A theoretical model to predict the thermal resistance of plain woven fabrics

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A single cell woven fabric model has been developed to predict the thermal resistance in a non-convective environment. The values from the model have been compared with those obtained from Alambeta in which the fabric is confined between two plates at a pressure of 200 Pa, thus having the negligible natural convection and no forced convection. The model gives good correlation with the experimental values.

Keywords: Alambeta, Conduction, Cotton, Thermal resistance, Woven fabric

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1 Introduction

Heat and mass transfer through textiles affect their comfort characteristics. Most of the work done in this field is mainly related to (i) understanding different modes of heat transfer; (ii) study of different fabric parameters which are important for effective fabric insulation; (iii) use of new materials which facilitate the use of garments for extreme as well as moderate climatic conditions; and (iv) development of new methods and mathematical models to determine thermal properties of textiles.

Many researchers have tried to obtain the contribution of various modes of heat transfer for determining the thermal insulation of fabric. Most of them have purported that the heat transfer through textiles takes place by conduction, convection and radiation. The consideration of each mode is different from one researcher to another. A few have combined two different modes of heat transfer and considered them as one. A number of workers have stated that the most important fabric factor which determines the thermal insulation of the fabric is fabric thickness; the magnitude of the thickness depends upon the pressure applied while carrying out the measurements. Black and Mathew have investigated the plain woven fabrics with different thread densities and found that the thermal insulation decreases with increasing thread density. To build a mathematical model, the fabric is considered as a porous structure. The effective conductivity for packed beds was predicted by many researchers, depending upon various approaches.

Chang has given a theoretical model based on combined series and parallel conduction for the effective thermal conductivity of wire mesh. A unit cell model was employed to simulate the mesh and the solid-fluid lumped volumes were taken to predict the effective thermal conductivity. Luikov et al. have considered an elementary cell to be a system of resistances and by using the analogy of electrical system they predicted the effective conductivity of the porous system. In all the cases, the fabric has been considered either as a porous layered sheet or a fibrous batting. In this study, an attempt has been made to find out a theoretical prediction of thermal resistance for plain woven fabrics in a non-convective environment.

2 Materials and Methods

2.1 Theoretical Model

Thermal transfer from the skin to environment through the fabric takes place individually by conduction, convection and radiation. The total heat transfer of the fabric is the algebraic sum of the amounts of heat transfer through each of these modes, which is given by the following equation:

\[ Q_{\text{total}} = Q_{\text{conduction}} + Q_{\text{convection}} + Q_{\text{radiation}} \]  \hspace{1cm} (1)

If the fabric is kept in a confined environment under a fixed pressure, the effects of forced convection can be neglected. The air inside the fabric can be assumed to be another insulating material with thermal conductivity and density values same as that of air. Therefore, the heat transfer through conduction and radiation only is considered.
2.1.1 Heat Transfer Through Plain Woven Fabric Structure

Consider a single cell of the plain woven fabric (Fig. 1). When observed closely, it can be found that there are mainly three areas from which conduction takes place: (i) through warp and weft yarn; (ii) through interlacements; and (iii) through air pores. The basic equation of conductivity as per the Fourier's Law is given below:

\[
\frac{dQ}{dr} = -kA \frac{dT}{dx}
\]

where \(dQ\) is the quantity of heat conducted in time \(dr\); \(T\), the temperature; \(x\), the thickness of the wall perpendicular to the direction of heat; \(A\), the area of the heated surface; and \(k\), the coefficient of thermal conduction.

When Eq. (2) is integrated over a time \(\tau\), then under the steady state conditions

\[
Q_{\text{condution}} = kA \left( \frac{T_1 - T_2}{\tau} \right)
\]

where \(T_1\) and \(T_2\) are the hot and cold temperatures respectively; and \(\tau\), the thickness of the material. Equation (3) can be used to calculate the total heat transfer due to conduction.

It can be observed from Fig. 2 that the interlacement regions consist of warp and weft yarns as well as air. They can be depicted in terms of series resistances. The yarns are also flattened (Fig. 3) so a single plain woven cell can be assumed to have a warp, a weft, an interlacement and an air gap. Based on Eq. (3), the conduction through various sources is given by the following equations:

\[
Q_{\text{cond.warp}} = \frac{\left( (p_2 - a_2) a_1 \right)\left( T_1 - T_2 \right)}{b_1 \text{sec} \theta_1 - t - b_1 \text{sec} \theta_1 \left( \frac{k_{\text{warp}}}{k_{\text{air}}} \right)}
\]

\[
Q_{\text{cond.wft}} = \frac{\left( (p_1 - a_1) a_2 \right)\left( T_1 - T_2 \right)}{b_2 \text{sec} \theta_2 - t - b_2 \text{sec} \theta_2 \left( \frac{k_{\text{wft}}}{k_{\text{air}}} \right)}
\]

where 1, 2 suffixes are meant for warp and weft respectively except for \(T_1\) and \(T_2\) which mean hot and cold plate temperatures; \(p\), the thread spacing; \(a\), the flattened diameter; \(b\), the flattened thickness; \(\theta\), the weaving angle; and \(\tau\), the thickness of fabric.

