Application of fractal in marine sciences: Study of the 2004 Sumatra earthquake (Mw 9.3) sequence in Andaman-Nicobar islands

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Aftershock sequence of the 26 December 2004, Sumatra-Andaman mega thrust event (Mw 9.3) that resulted a rupture of about 1300 km in the ocean floor, Sumatra to Andaman-Nicobar islands, is studied to evaluate the fractal dimension of the oceanic tectonic features. A large number of aftershocks (Mw ≥ 3.0) are recorded by temporary network that was established by the Geological Survey of India (GSI) in the Indian state of Andaman-Nicobar islands. The complex geological structures that include the Andaman trench, West Andaman fault and the backarc spreading zone, Andaman spreading Ridge (ASR), in the region generated a rupture area 800 × 300 km² below the Andaman – Nicobar islands. The Fractal dimension was estimated using correlation dimension method and the box counting method. Epicenters of 1100 well located earthquakes were considered for the analysis. A prominent N-S trending contour with fractal dimension between 0.90 - 1.30 indicates that the epicenters are linear, or almost one dimensional that correlates with the West Andaman fault. The box counting method estimated the fractal dimension 1.17 for this linear fault that lie between the trench and the back arc spreading zone in the ocean basin. The higher fractal dimension (>1.5) contours on both sides of the West Andaman fault indicate the extent of 2D heterogeneity of the Andaman Trench and the ASR. The fractal dimension values for the entire region suggests that the faults are spatially distributed in the whole region, and the whole region is seismically active.

[Key words: Fractal dimension, earthquake hypocenters, heterogeneity, seismogenic]

Introduction

Nature is a cluster of many patterns with different order of complexities, which are so irregular and fragmented that it is very difficult to describe their shapes by integer dimension. Mandelbrot ¹ observed that many natural phenomena possess self-similarity at many different scales. He used the term ‘fractal’ to describe such phenomena and called the scaling parameter fractal dimension D. Various tectonic discontinuous processes such as displacements on faults and continuous processes such as folding are extremely complex, but they satisfy fractal statistics.

Earthquakes are associated with fractals by virtue of the accompanying fracturing of rocks. The fractal nature of the spatial distribution of earthquakes was first demonstrated by Kagan & Knopoff². Hirata³ reported temporal variations in fractal dimension to quantify the seismic process, and Shimazaki & Nagahama⁴ demonstrated that active fault systems in Japan possess self-similarity with fractal dimensions of 0.5 to 1.6. Fractal dimension is an efficient statistical parameter to quantify the dimensional distribution of seismicity and with that the proportion of randomness and clusterization.

Geological Survey of India (GSI) made a temporary microearthquake network to monitor aftershocks in the Andaman-Nicobar islands⁵, the northernmost seismic station in the Andaman islands was about 1300 km away from the mainshock epicentre. The network recorded a large number of aftershocks that occur below the Andaman-Nicobar islands. The aftershock analysis revealed a rupture area of 800 × 300 km² below this region⁵. This rupture zone below the Andaman-Nicobar islands with large aftershock data gave an opportunity to examine the spatial variation of fractal dimension of the marine geological structures that ruptured and caused these aftershocks.

The devastating mega-thrust earthquake (Mw 9.3) of 26 December, 2004 occurred in the Indian Ocean at the interface of the India and Burma plates in the north Sumatra (Fig. 1). It resulted in devastation to
most coastal countries of the Southeast Asia and the adjoining regions. The Andaman-Nicobar (A & N) islands was one of the worst affected Indian states due to this killer tsunamigenic earthquake. In this study we made an attempt to map the fractal dimension of the study region to understand the seismogenic structures that caused cluster of aftershocks. In order to spatially map the fractal dimension, the study area was divided into grids with the grid size of 0.6° × 0.6° with an overlapping of 0.3°. Each grid was overlapped both in X and Y directions. This exercise generated 52 grids with at least 50 number of events in each grid. Taking centre of the grid as the the plotting point, contour map of estimated fractal dimensions $D_2$ was prepared

