

Field solution within the ionospheric anisotropic plasma in presence of thunderstorm and lightning

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In the presence of thunderstorm and lightning, Maxwell's equations for the ionospheric anisotropic plasma have been solved, considering the influence of geomagnetic field and time-varying random irregularities developing an ac magnetic field. This analysis has been used to study the electrodynamics of middle atmosphere during thunderstorm and lightning. The variations of thundercloud and lightning induced electric field have been numerically estimated for some specific ionospheric height range. The fluctuation of electron number density within the medium is also studied.

Keywords: Ionospheric plasma, Lightning discharges, Anisotropic plasma

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1 Introduction

Theoretical investigations of wave propagation through the anisotropic ionosphere have been conducted by different workers using various formalism¹⁻⁵. The dielectric tensor for the ionospheric medium under different physical situations is used to explore various states and properties of ionospheric plasma^{4,6-9}.

Studies on wave propagation through the ionospheric medium containing random irregularities have contributed a lot towards our knowledge and understanding in this subject during the past few decades^{9,10}, assuming the background medium to be either isotropic or anisotropic. The fluctuation of electron density during thunderstorm and lightning discharges affect wave propagation in the lower ionosphere. This is very common in the global atmosphere, more specifically, in the tropical latitude ($\pm 25^\circ$) and temperate latitude ($\pm 60^\circ$), where ionospheric effects due to thundercloud and lightning discharge fields are mostly pronounced¹¹. The existence of electric fields at higher altitudes following sudden separation of thundercloud charges¹² by lightning discharge at low altitudes heat the mesospheric electrons that produce ionization. The process may be enhanced further by the influence of zonal and meridional winds. Thunderstorm and lightning produce ELF/VLF waves, which interact with the medium in a non-linear manner, changing the electron collision frequency and ionizing

frequency¹³⁻¹⁷. These would introduce random perturbations, producing different types of irregularities in the medium. Fluctuation of electron density is detectable in the upper ionosphere in backscatter observations with powerful radar systems¹⁸. The expression of scattering cross-section has been deduced from the fluctuations in the refractive index, which is assumed to be due solely to the fluctuations of electron density.

During heavy thunderstorm, lightning induced enhancement of ionization introduce changes in electron density in the lower ionosphere, causing amplitude and phase perturbation on sub-ionospheric VLF propagation¹⁹.

The return current from the ground during lightning continues to move upward and terminate in the lower region of the ionosphere²⁰. The redistribution of charge and the electromagnetic pulse during discharge produce acceleration of electrons, heating and ionization in the medium, leading to a non-linear situation²¹. Electron density and collision frequency are also modified during heavy thunderstorm and lightning²².

The results of thunderstorm sounding rocket experiments support the generation of electrostatic emissions by lightning-induced Whistler-mode radiation above thunderstorms. The emissions are supposed to be responsible for the perturbation in density in the medium during lightning²³.

The excitation and propagation of lightning generated electromagnetic signals are highly influenced by the random electrical properties of the atmosphere. The signal waveforms and amplitudes get modified in space-time domain²⁴. The complexity of the dynamics of the ionosphere is further enhanced due to the influence of ionization and recombination processes, which give rise to charge separation²⁵. This initiates the polarized electric field to propagate along the magnetic field lines in the form of Alfvén waves.

Atmospheric electricity is also affected by geomagnetic storms along with thunderstorm and lightning discharges. Geomagnetic storms introduce the largest global atmospheric effects among different other manifestations. The lower ionosphere responds very sharply to geomagnetic storms. Electron density is enhanced significantly, specially in the auroral zone²⁶.

In the stated circumstances, the electron density may be considered to change by an amount ΔN from its ambient value N . The fluctuation of electron density affects wave propagation. Several workers considered theoretically the effects of irregularities on radio waves under different assumptions^{27,28}.

Propagation of ELF/VLF waves generated due to thunderstorm and lightning activities may introduce perturbation in the magnetic field over and above the static geomagnetic field. In this paper, the form of the dielectric tensor²⁹ under such circumstances has been used to derive the field solution of Maxwell's equations. The variations of electric field for a specific height range have been numerically evaluated. The fluctuation of electron number density with frequency during intra-cloud discharges is also studied. The results are presented graphically.