\[
Q_{\text{cond.int}} = \frac{(a_1 a_2)\left( T_1 - T_2 \right)}{k_{\text{warp}} + k_{\text{wft}} + k_{\text{air}}}
\]
Conduction through air pore
\[ Q_{\text{cond.pore}} = k_{\text{air}} \left( (p_1 - a_1)(p_2 - a_2) \right) \] ... (7)

Total heat transfer due to conduction
\[ Q_{\text{cond.tot}} = Q_{\text{cond.warp}} + Q_{\text{cond.weft}} + Q_{\text{cond.jn}} + Q_{\text{cond.pore}} \]

Conduction per unit area
\[ Q_{\text{condution}} = \frac{Q_{\text{cond.tot}}}{P_1P_2} \] ... (8)

Hence, the resistance of the fabric due to conduction only is given by the following equation:
\[ R_{\text{condution}} = \frac{(T_1 - T_2)}{Q_{\text{condution}}} \text{ K.m}^2/\text{W} \] ... (9)

2.1.2 Radiative Heat Transfer

If the same single cell model of fabric is considered, there are three ways in which heat transfer due to radiation takes place: (a) from skin to environment through air pores; (b) from yarns to environment; and (c) between yarns and skin (Fig. 1).

The basic equation of radiation heat transfer between two plates is
\[ Q_{\text{radiation}} = \frac{\sigma(T_1 - T_2)^4}{R_i} \] ... (10)

where \( R_i \) is the total resistance encountered during net radiation transfer through two surfaces. This value has following two components:

(i) surface resistance \( R_{\text{surface}} \) which depends upon the emissivity and the area of the surface, as given by \( R_{\text{surface}} = (1 - \varepsilon_i)/A_i \); and

(ii) space resistance \( R_{\text{space}} \) which depends upon the view factor of the two surfaces, as given by \( R_{\text{space}} = 1/A_iF_{ij} \).

where \( \varepsilon_i \) is the emissivity of the surface i; \( A_i \) the area of the surface; and \( F_{ij} \) the view factor between surfaces i and j.

In case of radiation heat transfer directly to the atmosphere, the environment can be assumed to be an all-absorbing black surface whose temperature is equal to the ambient temperature \( T_{\text{amb}} \). In that case, only \( R_{\text{surface}} \) is relevant. Also, the view factor in case of parallel planes is 1.

Considering all these factors, Eq. (10) now becomes
\[ Q_{\text{radiation}} = \frac{\sigma(T_1 - T_2)^4}{1 - \varepsilon_i + \frac{1}{A_i\varepsilon_i} + \frac{1 - \varepsilon_j}{A_j\varepsilon_j}} \] ... (11)

Therefore, the heat transfers due to radiation through the plain woven cell are given as:

Through air pore
\[ Q_{\text{rad.pore}} = \sigma a_i (p_2 - a_2) + p_1 a_2 \left( \frac{T_1 - T_2}{\varepsilon_{\text{skin}}} \right) \] ... (12)

Between yarns and environment
\[ Q_{\text{rad.yarn}} = \sigma a_i (p_2 - a_2) + p_1 a_2 \left( \frac{T_1 - T_2}{\varepsilon_{\text{yarn}}} \right) \] ... (13)

Between the skin and yarns
\[ Q_{\text{rad.skyarn}} = \sigma a_i (p_2 - a_2) + p_1 a_2 \left( \frac{T_1 - T_2}{\varepsilon_{\text{skin}}} \right) \] ... (14)

Total heat transfer due to radiation
\[ Q_{\text{rad.total}} = Q_{\text{rad.pore}} + Q_{\text{rad.yarn}} + Q_{\text{rad.skyarn}} \] ... (15)

Total heat transfer due to radiation per unit area
\[ Q_{\text{rad}} = \frac{Q_{\text{rad.total}}}{P_1P_2} \] ... (16)

Hence, the resistance of the fabric due to radiation only is given by the following equation:
\[ R_{\text{rad}} = \frac{Q_{\text{rad}}}{Q_{\text{radiation}}} \] ... (17)

If the modes of heat transfer are made analogous to a parallel circuit system, then the overall resistance of the fabric is given by the following relationship:
\[ R_{\text{total}} = \left( R_{\text{cond}} \right)^{-1} + \left( R_{\text{rad}} \right)^{-1} \] ... (18)

2.1.3 Effective Yarn Diameter

When single yarns are woven into fabric, the yarn during interlacement tends to flatten. The woven fabrics were observed under microscope with a magni-
cation of x 40. It can be observed in Figs 4a and 4b that the yarns do not have the same flattened diameter throughout the fabric. The diameter is more at the crown of the interlacement and less in the rest of the body. It has been assumed that the effective diameter in this case is the average of flattened diameter and normal diameter, as given below:

\[
d_{\text{eff}} = \frac{d_{\text{yarn}} + d_{\text{yarn}}}{2} = d_{\text{yarn}} \left[1 + \frac{1}{2e}\right] \tag{19}
\]

where

\[
e = \frac{d_{\text{yarn}}}{a_{\text{yarn}}} = \frac{b_{\text{yarn}}}{d_{\text{yarn}}} \tag{20}
\]

where \(d_{\text{eff}}\) is the effective diameter of the yarn; \(d_{\text{yarn}}\), the theoretical diameter of the yarn; \(a_{\text{yarn}}\), the flattened diameter; \(b_{\text{yarn}}\), the flattened thickness; and \(e\), the degree of flattening.

