Simulation of yarn stress relaxation and creep behaviors using genetic algorithm

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Eyring’s non-linear visco-elastic model has been used to describe two important time dependent mechanical behaviors for textile materials such as stress relaxation and creep. Three yarns suitable for the production of stretch fabrics, viz. 100% wool, wool–lycra and polyester–wool–lycra spun from siro spinning technology are considered in this study. The complex mathematical equations of Eyring’s model are handled by a nontraditional evolutionary algorithm such as genetic algorithm. The findings show that Eyring’s model not only simulates both stress relaxation as well as creep behaviors of the experimental yarns with reasonable degree of accuracy, but also deciphers the underlying molecular mechanism of the two behaviors for these yarns.

Keywords: Creep, Eyring’s model, Genetic algorithm, Stress relaxation, Visco-elasticity, Yarn

1 Introduction

The mechanical properties of a textile material are the time dependent phenomena because of its visco-elastic nature combining features of both elasticity and viscosity. Stress relaxation and creep are the two important time dependent mechanical behaviors of yarns manifesting the visco-elasticity. One way of analyzing the time dependent mechanical behavior is to use linear visco-elastic models composed of one or several elements such as ideal viscous dashpots obeying Newton’s law of viscosity and ideal elastic springs obeying Hook’s law. Maxwell and Voigt-Kelvin models are such types of models which consist of a single spring and a single dashpot in series and parallel respectively. However, neither of these models is adequate in explaining the general behavior of a visco-elastic material where it is necessary to describe both stress relaxation and creep. An arrangement in different orders of ideal springs and dashpots may lead to various linear visco-elastic models capable of describing the general behavior of a visco-elastic material. For an example a ‘standard linear solid’ or Zener model consisting of three elements provides an approximate description of both stress relaxation and creep. A four-element linear visco-elastic model enables even a better explanation of both the time dependent behaviors. However, the linear visco-elastic models are restricted to a linear dependence of stress i.e. if all the stress values of a given sequence are doubled, all the strain values will also be doubled. Moreover, a textile polymer usually shows a linear visco-elastic behavior for a short period of time at a given stress level, but its behavior becomes markedly non-linear for a long period of time at the same level of stress. Furthermore, a linear visco-elastic model does not provide any physical insight into the mechanism of time dependent behavior.

A non-linear visco-elastic model was developed by Eyring and coworkers on the assumption that the deformation of a polymer involves the motion of chain molecules or parts of a chain molecule over potential energy barriers. The most attractive feature of the Eyring’s model is that it offers the possible identification of molecular mechanism and hence helps in unraveling some aspects of structure-dependence of mechanical behavior. In addition, it provides a common basis to explain both stress relaxation and creep behaviors of polymers. Although the Eyring’s model was developed way back to 40’s decade of the last century, its applications have been limited to the textile materials due to the mathematical rigors involved in the computational works. Even in these few reported
works many assumptions are made to simplify the equations derived from Eyring’s model. With the advent of very high computational speed and mathematical techniques it has now been possible to solve this problem using the original equations. In this study, an attempt has been made to describe the stress relaxation and creep behaviors of the core-spun worsted yarns intended for stretch fabrics by using Eyring’s model with the help of genetic algorithm.

2 Eyring’s Visco-elastic Model

Eyring’s three elements visco-elastic model is shown in Fig. 1. For the spring in the right hand arm of the model, the stress-strain relationship is given by the following relationship:

$$\varepsilon_1 = \frac{\sigma_1}{E_1} \quad \ldots(1)$$

where \(\varepsilon_1, \sigma_1\) and \(E_1\) are the strain, stress and the modulus of the spring respectively. By differentiating Eq. (1) we have

$$\frac{d\varepsilon_1}{dt} = \frac{1}{E_1} \frac{d\sigma_1}{dt} \quad \ldots(2)$$

For the dashpot, the strain rate of the non-Newtonian fluid is represented by the hyperbolic-sine law of viscous flow, as shown below:

$$\frac{d\varepsilon_2}{dt} = A \sinh \alpha \sigma_1 \quad \ldots(3)$$

where \(\varepsilon_2\) and \(\sigma_1\) are the strain and stress of the dashpot respectively; \(A\) and \(\alpha\), the two constants of the non-Newtonian fluid. Constant \(A\) and \(\alpha\) are the indirect measures of activation free energy and activation volume of the flow respectively. As the spring and dashpot are in series, the total strain of the right arm is given by the following relationship:

$$\varepsilon = \varepsilon_1 + \varepsilon_2 \quad \ldots(4)$$

By differentiating Eq. (4) we have

$$\frac{d\varepsilon}{dt} = \frac{d\varepsilon_1}{dt} + \frac{d\varepsilon_2}{dt} \quad \ldots(5)$$

Substituting the relations from Eqs (2) and (3), Eq. (5) becomes

$$\frac{d\varepsilon}{dt} = \frac{1}{E_1} \frac{d\sigma_1}{dt} + A \sinh \alpha \sigma_1 \quad \ldots(6)$$