2. Mathematical formulation

The Lorentz force equation for the stated medium can be written as

$$\frac{\partial \mathbf{v}}{\partial t} = \frac{e}{m} \left(\mathbf{E} + \frac{\mathbf{v} \times \mathbf{B}}{c} \right) - \eta \mathbf{v} \quad \dots (1)$$

Here, e is the charge of the electron; m , the mass of the electron; \mathbf{v} , the average velocity of an electron; \mathbf{E} , the electric field; \mathbf{B} , the external magnetic field and η

is the collision factor. The dielectric tensor ($\bar{\epsilon}$) has been derived from Eq. (1) using appropriate relations²⁹.

In presence of ELF/VLF waves generated due to thunderstorm and lightning, the effective magnetic field \mathbf{B} can be written as

$$\mathbf{B} = \mathbf{B}_0 + \mathbf{B}_1 e^{j\omega t} \quad \dots (2)$$

\mathbf{B}_0 is the earth's magnetic field which is taken to be in the z-direction and $\mathbf{B}_1 e^{j\omega t}$ represents the magnetic field contributed by the presence of other types of waves²⁹.

Introducing dielectric polarization $\mathbf{P} (= N e \mathbf{r})$ and Eq. (2), the Eq. (1) under Fourier transform through the aid of Faltung theorem yields

$$e \mathbf{E} = \frac{m}{N} (j\omega \eta - \omega^2) \mathbf{P} - \frac{j\omega \mathbf{P}}{N e} \times [\mathbf{B}_0 + \mathbf{B}_1 \delta(\omega - \omega_0)] \quad \dots (3)$$

where δ is Dirac delta function and N the electron number density.

From Eq. (3), the expression for the dielectric tensor has been derived as

$$\begin{aligned} \left(\bar{\epsilon} \right) &= \begin{pmatrix} 1 - XX_1 & jXX_2 & 0 \\ -jXX_2 & 1 - XX_2 & 0 \\ 0 & 0 & 1 - \frac{X}{\omega^2} \end{pmatrix} \\ &+ \eta X \begin{pmatrix} jX_3 & -X_4 & 0 \\ X_4 & -jX_3 & 0 \\ 0 & 0 & -\frac{j}{\omega^3} \end{pmatrix} \\ &+ \frac{XX_1 \delta(\omega - \omega_0)}{\omega^2} \begin{pmatrix} -R_1 & jR_2 & R_3 - jR_4 \\ jR_2 & -R_1 & R_5 + jR_6 \\ R_3 + jR_4 & R_5 - jR_6 & 0 \end{pmatrix} \quad \dots (4) \end{aligned}$$

Here

$$X = \omega_p^2, \quad X_1 = + \frac{1}{\omega^2 - \omega_c^2}, \quad X_2 = + \frac{j\omega_c}{\omega(\omega^2 - \omega_c^2)^2},$$

$$X_3 = + \frac{\omega^2 - \omega_c^2}{\omega(\omega^2 - \omega_c^2)^2}, \quad X_4 = \frac{2\omega_c}{(\omega^2 - \omega_c^2)^2},$$

$$\begin{aligned}
R_1 &= \frac{2\omega^2 \omega_c \omega_{cx}}{\omega^2 - \omega_c^2}, \quad R_2 = \frac{\omega(\omega^2 + \omega_c^2) \omega_{cz}}{\omega^2 - \omega_c^2}, \\
R_3 &= \omega_c \omega_{cx}, \quad R_4 = \omega \omega_{cy} \\
R_5 &= \omega_c \omega_{cx}, \quad R_6 = \omega \omega_{cx}, \quad \omega_p^2 = + \frac{4\pi N e^2}{m}, \\
\omega_c &= + \frac{eB_0}{mc}, \quad \omega_{cx} = + \frac{eB_{lx}}{mc}, \quad \omega_{cy} = + \frac{eB_{ly}}{mc}, \\
\omega_{cz} &= + \frac{eB_{lz}}{mc}
\end{aligned}$$

