\\
+ \sum_{k\sigma} V_k (c_{k\sigma}^{\dagger} h_{i\sigma} + h.c.) + \frac{U}{2} \sum_{j\sigma} \sum_{l\sigma} n_{j,\sigma}^{l} n_{j,-\sigma}^{l} - \sum_{i\sigma} (c_{i\sigma}^{\dagger} h_{i\sigma} + h.c.) + \frac{U}{2} \sum_{i\sigma} \sum_{l\sigma} n_{i,\sigma}^{l} n_{i,-\sigma}^{l} - \sum_{i\sigma} (c_{i\sigma}^{\dagger} h_{i\sigma} + h.c.) + \frac{U}{2} \sum_{i\sigma} \sum_{l\sigma} n_{i,\sigma}^{l} n_{i,-\sigma}^{l} \quad \ldots (1)
\]

where symbols have their usual meanings.

Here, the authors are considering only strongly interacting (i.e. \( U \to \infty \)) case. In this case, the probability of \( f^2 \) configuration is very small. The variational value function which projects \( f^2 \) configuration out, may be written as in Refs (14, 15)
\[ \psi = \sum_{k, \sigma} [1 + A_{k\sigma} P_j b_{k\sigma}^\dagger b_{k\sigma}] | F > \]  

where  
\[ | F > = \prod_{k \leq \epsilon_f} c_{k\sigma}^\dagger | 0 > \]

is the Fermi sea of conduction electrons. \( A_{k\sigma} \) is the variational parameter and \( P_l = 1 - n_{l\sigma} \).

With this wave function, the variational parameter \( A_{k\sigma} \) and the number of conduction and \( f \)-electrons \( n_{l\sigma} \) and \( n_{f\sigma} \) at finite temperatures are, respectively, given by:

\[ A_{k\sigma} = \frac{1}{2V_k P_f^2} \left[ (\epsilon_k - \epsilon_f) P_j + \sqrt{(\epsilon_k - \epsilon_f)^2 + 4V_f^2 P_f^2} \right] \]

\[ n_{l\sigma} = \frac{1}{N} \sum_k \frac{f_{k\sigma}}{1 + A_{k\sigma}^2 P_f^2} \]

\[ n_{f\sigma} = \frac{1}{N} \sum_k \frac{A_{k\sigma} P_f^3 f_{k\sigma}}{1 + A_{k\sigma}^2 P_f^2} \]

where \( f_{k\sigma} = f(E_{k\sigma} - \mu) \) is the Fermi function for the lower branch of the quasi-particle spectrum \( E_{k\sigma} \).

\( E_{k\sigma} \) is given by:

\[ E_{k\sigma} = \frac{1}{2} \left[ (\epsilon_k + \epsilon_f) P_j - \sqrt{(\epsilon_k - \epsilon_f)^2 + 4V_f^2 P_f^2} \right] \]

Let \( N'(\epsilon_{k\sigma}) \) denote the density of the unperturbed conduction band, \( N'(E_{k\sigma}) \), the total density of lower quasi-particle states, \( N'(E_{k\sigma}) \) the density of the lower part of the perturbed conduction states.

Then:

\[ N'(E_{k\sigma}) = N'(E_{k\sigma}) n_{l\sigma} = \frac{N'(E_{k\sigma})}{1 + A_{k\sigma}^2 P_f^2} \]

\[ N'(E_{k\sigma}) = \frac{N'(\epsilon_{k\sigma})}{(d E_{k\sigma}/d \epsilon_{k\sigma})} \]

Hence: \( N'(E_{k\sigma}) = \frac{1}{(1 + A_{k\sigma}^2 P_f^2)} \left( \frac{1}{d E_{k\sigma}/d \epsilon_{k\sigma}} \right) \)

\[ \frac{d E_{k\sigma}}{d \epsilon_{k\sigma}} = \frac{1}{m} \frac{1}{\sqrt{(\epsilon_{k\sigma} - \epsilon_f P_j)^2 + 4V_f^2 P_f^2}} \]

Fig. 1(a) — Variation of Hall constant \( R_H \) with temperature for tight binding conduction band \( (\epsilon_f = -\cos k) \) for two different positions of \( f \)-level \( \epsilon_f \) Curve I for \( \epsilon_f = 0 \); Curve II for \( \epsilon_f = 0.2 \), \( V = 0.25 \)

2.1 Magnetic susceptibility

In the presence of a static magnetic field \( B \), the \( f \)-level is \( \epsilon_f - g \sigma \mu_B B \). The magnetic susceptibility is given by:

\[ \chi(T) = g \sigma \mu_B \frac{\partial}{\partial B} \left[ n_{f\sigma} - n_{l\sigma} \right] \]

Putting \( n_{f\sigma} \)'s from Eq. (5) one gets the expression of susceptibility\(^{16,17}\) for \( U \to \infty \) in units\(^{2}\) of \( (g \sigma \mu_B) \).

