Computational scheme and effect of complex multi-islands terrain in numerical calculation of waves in Archipelago

Ke-feng Mao1,2, Xue-feng Yao3 & Zhong-le Xiao3

1Institute of Meteorology and Oceanography, PLA University of Science and Technology, Nanjing 211101, China
2State Key Laboratory of Satellite Ocean Environment Dynamics, Second Institute of Oceanography, State Oceanic Administration, Hangzhou 310012, China
3No.96631 army of PLA, Beijing, 102208, China

[E-mail: oceanlgdx@163.com]

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In view of complex multi-island terrain in archipelagic sea areas, the paper designs a computational scheme to optimize computational grid by integrated utilization of water depth data and high-resolution coastline data with an introduction of topographic subgrid-scale effect. Numerical tests under the two typical weather conditions of cold air and typhoon in Ryukyu Islands were conducted using the WAVEWATCH-Ⅲ model. Results were compared with measured data. Results showed that different computational schemes of complex multi-island terrain were the major sources of errors of wave numerical simulation in archipelagic sea areas. Scheme in this paper gave full consideration to the effect of coastal and multi-island terrain on wave propagation, and numerical calculation results were obviously improved.

[Keywords: Ocean waves, Multi-islands terrain, Subgrid, Archipelagic sea area]

Introduction

Ocean wave is one of the most important dynamic factors in marine environment, and its calculation has always been the focus of oceanography and ocean engineering. At present, WAM1 and SWAN2 are developed based on the wave calculation model consisting of spectral wave energy balance equation (Formula 1).

\[
\frac{\partial N}{\partial t} + c_g \nabla N = S \quad \ldots(1)
\]

WAVEWATCH-Ⅲ3 is a representative of the third-generation wave numerical model. In the energy balance equation, \(N\) is wave action density spectrum, which is the function of frequency \(f\), propagation direction \(\theta\), time \(t\) and geographic space. \(c_g\) is group velocity, and \(S\) represents energy source and sink. It considers the energy exchange process, such as energy dissipation caused by absorption of wave energy, wave breaking and white capping, eddy viscosity caused by non-linear interaction among waves, bottom friction at low water depth. When numerical calculation is conducted according to Formula (1), the choice of resolution is a very important issue for the calculation of spectral space \((f, \theta)\) or \((k, \theta)\) and geographic space, which has great influence on precision and efficiency of wave calculation. Multi-islands and multi-reefs in Archipelagoes, such as Ryukyu Islands, Zhoushan Archipelago, have different oceanic conditions and complex terrain. Many factors in sea areas and channels may affect sea waves. Besides wind, the main dynamic mechanism producing sea waves, there are such factors as length and width of water areas in upwind direction, bottom topography and water level which determines the wave height4. To calculate sea waves in archipelagoes more accurately, the grid resolution of wave model should be improved to reflect island terrain distribution, bottom topography and water depth changes. However, the increase of resolution is subject to the limitations of resolution of underwater topography data and wind fields data and computational efficiency. Tolman and Chawla5 pointed out that islands and obstacles not revealed in computational grid of a certain resolution are one of the main error sources of wave calculation and prediction. In order to solve this problem, Holthuijsen6 proposed that the obstacle effect of subgrid-scale obstacle on wave energy propagation...
should be considered in SWAN model. Tolman applied this method in WAVEWATCH-III model by considering the subgrid-scale effect of islands and local sea ice on wave propagation. But what is worthy of further discussion is how to deal with multi-island distribution and complex coastline features in wave numerical calculation, how to design subgrid computational scheme and what is the effect of such methods on wave numerical calculation. In consideration of this, this paper designs a method to optimize the computational grid for wave numerical calculation by integrated use of water depth data and high-resolution coastline data. Meanwhile, the topographic subgrid-scale effect is introduced to conduct numerical test with WAVEWATCH-III model. The effect of this scheme on wave numerical calculation in archipelagic sea area is analyzed by comparing with the measured data.

