Numerical solution of hydromagnetic unsteady free convection flow past an infinite porous plate

Atul Kumar Singh

Department of Mathematics, VSSD College, Kanpur 208 002

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Unsteady free convection flow of an incompressible, electrically conducting, viscous fluid past an infinite vertical porous flat-plate under the influence of uniform magnetic field, applied normal to the flow, has been studied by finite difference method. The uniform suction is considered normal to the plate. The results for the velocity distribution, the temperature distribution and the skin-friction have been investigated and discussed for different values of the parameters encountered in the equations.

1 Introduction

The study of flows, past a porous plate has applications in the fields of engineering, technology, bio-physics, astrophysics, space dynamics and so on. At present, it is usual to use an engineering process in which, the fluid supports an exothermic chemical or nuclear reaction. For economic use, an appropriate process design requires accurate correlation for the heat transfer coefficient at the boundary surface. Therefore, due to increasing importance in technological and physical problems, free convection flows of viscous fluids have received attention of the workers.

Free convection flow, past an infinite vertical plate, in the presence of magnetic field has been studied by several workers\(^{1-4}\). Soundalgekar\(^{5}\) studied free convection effects on MHD flow, past a vertical porous plate. Unsteady case of this problem for flat-plate has been studied by Singh\(^{6}\), while unsteady case with porous plate has been discussed by Singh\(^{7}\). Malashetty & Leela\(^{8}\) have investigated a similar case for two-phase flow, under some changed boundary conditions. Shende & Soundalgekar\(^{9}\) have studied unsteady, forced and free convection MHD flow of an incompressible, viscous, conducting fluid, past an infinite vertical porous plate with variable suction and oscillatory wall temperature. Recently, Singh\(^{10}\) has discussed MHD effects on convective flow of an electrically conducting, incompressible, viscous fluid taking into account the induced magnetic field.

Several authors including, Agrawal & Rani\(^{11}\), Shah & Verma\(^{12}\), Das et al\(^{13}\) have discussed MHD free convection flow by finite difference method. Recently, Sriekanth et al.\(^{14}\) have studied unsteady, transient free convection flow of an incompressible, dissipative, viscous flow, past an infinite vertical plate, under the influence of uniform transverse magnetic field. However, in engineering and technology, there are occasions where the plate is porous. Hence, it is proposed to study the flow, past an infinite vertical porous plate, taking into account constant suction velocity, normal to the plate. Finite difference method (Fletcher\(^{15}\)) has been employed to obtain velocity field and temperature distribution. Numerical values of skin-friction have also been obtained, by applying Newton’s interpolation formula for four points.

2 Formulation of the Problem

Consider the unsteady free convection flow of an incompressible, slightly conducting, viscous fluid, past an infinite, vertical, porous plate under the influence of uniform magnetic field of strength \(H_0\) applied normal to the plate. The \(x’\)-axis is taken along the plate in a vertically upward direction and the \(y’\)-axis normal to it. Let \(u’\) be the velocity of the fluid along the plate and \(v’ = -v_n\) is the constant suction velocity normal to the plate (where \(-v_n\) is positive constant and the negative sign indicates that, the suction is towards the plate). Therefore, the velocity and the magnetic field are given by \(\vec{q} = (u', -v_n, 0)\) and \(\vec{H} = (0, H_n, 0)\). Initially, when the
time $t' \leq 0$, the plate and the fluid are assumed to be at the same temperature $T'_w$. When the time $t' > 0$, the plate temperature is raised to $T'_w$ and is maintained constant. This rise in temperature causes the creation of free convection currents due to temperature difference $T'_w - T'_w$. The fluid being slightly conducting, the magnetic Reynolds number is much less than unity and hence, the induced magnetic field can be neglected in comparison with the applied magnetic field (Sparrow & Cess). Under these conditions, the flow variables are functions of $y'$ and $t'$ alone. Therefore, for these assumptions and usual Boussinesq's approximation (Gebhart & Pera), the equations governing the flow are:

$$\frac{\partial u'}{\partial t'} - \frac{\partial v'}{\partial x'} = \frac{\partial^2 u'}{\partial y'^2} + \frac{\partial^2 v'}{\partial y'^2}$$