2.2 Electrical resistivity

The authors are not interested here in the absolute value of electrical resistivity \( \rho(T) \), but only in the variation of \( \rho(T) \) with temperature. The electrical conductivity formula given by Mott Ref. \( (21) \) [considered in Ref. \( (19) \) by the authors] has been used.
\[ \sigma(T) = \int \left[ -\frac{\partial f_\sigma}{\partial E_\sigma} \right] \bar{\sigma}(E_\sigma) dE_\sigma \] \quad \text{(12)}

where \( \bar{\sigma}(E_\sigma) \) can be written as:

\[ \bar{\sigma}(E_\sigma) = \sigma_0 \left| N'(E_\sigma) \right|^2 \] \quad \text{(13)}

with \( \sigma_0 = \frac{2\pi e^2 h^3}{m^2} \left| D_E \right|^2 \alpha \nu \) \quad \text{(14)}

The symbols in \( \sigma_0 \) are given in Ref. (21). \( \sigma_0 \) has been taken as a constant in the calculations.

The Hall constant due to intrinsic skew scattering is given by the expression [Fert formula Ref. (1)]

\[ R_{\text{H}}(T) = \gamma \chi(T) \rho(T) \] \quad \text{(15)}

where \( \chi(T) \) is the normalized susceptibility \( \chi = \chi/C \) (\( C \) being the Curie constant).

The coefficient \( \gamma \) takes on different values \( \gamma_1 \) and \( \gamma_2 \) above and below \( T_c \). In this work, Hall constant has been calculated for the compound CeAl. For this compound, \( \gamma_1 = 0.075 \) K/T for \( T > T_K \) and \( \gamma_2 = 0.097 \) K/T for \( T < T_K \), \( T_K = 40 \) K (Ref. 1).

Eq. (15) can account for the main features of the Hall effect at the onset of coherence effects and is valid for both the temperature regions \( T >> T_K \) as well as \( T_{\text{coh}} \leq T \leq T_K \). The Hall constant term having the form \( \chi(1 - \chi/T) \) used by CAR (Refs. 4 and 5) and Panwar & Singh (Ref. 20) is valid only well above the Kondo temperature \( T_K \) in the incoherent regime, where the heavy fermion systems can be pictured as a collection of independent resonant scattering centers.

3 Results and Discussion

In these calculations, the authors have considered a tight binding conduction band with conduction band width \( W = 2.0 \) eV. The total number of electrons per site \((n_f + n_g)\) has been taken to be 1.5. In Fig. 1(a and b), the temperature dependence of Hall constant \( R_{\text{H}}(T) \) for different position of level \( E_1 = 0.0 \) and \( 0.2 \) eV and for different values of hybridization \( V = 0.25 \) and \( 0.5 \) eV, has been shown, respectively.

Obviously, for \( \varepsilon_f = 0.0 \) eV, the authors obtain the typical behaviour of \( R_{\text{H}}(T) \) with a rapid increase \(-T^2\) at low temperature, a maximum near \( T_{\text{max}} = 30K \) and then a sharp decrease towards the higher temperature. Further, on increasing \( \varepsilon_f \) i.e. \( \varepsilon_f = 0.2 \) eV, the value of Hall constant \( R_{\text{H}}(T) \) decreases in the given temperature range, from 0 to 500 K and the maxima at low temperature also disappears. Further, as \( V \) increases [Fig. 1(b)], the maxima in \( R_{\text{H}}(T) \) at low temperature becomes more wide and is less pronounced and is shifted towards higher temperature regime. This behaviour of \( R_{\text{H}}(T) \) i.e. the positive Hall effect [Fig. 1(a and b)] has been the characteristics of heavy fermion cerium compounds e.g. CeAl. Similar behaviour of \( R_{\text{H}}(T) \) has been observed in other heavy fermion compounds like, CeCu_6, CeRu_2Si_2, UPt_3, etc.

The Hall constant \( R_{\text{H}}(T) \) obtained from the expression \( \chi(1 - \chi/T) \) (as done by CAR Refs (4, 5) and Panwar & Singh\(^{20}\) is negative and cannot account for the positive Hall effect observed in most heavy fermion compounds. This term is valid only in the incoherent regime. While \( R_{\text{H}}(T) \) expressed by Eq. (15) is equally valid in both the coherent and
incoherent regime and shows the possible cross-over between the two preceding regimes. It is also found that, Eq. (15) can account for the main features of the Hall effect at the onset of coherence effects. The authors expect large anomalous Hall constants to occur, generally, in the heavy fermion systems. At low temperatures in very pure compounds, true heavy bands will be formed. In this regime, the current carried by each Bloch state will have a skew scattering component, differing widely between different Fermi surface regions so that, the resultant Hall effect can only be derived from a detailed band structure and transport theory. In near future, the authors plan to take into account these factors and calculate the Hall constant at low temperatures.

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References