Materials and Methods

When wave numerical calculation is performed using WAVEWATCH-III model, the computational grid uses “dry or wet” to identify whether the grid points represent land or sea. The “wet” grid of the sea is the effective computational grid while the “dry” grid of land demarcates the boundary of computation. Using this method the multi-island distribution and coastal terrain can be reflected on the scale of computational grid. If only this method is used, the island terrain distribution described would be more refined with the improvement of computational grid resolution. But the resolution of computational grid cannot be improved infinitely. Under the certain resolution level, terrain features, such as islands that are smaller than the computational grid scale, cannot be described. As shown in Fig. 1, there are several islands similar to that marked by A. At this computational resolution, the computational grid points surrounding it are “wet”, and it is neglected in computation. Thus, the topographic subgrid-scale effect should be taken into account for wave numerical calculation in archipelagic sea area. Only by this means, the topographic effect of islands smaller than the computational grid resolution can be included.

As early as the initial debugging of WWATCH1.15, Tolman already pointed out that the island groups not captured by computational grid are the main reasons for the error wave simulation. Tolman adopted the computational scheme in which the topographic subgrid-scale effect is considered in WAVEWATCH-III. Main idea is to restrain energy flux between computational grid points by introducing the obstacle effect of subgrid-scale islands on sea waves. Specific process is as follows: the splitting algorithm is used for numerical solution of formula (1). Depth changes over time and corresponding wave number grid are considered; wave number grid is constant and depth is quasi-stable if the effect of the water surface changes is temporarily neglected. Then, propagation of wave spectrum in physical space, propagation terms in the wave number space on the left side of formula (1) and the source function term on the right of formula (1) can be calculated. In the spherical coordinates, the propagation of waves in the geographic space is expressed as (φ longitude, λ latitude):

\[
\frac{\partial \hat{N}}{\partial t} + \frac{\partial}{\partial \phi} \hat{N} + \frac{\partial}{\partial \lambda} \lambda \hat{N} = 0 \quad \hat{N} = N c_s^2 \cos \phi \quad \text{(2)}
\]

ULTIMATE QUICKEST scheme is used in WAVEWATCH-III in this paper. According to this scheme, the zonal propagation and radial propagation are calculated separately and calculation sequence is arbitrary. In the direction of φ, the flux between i th and i-lth grid points is \( F_{i,-} \).

Fig. 1—Computation grid and islands terrain distribution
\[ F_{i,-} = \left[ \phi_b N_b \right]_{j,l,m}^n \Rightarrow \phi_b = 0.5 \left( \phi_{i-1} + \phi_i \right) \quad \ldots(3) \]

\[ N_b = \frac{1}{2} \left[ \left( 1+C \right) N_{i-1} + \left( 1-C \right) N_i \right] - \left( \frac{1-C^2}{6} \right) C_u \Delta \phi^2 \quad \ldots(4) \]

\[ C_u = \begin{cases} 
\left( N_{i-2} - 2N_{i-1} + N_i \right) \Delta \phi^{-2} & \text{for } \phi_b \geq 0 \\
\left( N_{i-1} - 2N_i + N_{i+1} \right) \Delta \phi^{-2} & \text{for } \phi_b < 0 
\end{cases} \]

\[ C = \frac{\phi_b \Delta t}{\Delta \phi} \quad \ldots(5) \]

Where j, l and m represent latitude \( \lambda \), spectral space \( \theta \) and number of discrete grids in the k direction, respectively; \( n \) is time horizon, \( C_u \) is curvature of the action density, \( C \) is the CFL quantity with symbols, \( \phi_b \) is the propagation velocity between two grid points on the grid point boundary, \( \phi_i \) is the propagation velocity in the grid points, \( N_b \) is the action on grid point boundary, \( N_i \) is the action of the \( i \)th grid point.