**Methodology**

**Tectonic setting**

The N-S trending Sagaing transform fault demarcates the eastern margin of the Burma microplate. It separates the central low lands from the eastern high lands of Burma, and the fault continues to the Andaman Sea Rift system, known as the Andaman Spreading Ridge (ASR) (Fig. 1). The Andaman Sea basin is considered to be a complex back arc spreading centre; it is categorized as a ‘pull apart’ or ‘rip off’ basin rather than a typical back-arc-extensional basin. Seismic reflection studies across the trench slope indicate folding and thrusting in the accretionary prism, where a major component of convergence occurs normal to the trench axis. Among the series of thrusts/faults in the Andaman subduction zone, the West Andaman Fault (WAF) is most prominent (Fig. 1). The region between the Andaman- Nicobar islands and the volcanic arc is the foredeep sedimentary trough.

Subduction along the Andaman-Sumatra trench system has given rise to a discontinuous belt of submarine ridges and volcanic seamounts. The andesite volcanoes of the Barren and Narcondam islands are prominent among them. The Narcondam is now extinct, but the Barren is still marked by an active volcano; it erupted last during March, 1991 after lying dormant for about two centuries. Further south, this volcanic belt is represented by the Barisan range in Sumatra, and in the north, the trend is correlated with the central molasse basin of Burma. Along the Sumatra subduction zone, plate convergence is partitioned into dip-slip and right-lateral strike-slip components, the former being accommodated by slip on the subduction interface and the latter by the Sumatran Fault System. The present-day tectonic processes are controlled by three major fault systems, the most prominent being the subduction thrust, which outcrops in the Sunda trench. The trench-parallel Sumatra Fault that runs through the entire length of the island from Banda Aceh to Sunda Strait accommodates oblique convergence through strike-slip faulting.

**Data source**

The data base of the present study constitutes the high precision digital data set obtained by six three-component digital short period seismograph (4-Reftek and 2-Kinemetrics) stations established in the A & N Islands, the temporary seismic stations were established at Car Nicobar, Hutbay, Port Blair,
Rangat, Diglipur, and at the Narcondum volcanic islands. Figure 2 shows the epicentral distribution of the aftershocks.

**Fractal dimension estimate**

Correlation integral method: The correlation integral method is widely applied in seismology, especially to spatial distribution of earthquake epicentres. This technique is preferred to box counting algorithm because of its greater reliability and sensitivity to small changes in clustering properties. The correlation integral is related to the standard correlation function as given by:

\[ C_r \sim r^{D_2} \]

where \( D_2 \) is a fractal dimension, more strictly the correlation dimension. Grassberger & Procaccia introduced a practical algorithm for the measure of the correlation dimension, commonly referred to as the Grassberger-Procaccia algorithm (GPA). The slope of the graph between \( C_r \) against \( r \) on a double logarithmic coordinate gives the fractal dimension \( D_2 \). The distance \( r \) between two events, \( \theta_1, \phi_1 \) and \( \theta_2, \phi_2 \), was calculated as given by Hirata:

\[ r = \cos^{-1} [\cos \theta_1 \cos \theta_2 + \sin \theta_1 \sin \theta_2 \cos (\phi_1 - \phi_2)] \]

In the practical calculation, the fractal dimension analysis is based on a power law which is turned into a linear law after logarithmic transformation. Therefore, sufficient data points are the key to a reliable estimate of fractal dimension based on the ensuing linear regression. Smith suggested the minimum number of points or events required for a reliable calculation of a correlation dimension as:

\[ N_{min} \geq \{R(2-Q)/r(1-Q)\}^\mu \]

where \( Q \) is a quality factor and \( 0 < Q < 1; R = r_{max}/r_{min} \), where \( r \) is a scale to calculate \( C_r \) and \( \mu \) is the greatest integer, less than the obtained fractal dimension of the set. The smallest topological dimension in which the distribution of epicenters embeds is 2, therefore the value of \( D_2 \) will be \(<2\), hence \( \mu = 1 \). If \( Q = 0.95 \) and \( r = 4 \), we will have \( N_{min} \geq 42 \).