In the analysis, the positive ions in the lower ionosphere are considered to be stationary and only the electrons will affect the plasma waves. The situation may be governed by the following Maxwell's equations:

$$\left. \begin{aligned}
\nabla \times \mathbf{E} &= -\frac{1}{c} \frac{\partial \mathbf{H}}{\partial t} \\
\nabla \times \mathbf{H} &= -\frac{(\bar{\epsilon})}{c} \frac{\partial \mathbf{E}}{\partial t} \\
\nabla \cdot \mathbf{H} &= 0
\end{aligned} \right\} \dots (5)$$

The wave equation for the electric field can be given by

$$\nabla^2 \mathbf{E} = -\frac{\omega^2}{c^2} (\bar{\epsilon}) \mathbf{E} \quad \dots (6)$$

The fluctuation in electron number-density for the stated situation may be considered as

$$X = \langle X \rangle + \Delta X \quad \dots (7)$$

$$\text{where} \quad \Delta X \ll \langle X \rangle$$

For planar stratification, introducing Eq. (7) in Eq.(6), one can get the following coupled equations:

$$\begin{aligned}
\nabla^2 E_x + k_o^2 \{ 1 - \langle X \rangle [X_l + j\eta X_3] \\
+ \frac{X_1 R_1}{\omega^2} \delta(\omega - \omega_o)] \} E_x &= \frac{\omega^2}{c^2} \langle X \rangle \{ \eta X_4 - j [X_2 \\
+ \frac{X_1 R_1}{\omega^2} \delta(\omega - \omega_o)] \} E_y +
\end{aligned}$$

$$\begin{aligned}
+ \frac{\omega^2}{c^2} \Delta X \{ \{ X_1 [1 + \frac{R_1}{\omega^2} \delta(\omega - \omega_o)] + j\eta X_3 \} E_x + \\
+ \{ \eta X_4 - j [X_2 + \frac{X_1 R_1}{\omega^2} \delta(\omega - \omega_o)] \} E_y \} \dots (8)
\end{aligned}$$

$$\begin{aligned}
\nabla^2 E_y + k_o^2 \{ 1 - \langle X \rangle [X_l + j\eta X_3 + \frac{X_1 R_1}{\omega^2} \delta(\omega - \omega_o)] \} E_y \\
= \frac{\omega^2}{c^2} \langle X \rangle \{ \eta X_1 - j X_2 - \frac{X_1 R_1}{\omega^2} \delta(\omega - \omega_o) \} E_x + \\
+ \frac{\omega^2}{c^2} \Delta X \{ \{ -\eta X_1 + j X_2 + \frac{X_1 R_1}{\omega^2} \delta(\omega - \omega_o) \} E_x + \\
+ \{ X_1 [1 + \frac{R_1}{\omega^2} \delta(\omega - \omega_o)] + j\eta X_3 \} E_y \} \dots (9)
\end{aligned}$$

In the absence of ΔX , two usual characteristic modes of wave propagation are obtained and as a far field solution, one may write

$$E_{0x} = \frac{1}{r} e^{j\mathbf{k} \cdot \mathbf{r}}, \quad E_{0y} = \frac{j}{r} e^{j\mathbf{k} \cdot \mathbf{r}}$$

Under the stated disturbed situation, the electric fields given by Eqs (8) and (9) may be represented as

$$E_x = E_{ox} e^{-j\phi} \text{ and } E_y = E_{oy} e^{-j\phi} \quad \dots (10)$$

where ϕ is a small order quantity. For the two modes, ϕ will be replaced by $\phi^{(1)}$ and $\phi^{(2)}$. Introducing Eq. (10) in Eqs (8) and (9), one obtains

$$\begin{aligned}
E_{ox} \nabla^2 \phi + 2 \nabla E_{ox} \cdot \nabla \phi \\
= \frac{j\omega^2}{c^2} \Delta X \{ \{ X_1 [1 + \frac{R_1}{\omega^2} \delta(\omega - \omega_o)] + \\
+ j \eta X_3 \} E_{ox} + \{ \eta X_4 - j [X_2 + \frac{X_1 R_1}{\omega^2} \delta(\omega - \omega_o)] \} E_{oy} \} \dots (11)
\end{aligned}$$

$$\begin{aligned}
E_{oy} \nabla^2 \phi + 2 \nabla E_{ox} \cdot \nabla \phi \\
= \frac{j\omega^2}{c^2} \Delta X \{ \{ -\eta X_4 + j X_2 + \frac{X_1 R_1}{\omega^2} \delta(\omega - \omega_o) \} E_{ox} + \\
+ \{ X_1 [1 + \frac{R_1}{\omega^2} \delta(\omega - \omega_o)] + j\eta X_3 \} E_{oy} \} \dots (12)
\end{aligned}$$