When \( |C| \leq 1 \), the scheme has a stable solution. So the scheme in \( \phi \) direction can also be expressed as:

\[ \tilde{N}_{i,j,l,m}^{n+1} = \tilde{N}_{i,j,l,m}^n + \frac{\Delta t}{\Delta \phi} \left[ F_{i,-} - F_{i,+} \right] \quad \ldots(6) \]

Propagation in \( \lambda \) direction can also be obtained by changing subscript and corresponding increment. For numerical calculation and other contents see WAVEWATCH-III technical manual\(^3\). In Formula (6), there may be a small island or obstacle that is not identified by the grid within the unit grid of the \( i \)th computational grid point, as shown in Fig. 2. In order to consider its obstacle effect to wave propagation in this grid, the energy flux penetration coefficients \( \alpha_{\phi}^\lambda \alpha_{\phi} \) in the two directions of the grid boundary are defined, the coefficient range being 0 ~ 1, where 0 indicates that the effect of subgrid-scale island is shown as closed boundary and wave energy flux can be completely impeded; 1 indicates no subgrid-scale islands. The coefficient is used to calculate the topographic subgrid-scale effect of island. Thus formula (6) is rewritten as:

\[ \tilde{N}_{i,j,l,m}^{n+1} = \tilde{N}_{i,j,l,m}^n + \frac{\Delta t}{\Delta \phi} \left[ \alpha_{\phi_i} F_{i,-} - \alpha_{\phi_i} F_{i,+} \right] \quad \ldots(7) \]

Based on existing researches, the paper designs a computational scheme to realize integrated use of water depth data and high-resolution coastline data to optimize the computational grid, with the consideration of topographic subgrid-scale effect. First, calculate the sum of included angles between the grid and polygon\(^8\) to distinguish between the dry and wet attributes of the computational grid, that is, land or sea. Determine the location relations between “wet” marine computational grids and small islands that are not distinguished by the computational grid, and search out the computational grids that require the calculation of the energy flux penetration coefficient; the next step is to calculate the energy flux penetration coefficient \( \alpha_{\phi}^\lambda \alpha_{\phi} \) of this grid.

Specific procedure is as follows: the entire mainland and any island are seen as a polygon with \( n \) sides using some coastlines, islands, data provided by the sea chart, with the vertex sequence of \( p_i (x_i,y_i) (i = 1,2,\cdots,n, p_1 = p_{n+1}) \). Grid of wave numerical computation is taken as \( p (x_p,y_p) \), the point to be determined. Connect P point and each vertex of the polygon and calculate the sum of included angles. Clockwise and counterclockwise directions are positive and negative, respectively, as shown in Fig. 3a & b., If

\[ \sum_{i=1}^{n-1} \angle p_i p_{i+1} + \angle p_n p_1 = \begin{cases} 360^\circ & \text{if clockwise} \\
0^\circ & \text{if counterclockwise} \end{cases} \quad \ldots(8) \]
In Equation 8 when it is 360° Point P is inside the polygon and when it is 360° Point P is outside the polygon. To solve \( \angle p_ip_{i+1} \), \( p(x_p, y_p) \) and \( p_i(x_i, y_i) \) are seen as a vector \( \vec{v}_i \). If the azimuth angle is \( \beta_i \), the included angle \( \alpha_i \) of vector \( \vec{v}_i \) and \( \vec{v}_{i+1} \) can be calculated according to the difference of their azimuth angles.

\[
\alpha_i = \begin{cases} 
\beta_{i+1} - \beta_i, & |\beta_{i+1} - \beta_i| \leq 180^\circ \\
\beta_{i+1} - \beta_i + 360^\circ & |\beta_{i+1} - \beta_i| < -180^\circ \\
\beta_{i+1} - \beta_i - 360^\circ & |\beta_{i+1} - \beta_i| > 180^\circ 
\end{cases}
\] ...

(9)

Thus it can be judged: when \( \sum_{i=1}^{n} \alpha_i = 360^\circ \) P point is inside the polygon; when \( \sum_{i=1}^{n} \alpha_i = 0^\circ \) P point is outside of the polygon. It should be noted that this algorithm may not apply to concave or convex polygons formed by land and islands where there are lakes. As shown in Fig. 3c, the computational grid in the island closed polygon is judged as “dry”, while the grid outside of it is “wet”. In this way, the coastline and multi-island topographic features can be fully reflected on the scale of computational grid. According to the location relations between the “wet” marine computational grid and small islands not distinguished by the computational grid, the grid units that require the calculation of energy flux penetration coefficients \( \phi \) and \( \lambda \), are searched and calculated.