For the spring in the left hand arm of the model, the stress-strain relationship is given by the following equation:

$$\varepsilon = \frac{\sigma_2}{E_2} \quad \ldots(7)$$

where \(\varepsilon, \sigma_2\) and \(E_2\) are the strain, stress and the modulus of the left hand spring respectively. By differentiating Eq. (7) we have

$$\frac{d\varepsilon}{dt} = \frac{1}{E_2} \frac{d\sigma_2}{dt} \quad \ldots(8)$$

As the right and left arms of model are in parallel, the total stress \(\sigma\) is given by the following equation:

$$\sigma = \sigma_1 + \sigma_2 \quad \ldots(9)$$

By differentiating Eq. (9) we have

$$\frac{d\sigma}{dt} = \frac{d\sigma_1}{dt} + \frac{d\sigma_2}{dt} \quad \ldots(10)$$

Eliminating the relations from Eqs (6) – (8), Eq. (10) becomes

$$\frac{d\sigma}{dt} = \frac{d\varepsilon}{dt} (E_1 + E_2) - E_1 \sinh(\sigma - E_2 \varepsilon) \alpha \quad \ldots(11)$$

For stress relaxation, \(\varepsilon = \) constant, hence \(\frac{d\varepsilon}{dt} = 0\).

From Eq. (11) it can be deduced that

$$\sigma(t) = E_2 \varepsilon + \frac{2}{\alpha} \tanh^{-1}\left\{ e^{-a \varepsilon_1 t} \left[ \tanh\left( \frac{E_2 \varepsilon}{2} \alpha \right) \right] \right\} \quad \ldots(12)$$

at \(t = 0, \sigma(0) = (E_1 + E_2)\varepsilon\); and at \(t = \infty, \sigma(\infty) = E_2 \varepsilon\).
For creep, $\sigma = \text{constant}$, hence $\frac{d\sigma}{dt} = 0$. From Eq. (11) it can be shown that

$$\varepsilon(t) = \frac{\sigma}{E_2} - \frac{2}{\alpha E_2 \tanh^{-1}} \left( e^{-\frac{\alpha E_2 \varepsilon(t)}{E_1 + E_2}} \tanh \left( \frac{\sigma - E_2 \varepsilon(t)}{2} \right) \right)$$

...(13)

At $t = 0$, $\varepsilon(0) = \frac{\sigma}{(E_1 + E_2)}$ and at $t = \infty$, $\varepsilon(\infty) = \frac{\sigma}{E_2}$

Eqs (12) and (13) represent the expressions of stress relaxation and creep respectively as a function of time for the Eyring’s model.

3 Materials and Methods

3.1 Materials

Three yarns, viz. 100% wool (W), wool–lycra (W-L) of 97:3 blend ratio and polyester–wool–lycra (P-W-L) of 52:45:3 blend ratio, each having 36 tex nominal count, spun from siro spinning technology were used in this study. Usual commercial method of worsted spinning was used to produce two rovings one of which is 100% Australian merino wool of 22 \(\mu\)m and another of its blend with 3 denier polyester staple fibre. A total of 100% wool yarn was spun by feeding two rovings in each drafting unit of ring frame. By adopting the same technique, other two blended yarns, viz. W-L and P-W-L, were also spun by feeding a lycra filament (40 denier) at the nip of the delivery roller such that lycra at the core gets enwrapped with a drafted sheath of staple fibres from twin rovings of pure wool and polyester-wool blend respectively. These worsted core-spun yarns are chosen because they are suitable for the production of stretch fabrics for which stress relaxation and creep behaviors have great practical significance.

3.2 Methods

All the yarn samples were conditioned for 24 h at the standard atmospheric condition of 65% RH and 27°C before the experiments.

3.2.1 Determination of Stress Relaxation

Stress relaxation phenomenon of these yarns was observed by holding each specimen between two jaws with an initial separation of 200 mm in the Instron tensile tester at a pretension of 0.5 cN/tex. Each specimen was then extended up to a strain level of 15% by moving the upper jaw and was constrained to remain at that strained condition by stopping the upper jaw. The stress values were recorded over 1h at regular intervals.

3.2.2 Determination of Creep

The measurement of creep of all yarns was carried out on a specially designed set up by suspending a 200 mm length of sample from a hook fixed to a wooden stand. Each sample was given a pretension of 0.5 cN/tex by a paperclip. After taking the initial reading, a predetermined load equal to 60% of the average breaking load of each yarn was suspended from the free end of the sample. The extension of the sample was measured by a traveling microscope at different intervals of time starting from 30 s onward till 1 h.