Replacing E_{oy} by jE_{ox} in Eqs (11) and (12)

$$\begin{aligned} & E_{ox} \nabla^2 \phi + 2 \nabla E_{ox} \cdot \nabla \phi \\ &= \frac{j\omega^2}{c^2} \Delta X [X_1 + X_2 + \frac{X_1(R_1+R_2)}{\omega^2} \delta(\omega - \omega_0) + \\ &+ j\eta (X_3 + X_4)] E_{ox} \quad \dots (13) \end{aligned}$$

$$\text{Substituting } \phi = \frac{U}{E_{ox}},$$

$$\begin{aligned} \nabla^2 U + K^2 U &= \frac{j\omega^2}{c^2} \Delta X [X_1 + X_2 + \\ &\delta(\omega - \omega_0) + j\eta (X_3 + X_4)] E_{ox} \quad \dots (14) \end{aligned}$$

The solution of Eq. (14) can be derived as

$$\begin{aligned} U &= \frac{j}{k^2} \frac{\omega^2}{c^2} \Delta X [X_1 + X_2 + \frac{X_1(R_1+R_2)}{\omega^2} \delta(\omega - \omega_0) + \\ &+ j\eta (X_3 + X_4)] \frac{e^{-jk.r}}{r} \quad \dots (15) \end{aligned}$$

Thus

$$\begin{aligned} \phi &= \frac{j}{k^2} \frac{\omega^2}{c^2} \Delta X [X_1 + X_2 + \frac{X_1(R_1+R_2)}{\omega^2} \delta(\omega - \omega_0) + \\ &+ j\eta (X_3 + X_4)] \end{aligned}$$

Substituting ϕ , one obtains

$$\begin{aligned} E_x &= \frac{1}{r} e^{jk.r} \exp \\ &\left[\frac{\omega^2}{k^2 c^2} \Delta X \left\{ X_1 + X_2 + \frac{X_1(R_1+R_2)}{\omega^2} \delta(\omega - \omega_0) + j\eta (X_3 + X_4) \right\} \right] \quad \dots (16) \end{aligned}$$

In a similar manner, the other field components can be determined.

3 Perturbed electric field

For the first order variables, Maxwell's equations yield the wave equation as

$$\nabla(\nabla \cdot \mathbf{E}_1) - \nabla^2 \mathbf{E}_1 = \frac{\omega_0^2}{c^2} (\bar{\bar{\epsilon}}_0) \mathbf{E}_1 \quad \dots (17)$$

where $(\bar{\bar{\epsilon}}_0)$ is the unperturbed component of the dielectric tensor given by Eq. (4).

Energetic intra-cloud discharges are isolated lightning events that occur during thunderstorms. These produce powerful HF/VHF radiation and distinctive electric pulses³⁰⁻³⁶.

Numerical computation of Eq. (17) has been carried out for the lower region of the ionosphere under the stated physical situation. The results are given in Table 1 for an arbitrary value of frequency. The IRI data, published from National Physical Laboratory, New Delhi, 1979, are taken for numerical calculation. It is seen that the perturbed electric field in the E-region of the ionosphere is nearly 1 mVm^{-1} . The variation is due to fluctuations in the magnitude and nature of disturbances causing the perturbation.

4 Derivation of ΔN

The expression for electron density fluctuation may be deduced from the equation of continuity. Due to fluctuation

$$\rho = (N + \Delta N) m \quad \dots (18)$$

Thus, the continuity equation yields

$$\frac{\partial^2}{\partial t^2} (\Delta N) = N \nabla \cdot \mathbf{v} - \mathbf{v} \cdot \nabla N$$

$$\frac{\partial^2}{\partial t^2} (\Delta N) = \frac{e}{m} (\mathbf{E} \cdot \nabla N) - \frac{4\pi N e^2}{m} \Delta N$$

Using Eq. (18), one can obtain

$$\Delta N = + \frac{e}{m(\omega_p^2 - \omega^2)} E_x \frac{dN}{dx} = \frac{e}{m(\omega_p^2 - \omega^2)} E_x \frac{N}{h} \quad \dots (19)$$

Here N/h is taken to be constant

E_x is the x -component of the electric field.