As shown in Fig. 4, \( \alpha_{\phi} = 1 - \frac{\Delta L_{\phi}}{\Delta \phi} \) in the direction of \( \phi \) and \( \alpha_{\lambda} = 1 - \frac{\Delta L_{\lambda}}{\Delta \phi} \) in the direction of \( \lambda \). \( \Delta L_{\phi} \) is the width of island in the \( \phi \) direction, \( \Delta L_{\lambda} \) is the width of grid unit in the \( \lambda \) direction, \( \Delta \phi \) is the width of island in the direction of \( \phi \), \( \Delta \phi \) is the width of grid unit in the direction of \( \phi \).

**Result**

The sea area around the Ryukyu Islands (Fig. 4a) is the only route for China to go out of eastern Pacific Ocean, and an important waterway for countries in the eastern Pacific and Oceania to travel between the East China Sea and the Yellow Sea. Ryukyu Islands have a great strategic significance in military. Eastern deep sea area of continental shelf of the East China Sea is divided by the Ryukyu ridge into a narrow deep sea area—Okinawa Trough in between the continental shelf of the East China Sea and the Ryukyu ridge. In the 600 nM water area from the shallow seas of southeast continental shelf to the northwest Pacific Ocean in the east connected by the Okinawa Through, and from the Kyushu Japan to Ryukyu and Taiwan, there are many channel, waterways and island chains including Iriomote-jima-Ishigaki-jima-Miyko Island, Okinawa Island, Amami Oshima Island, etc. Complex multi-island terrain results in the need for special treatment in wave simulation and forecast in this area. For this reason, the sea areas of Ryukyu Islands were selected for the
Numerical simulation for 1 month was conducted over the typical weather process of winter monsoon and typhoon of the Ryukyu Islands to test the efficiency of this scheme. The results then were compared with observed data.

Major weather process influencing the Ryukyu Islands in winter is the cold air gale, and northeast monsoon is prevailing in this area. In this article, we chose the period from 02:00 of January 1 to 23:00 of January 31 in 2004 to conduct test 1 and test 2. Typhoon is the major weather disaster in the Ryukyu Islands, and disastrous waves would occur under the influences of typhoon. We chose the period (influenced by three typhoons) from 02:00 of October 1 to 23:00 of October 31 in 2004 to perform test 3 and test 4. Range of wave numerical calculation can be seen in Fig. 4a: 20°N~35°N, 116°E~132.75°E, and grid interval is 15’. WAVEWATCH-III considers the physical processes such as interaction between wind and waves, wave-wave interaction, whitecapping, and bottom friction; time step is 120 seconds. QuickSCAT/NCEP mixed wind field was adopted in the January of 2004, while wind field was synthesized with the Holland’s typhoon wind field model and QuickSCAT/NCEP mixed wind field. Simulation test 1 and test 3 were conducted by adopting the computational grid that is optimized by integrated use of the water depth data and high-resolution coastline data with the consideration of topographic subgrid-scale effect. Test 2 and test 4 directly adopted the ETOPO2 water depth data without considering topographic subgrid-scale effect. In Fig. 4a, the red point indicates the islands that cannot be distinguished by the 15’ computational grid and where the subgrid scale is adopted. In order to compare with the observed wave data of the Ryukyu Islands, Fig. 4b shows the distribution of islands in the nearby sea areas, where the blue point indicates the continental computational grid, the red point indicates the subgrid point, and the green square indicates the two wave measuring stations, NAHA Station and NASE Station.