3.2.3 Curve Fitting Using Genetic Algorithm

In least square method of curve fitting, the sum of square of the distances from the theoretical points to the experimental ones is required to be minimized. The intricate nature of Eq. (12) makes it difficult to obtain the best fitted curve on the experimental stress relaxation data by means of classical optimization method. Non-traditional searched based optimization technique such as genetic algorithm (GA)\textsuperscript{10,11} is an appropriate method to solve such type of complex problem. GA mimics nature’s evolutionary principles to drive its search towards an optimal solution. One of the striking differences between GA and classical optimization algorithms is that the latter use a point-by-point approach, whereas the GA works with a population of solutions instead of a single solution. GA proceeds by randomly generating an initial population of individuals, which should ideally cover the domain to explore. Each individual is represented by a binary coded string or chromosome encoding a possible solution in the data space. At every iteration step or generation, the individuals in the current population are tested according to the fitness function. To form a new population (the next generation), good individuals are selected according to their fitness, which is termed as reproduction. New individuals in the search space are generated by two operations, namely crossover and mutation. Crossover concerns two selected individuals (parents) that exchange parts of their chromosome to form two new individuals (offsprings). The mutation
operation is used as a means to achieve a local change around the current solution, i.e. if a solution gets stuck at the local minimum, mutation may help it to come out of this situation and consequently, it may jump to the global basin.

The sum of squares of the distances from the theoretical points obtained with Eq. (12) to the experimental points of stress relaxation for each yarn was minimized using GA by means of MATLAB (version 7.7) coding on a 2.6 GHz PC to determine the optimum values of $E_1$, $E_2$, $A$ and $\alpha$.

4 Results and Discussion

The experimental and fitted stress relaxation curves obtained with Eq. (12) for W, W-L and P-W-L yarns are depicted in Fig. 2. The experimental curves are shown by the solid lines and the corresponding fitted curves are shown by the dotted line. Invariably for each case, a high degree of coefficient of determination ($R^2$) justifies a good fit to the experimental data. A least square method of curve fitting using GA on the experimental stress relaxation data with Eq. (12) computes the optimum values of $E_1$, $E_2$, $A$ and $\alpha$ for the Eyring’s model. Table 1 shows the optimum values of these constants for three different yarns. The creep curves are then constructed with Eq. (13) using the values of four constants obtained from the stress relaxation experiment. The experimental creep curves along with those predicted by the Eyring’s model for W, W-L and P-W-L yarns are illustrated in Fig. 3. High values of $R^2$ substantiate the fact that the Eyring’s model is able to predict the yarn creep curves reasonably well.

It is evident from Table 1 that the values of $E_1$, $E_2$ and $A$ are higher for P-W-L yarn, followed by W and W-L yarns, whereas it shows a opposite trend for $\alpha$. The higher values of $E_1$ and $E_2$, which correspond to the spring constants of the Eyring’s model, strengthen the elastic part of the yarn structure. Constants $A$ and $\alpha$ correspond to the activation free energy and activation volume of the flow respectively for the non-Newtonian dashpot of Eyring’s model. A stiff dashpot shows higher value of $A$ and lower value of $\alpha$ in comparison to a slack dashpot. As the viscous part overwhelmingly dominates in lycra, in model equivalent of W-L yarn, the values of $E_1$, $E_2$ and $A$ drop but $\alpha$ rises.

<table>
<thead>
<tr>
<th>Constants</th>
<th>W yarn</th>
<th>W-L yarn</th>
<th>P-W-L yarn</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_1$, cN/tex</td>
<td>25.03</td>
<td>24.97</td>
<td>25.912</td>
</tr>
<tr>
<td>$E_2$, cN/tex</td>
<td>25.42</td>
<td>23.82</td>
<td>27.288</td>
</tr>
<tr>
<td>$A$, s$^{-1}$</td>
<td>6.9 x 10$^{-7}$</td>
<td>6.8 x 10$^{-7}$</td>
<td>9 x 10$^{-6}$</td>
</tr>
<tr>
<td>$\alpha$, tex/cN</td>
<td>3.10</td>
<td>4.29</td>
<td>3.0246</td>
</tr>
</tbody>
</table>

Fig. 2 – Experimental and fitted stress relaxation curves for (a) W, (b) W-L and (c) P-W-L yarns
crystallinity and modulus of polyester in P-W-L yarn helps to strengthen the elastic part of the structure so that in its model equivalent as shown in Fig. 1, $E_1$, $E_2$ and $A$ acquire higher values but $\alpha$ goes down. Therefore, in P-W-L yarn the influence of lycra is overshadowed by that of polyester, and being in high blend proportion it reinforces the elastic part in yarn structure.

5 Conclusion
The stress relaxation and creep behaviors of the W, W-L, and P-W-L yarns have been described using Eyring’s nonlinear visco-elastic model. The advent of nontraditional search based optimization technique such as GA makes it easy to solve the complex curve fitting problem with the Eyring’s equations. Eyring’s model provides a common basis to simulate both stress relaxation as well as creep behaviors of worsted core spun yarns with reasonable degree of accuracy. Further, it can decipher the underlying molecular mechanism of stress relaxation and creep behaviors for different yarns from its parameters.

References