Box counting method: In the box counting method a fault system is assumed to be enclosed by a square region with a side length say \( R_0 \). The same region can be divided into several square boxes \((R_0/r^2)\) of side length \( r \). Let \( N(r) \) be the number of boxes that enclose the fault system. If a fault system has a self-similar structure, it is represented as:

\[ N(r) = r^{-D} \]

where \( D \) is a fractal dimension more specifically the capacity dimension. Practically, if \( N(r) \) is plotted against \( r \) on double logarithmic scale. The slope \((-D)\) of the least-square fit line estimates the value of fractal (capacity) dimension. In this study the box-counting method is used to obtain the fractal dimension of the active West Andaman fault system, that generated maximum number of aftershocks in the region.
Results and Discussion

Fractal dimension $D_2$ (i.e. the correlation dimension), in general we call it $D$ here, provides a measure of the degree of fractal clustering of points in the space. Tosi\textsuperscript{15} illustrated that possible values of fractal dimension are bound to range between 0 and 2, which is dependent on the dimension of the embedding space. Interpretation of such limit values is that a set with $D_2 \sim 0$ has all events clustered into one point. At the other end of the scale, $D_2 \sim 2$ indicates that the events are randomly or homogenously distributed over a two-dimensional embedding space and $D_2 \sim 1$ implies that the events are distributed along a line.

The $D_2$ contours show a prominent N-S trend with fractal dimension of $\sim 1$ (0.90-1.30). This corresponds to the active West Andaman Fault (WAF) in the region (Fig. 3). The $D_2 \sim 1.0$ infers a linear nature of the fault system. The fractal dimension values increase ($D_2 > 1.5$) on the both sides of the WAF. The higher $D_2$ values correspond with the active Andaman Trench to the west and the Andaman spreading Ridge (ASR) to the east. The fractal dimension map clearly demarcates the fractal nature of the complex geological structures beneath the ocean. The WAF is a linear fault system with $D_2 \sim 1.0$, and the Andaman Trench and the ASR as two dimensional geological structures with $D_2 \sim 2.0$.

The WAF is geologically, as well as seismically, a well identified active fault and the fractal contours show a remarkable linear trend (Fig. 3). The box counting method is suitable to estimate fractal

Fig. 3—Contours of fractal dimension in the study region
dimension of such well identified linear fault. Here we have taken $R_0 = 400$, changing the length $r$ of each box from 12.5 km ($=R_0/(2^5)$) to 200 km ($=R_0/5$) on the map (Fig. 3). The number of boxes that the fault line enter $N(r)$ was plotted against log $r$. The graph is almost linear between the chosen values of $r$, which indicated that the fault system has a self-similar structure over this range of scales. The fractal dimension $D$ was calculated by the least square method. The value is 1.17 which is the fractal dimension of a line. The fractal dimension values for the entire region vary between 0.90 and 1.60 suggesting that the faults are spatially distributed in the whole region, and the whole region is seismically active. The adjoining areas show fractal dimension ~1.6 that implies heterogeneous activity in the surrounding complex tectonic structures in the sea basin. The fractal dimension mapping has been useful to assess the fractal characteristics of the sea basin structures.

**Conclusion**

The Andaman-Nicobar region is one of the most seismically vulnerable areas in the southeast Asia. The region is associated with complex active marine structures, like deep oceanic trench, back arc spreading ridge, active faults that are linked with the inland faults etc. The fractal dimension map is related to the seismically active faults/structures; the linear and extent of 2-D heterogeneity structures in the Andaman sea basin are identified. The West Andaman fault, is reflected as having a fractal dimension value ~1.0 that implies cluster of activity along the fault line. The higher fractal values trending towards 2.0 are obtained for the Andaman Trench and the Andaman Sea Ridge on both sides of the linear West Andaman fault, that indicates two dimensional complex sea basin structures that are also seismically active.

**References**