Discussion

In the observed data of the two measuring stations, the significant wave height was at an interval of 2 hours. The error of the simulated significant wave height of the two stations in January and October of 2004 was calculated with the data of test 1, test 2, test 3 and test 4. Results are shown in Table 1 the skewness of significant wave height, root mean square error, mean absolute error, mean squared relative error, and average relative error of test 1 and test 3 were obvious lower than test 2 and test 4 respectively. As a result, the skewness of test 1 was lower than that of test 2 by about 0.42 m, and the average relative error decreased by about 18%; the skewness of test 3 was lower than that of test 4 by 0.31 m, and the average relative error decreased by
about 12%. This indicates that the use of computational grid optimized by integrated use of water depth data and high-resolution coastline data with the consideration of topographic subgrid-scale effects has greatly improved the precision of simulated significant wave height. This can also be demonstrated in the scatter plot of simulated and observed data of significant wave height at the two stations: the simulated significant wave heights (green line) of test 2 and test 4 were obvious larger than the observed value, while the simulated data of test 1 and test 3 (blue line) were lower than that of test 2 and test 4 and closer to the observed values. Therefore, in this two buoy stations, the simulated results of test 1 and test 3 (blue line) were superior to that of test 2 and test 4 (green line). Reasons are as follows: As Fig. 4b shows, the area of Kerama-syotou in the western sea areas of the NAHA Station is so small, and the grid interval of numerical test is 15’. Computational gridpoints of the nearby sea areas are all "wet", which fail to reflect the influence of terrain features on computation of waves. Same situation applies to part of the small Tokara Islands in the north of NAHA Station and in the northeast of NASE Station. Thus this is one of the major reasons that result in large error of simulated significant wave height in test 2 and test 4. By adopting the computational scheme of complex multi-island terrain, many grid points considering the topographic subgrid-scale effects were found in the sea area around these islands. As the red point shows in Fig. 5b, the error of simulated significant wave height in test 1 and test 3 was greatly decreased. That is because the obstacle effect of these small islands to waves has been taken into consideration. Figure 8 shows the distribution diagram of monthly average difference of significant wave height of the two tests within the area. Results showed clear negative systematic deviation after using the method in this paper. Largest average difference of significant wave height in Figure 8a was around 0.6 m in January, or 0.8 m in October in Fig. 8b. Compared to Fig. 4, the places with obvious average monthly difference of significant wave height were concentrated near the subgrid points. Distribution of monthly average difference of significant wave height in Fig. 8a & b was basically similar, in spite of the slight difference. Therefore, the island subgrid-scale effect has obvious influence on error of simulated significant wave height, though the influence is local. This may be caused by obstruction by island subgrid-scale effect of wave propagation. There is some inconsistency in the distribution of average monthly difference of significant wave height between January and October, which may result from the different prevailing sea surface wind in the two months and different wave propagation.

<table>
<thead>
<tr>
<th>Test</th>
<th>Skewness (Unit: m)</th>
<th>Root mean square error (Unit: m)</th>
<th>Mean absolute error (Unit: m)</th>
<th>Root-mean relative error</th>
<th>Average relative error</th>
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</thead>
<tbody>
<tr>
<td>Test 1</td>
<td>0.059</td>
<td>0.271</td>
<td>0.250</td>
<td>0.178</td>
<td>0.164</td>
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<tr>
<td>Test 2</td>
<td>0.480</td>
<td>0.534</td>
<td>0.528</td>
<td>0.350</td>
<td>0.346</td>
</tr>
<tr>
<td>Test 3</td>
<td>0.051</td>
<td>0.629</td>
<td>0.422</td>
<td>0.352</td>
<td>0.236</td>
</tr>
<tr>
<td>Test 4</td>
<td>0.361</td>
<td>0.859</td>
<td>0.635</td>
<td>0.481</td>
<td>0.355</td>
</tr>
</tbody>
</table>
Fig. 6a & b—Comparison between simulated and observed data of significant wave height of the two stations in January

Fig. 7a & b—Comparison between simulated and observed data of significant wave height of the two stations in October
Conclusions

In archipelagic sea area, a different computational scheme of complex multi-island scheme is one of the major sources of error in wave numerical simulation. Water depth data and high-resolution coastline data can be utilized in combination with the consideration of topographic subgrid-scale effect to optimize the computational grid. The land and sea are distinguished by "dry" or "wet" grid. By this method, the obstacle effect of islands smaller than the scale of computational grid to the waves can be better reflected. January and October when two typical weather processes of cold air and typhoon occur in Ryukyu Islands were selected for numerical simulation for one month, respectively. The simulated results were compared with the measured data. It was indicated that our computational scheme of the complex multi-island terrain can improve the precision of wave simulation and reduce the mean relative error of simulated significant wave height by about 12%-18%. Island subgrid-scale effect has obvious effect on the error of simulated significant wave height in archipelago, but the influence is local. This may due to the obstruction by island subgrid-scale effect of swell propagation.

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Reference