$$\frac{\partial T'}{\partial t'} - \frac{\partial v'}{\partial x'} = \frac{K}{\rho C_p} \frac{\partial^2 T'}{\partial y'^2} \ldots (2)$$

where $u'(y,t')$ is the velocity of the fluid, $g$ the acceleration due to gravity, $K$ the thermal conductivity of the fluid, $\rho$ the density of the fluid, $\dot{\omega}$ the kinematic coefficient of viscosity, $\beta$ the volumetric coefficient of thermal expansion, $T_w$ the temperature of the plate, $T'_w$ the temperature of the fluid, far away from the plate, $\mu$ the viscosity of the fluid, $\sigma$ the electrical conductivity of the fluid, $\mu_s$ the magnetic permeability, $C_s$ the specific heat at constant pressure, $H_0$ the magnetic field induction, and the other symbols have their usual meaning.

The initial and boundary conditions are:

$t' \leq 0, \quad u'=0, \quad T'=T'_w$ \quad for all $y'$

$t' \geq 0, \quad u'=0, \quad T'=T'_w \quad at \quad y'=0'$

$u'=0, \quad T'=T'_w \quad as \quad y' \to \infty \ldots (3)$

The following dimensionless quantities are introduced by the authors:

$$t = \frac{t'}{T_g}, \quad y = \frac{y'}{L}, \quad u = \frac{u'}{U_0}, \quad \dot{\omega} = \frac{T-T'_w}{T'_w-T'_w}, \quad \Delta T = T-w-T'_w$$

$$U_0 = \left( \frac{\partial g \beta \Delta T}{\partial T'} \right)^{1/3} \quad (Reference \ velocity),$$

$$L = \left( \frac{g \beta \Delta T}{\partial^2 T'} \right)^{1/3} \quad (Reference \ length),$$

$$T_g = \left( \frac{g \beta \Delta T}{\partial^2 T'} \right)^{2/3} \quad (Reference \ time),$$

$$v_0 = \frac{\nu_0 U_0 T_g}{\Delta T} \quad (Suction \ parameter)$$

$$P_r = \frac{\mu C_s}{\rho} \quad (Prandtl \ number)$$

$$M = \frac{\mu_t H_0^2 T_g}{\rho} \quad (Magnetic \ parameter)$$

Eqs (1) and (2) then reduce to:

$$\frac{\partial \dot{\omega}}{\partial t'} - v_0 \frac{\partial \dot{\omega}}{\partial y'} = \theta + \frac{\partial^2 \dot{\omega}}{\partial y'^2} - Mu \ldots (4)$$

$$\frac{\partial \theta}{\partial t'} - v_0 \frac{\partial \theta}{\partial y'} = \frac{1}{P_r} \frac{\partial^2 \theta}{\partial y'^2} \ldots (5)$$

with the initial and boundary conditions:

$t' \leq 0, \quad u'=0, \quad \dot{\omega}=0 \quad for \ all \ y'$

$t' \geq 0, \quad u'=0, \quad \dot{\omega}=1 \quad at \quad y'=0$

$u'=0, \quad \dot{\omega}=0 \quad as \quad y' \to \infty \ldots (6)$

3 Method of Solution

Defining a new dimensionless variable:

$$Y = \frac{y}{1+Y}$$

Eqs (4) and (5) become:

$$\frac{\partial \dot{\omega}}{\partial t'} = \theta + (1-Y)^2 \frac{\partial^2 \dot{\omega}}{\partial Y^2} + (1-Y)^2 \left[ v_0 (1-Y) \frac{\partial \dot{\omega}}{\partial Y} - Mu \right] \ldots (7)$$

$$P_r \frac{\partial \theta}{\partial t'} = -(1-Y)^2 \left[ 2(1-Y) - v_0 P_r \right] \frac{\partial \theta}{\partial Y} + (1-Y)^2 \frac{\partial^2 \theta}{\partial Y^2} \ldots (8)$$

With the initial and boundary conditions given in Eq. (6).