Table 1—Numerical results of the perturbed electric field (at frequency = 2.2 MHz)

Height, km	Perturbed electric field, mVm^{-1}
85	0.85
90	0.88
95	0.90
100	0.96
105	0.99
110	1.08
115	1.10
120	1.12

5 Results and discussion

Wave propagation through a medium in presence of VLF/ELF waves during strong thunderstorm and lightning discharges is studied in the anisotropic background. Mode coupling equations are obtained, the solution of which has been derived. Numerical computations using Eq. (16) are made to calculate the effective electric field for a specific height range of the ionosphere (Fig. 1). The CIRA 72 data (COSPAR International Reference Atmosphere 72 data), the EISCAT data (European Incoherent Scatter data) and the data obtained from C2 recorder at Haringhata field station (Lat. 22°58' N, Long. 88°34' E, University of

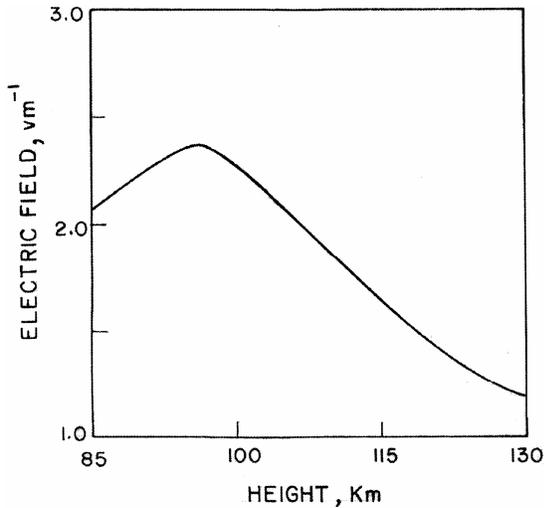


Fig. 1—Variation of electric field for the ionospheric height range between 85 and 130 km

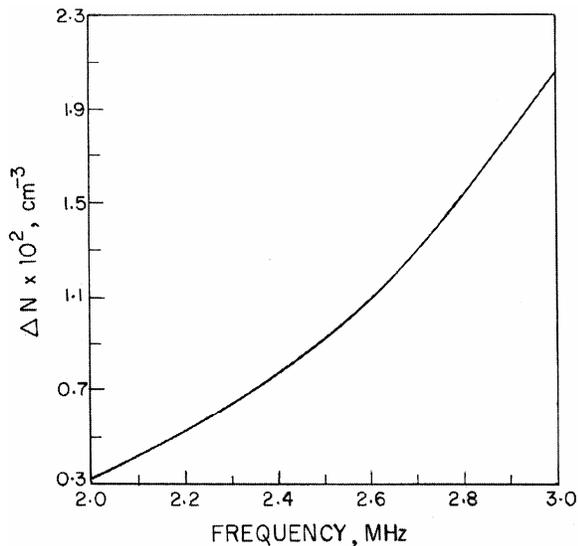


Fig. 2—Variation of ΔN with ω for the ionospheric height range between 85 and 130 km

Calcutta) are used in the numerical analyses. The calculated electric field at 85 km is about 2.05 V/m. It increases up to a height of 92 km and then falls to 1.2 V/m at 130 km. Data are chosen for E-layer height range. The variation may be attributed due to non-linearity of the medium.

From the numerical analysis using Eq. (19), the variation of ΔN with ω is plotted in Fig. 2 for the E-layer of the ionosphere. The parametric values have been chosen as:

$$\omega_p^2 = 1.76 \times 10^{12}, \quad \omega_c = 1.22 \text{ MHz},$$

$$N = 1.5 \times 10^5 \text{ cm}^{-3}.$$

The propagation frequency is chosen between 2 and 3 MHz. The increase in electron number density with frequency may be caused in presence of energetic intra-cloud discharges producing powerful radio frequency radiation. The asymptotic behaviour of the graph introduces a cut-off in the frequency range for a particular layer.

In brief, it can be said that the present analysis can be used to investigate the middle atmospheric electrodynamic phenomena during thunderstorm and lightning.

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