Eqs (7) and (8) are coupled non-linear partial differential equations and are to be solved by using the initial and boundary conditions given in Eq. (6). An implicit finite difference method is used to solve
these with finite difference method, Eqs (7) and (8) become:

\[
\begin{align*}
\frac{u_{i,j+1} - u_{i,j}}{\Delta t} &= \theta_{i,j} + (1 - Y_{i,j}) \left\{ v_0 - 2(1 - Y_{i,j}) \left( \frac{u_{i+1,j+1} - u_{i,j+1}}{\Delta Y} \right) \right\} \\
&+ (1 - Y_{i,j}) \left\{ \frac{u_{i+1,j+1} - 2u_{i,j} + u_{i-1,j}}{(\Delta Y)^2} \right\} - M u_{i,j} \\
\end{align*}
\]

\[\cdots (9)\]

\[
\begin{align*}
P_f \left[ \frac{\theta_{i,j+1} - \theta_{i,j}}{\Delta t} \right] &= \\
- (1 - Y_{i,j}) \left\{ 2(1 - Y_{i,j}) - v_0 P_f \left( \frac{\theta_{i+1,j+1} - \theta_{i,j+1}}{\Delta Y} \right) \right\} \\
&+ (1 - Y_{i,j}) \left\{ \frac{\theta_{i+1,j+1} - 2\theta_{i,j} + \theta_{i-1,j}}{(\Delta Y)^2} \right\} \\
\end{align*}
\]

\[\cdots (10)\]

In the initial and boundary conditions given in Eq. (6), the continuous variables get replaced by their discrete equivalents, as follows:

\[
\begin{align*}
u(0,0) &= 0 \quad \theta(0,0) &= 0 \quad \text{for all } i=0 \\
u(i,0) &= 0 \quad \theta(i,0) &= 0 \quad \text{for all } i \neq 0 \\
u(0,j) &= 0 \quad \theta(0,j) &= 1 \quad \text{for all } j \\
u(i,j) &= 0 \quad \theta(i,j) &= 0 \quad \text{for all } j \\
\end{align*}
\]

\[\cdots (11)\]

Here, the index \(i\) refers to \(Y\) and \(j\) refers to \(t\). The values of \(\Delta Y\) and \(\Delta t\) are chosen to be \(\Delta Y = 0.1\) and \(\Delta t = 0.0025\). The computation is carried out and the numerical values of \(u\) and \(\theta\) are computed. First, the velocity at the end of time-step, viz, \(u(i,j+1)\) is computed from Eq. (9) in terms of velocities and temperatures at points on the earlier time-step. Then \(\theta(i,j+1)\) is computed from Eq. (10). The procedure is repeated until \(t=1\). The infinity is assumed at \(Y=9.2\). To prove convergence of the finite difference scheme, the computation is carried out for slightly changed values of \(\Delta Y\) and \(\Delta t\) by running the same program. No significant change was observed in the values of \(u\) and \(\theta\). Thus, it is concluded that, the finite difference scheme is convergent and stable.

4 Skin-friction

The dimensionless skin friction (\(\tau\)) is:

\[
\tau = -\left( \frac{du}{dY} \right)_{Y=0}
\]

The numerical values of \(\tau\) are calculated by applying Newton’s interpolation formula for four points.
observed that, an increase in magnetic parameter decreases the velocity field while an increase in time parameter increases the velocity field.

Fig. 2 shows the temperature profiles $\theta$ versus $Y$ for various values of Prandtl number ($P_r=0.71$, 0.025 and 7.00), and time parameter ($t=0.1, 0.2$ and 0.3) at $v_0=0.1$. It is observed that, temperature field decreases with increase in Prandtl number ($P_r$) or time parameter ($t$). Also, the authors conclude that, $\theta$ decreases with increase in $Y$. It is interesting to note that, temperature field decreases more rapidly for water ($P_r=7.00$), in comparison to air ($P_r=0.71$).

Table 1 shows numerical values of the skin-friction for various values of Prandtl number ($P_r$), magnetic parameter ($M$) and suction parameter ($v_0$) for different values of time ($t=0.1, 0.2, 0.3$ and $0.4$). It is observed that, an increase in time results in an increase in the skin-friction. Further, it is noted that, an increase in Prandtl number ($P_r$) or magnetic parameter ($M$) decreases the skin-friction ($f$) while reverse effect is noted for an increase in suction parameter ($v_0$).

**References**

15. Fletcher C A J, Computational techniques for fluid dynamics, (Springer-Verlag, Berlin).