In case of two-ply yarns, the resultant diameter, as per convention, is equal to 1.44\(d\) where \(d\) is the diameter of individual yarn. When a yarn is woven into a dense fabric, the yarns try to push each other to accommodate more yarns per unit space. This leads to rolling of the individual yarns on top of each other in a two-ply yarn. The diameter of the yarn then becomes equal to the diameter of individual yarns. Hence, Eq. (18) is followed in case of two-ply yarns as well except that the value of \(d_{\text{yarn}}\) is equal to the individual ply diameter.

2.1.4 Effective Thermal Conductivity of Yarn

From Fig. 5, it can be assumed that the solid-fluid matrix in the yarn complies with the system of series arrangement. In that case, the effective conductivity is given by

\[
k_e = \frac{k_s}{\phi k_f + 1 - \phi} \tag{21}
\]

where \(k_e\) is the effective conductivity; \(k_s\), the conductivity of the solid; \(k_f\), the conductivity of the fluid; and \(\phi\), the porosity of the yarn.

2.2 Experimental

Twelve plain woven fabrics of different constructions were taken. The fabric parameters, like warp and weft linear densities, thread densities, thickness and mass per unit area, are given in Table 1. These fabrics were tested for thermal resistance on Alambeta. A line diagram of the Alambeta instrument is given in Fig 6. The fabric is kept between hot and cold plates; the hot plate comes in contact with the fabric sample at a pressure of 200Pa. As soon as the hot plate touches the surface of the fabric, the amount of heat flow from the hot surface to the cold surface through the fabric is detected by heat flux sensors. There is also a sensor to measure the thickness of the fabric. These values are then used to calculate the thermal resistance of the fabric. This instrument also gives the transient heat transfer as well as \(Q_{\text{max}}\) values.
3 Results and Discussion

3.1 Comparison of Theoretical Model with Experimental Values

The values obtained from the single cell fabric model were compared with the resistance values obtained from Alambeta (Table 2). The average deviation between Alambeta and the theoretical model is 7.92% with a minimum and maximum deviation of 1.45% and 14.77% respectively. It can be seen from Fig. 7 that the $R^2$ value is high which indicates a strong correlation between the two values.

In this case, convective heat transfer is not taken into consideration as the two plates are separated only by the thickness of the fabric which is of the order of 1 mm and therefore, there is negligible natural convection. Furthermore, as there was no open air space above the fabric sample, forced convection could also be ruled out.

It can be seen from Fig. 8 that the thermal resistance values of some samples obtained by the experimental data are lower than those obtained from the model. These are mainly fabrics with 2-ply warp yarns. This is because the $d_{\text{yar}}$ in Eq. (18) is obtained from theoretical individual ply diameter. But when these fabrics are observed under the microscope, the diameters are much larger than $d_{\text{eff}}$ at various places. Hence, the conductive heat losses are much higher.

### Table 1—Specifications of plain woven cotton fabric samples

<table>
<thead>
<tr>
<th>Sample No.</th>
<th>Warp count Ne</th>
<th>Weft count Ne</th>
<th>Ends/pick inch</th>
<th>Picks/pick inch</th>
<th>Thickness mm</th>
<th>Fabric weight g/m²</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>39.0</td>
<td>2/28</td>
<td>142</td>
<td>56</td>
<td>0.32</td>
<td>173</td>
</tr>
<tr>
<td>2</td>
<td>38.0</td>
<td>30.0</td>
<td>140</td>
<td>80</td>
<td>0.26</td>
<td>150</td>
</tr>
<tr>
<td>3</td>
<td>39.0</td>
<td>38.0</td>
<td>140</td>
<td>94</td>
<td>0.22</td>
<td>144</td>
</tr>
<tr>
<td>4</td>
<td>36.0</td>
<td>2/70</td>
<td>144</td>
<td>92</td>
<td>0.24</td>
<td>144</td>
</tr>
<tr>
<td>5</td>
<td>37.0</td>
<td>19.0</td>
<td>136</td>
<td>68</td>
<td>0.31</td>
<td>161</td>
</tr>
<tr>
<td>6</td>
<td>37.0</td>
<td>20.0</td>
<td>140</td>
<td>66</td>
<td>0.3</td>
<td>166</td>
</tr>
<tr>
<td>7</td>
<td>38.0</td>
<td>36.0</td>
<td>132</td>
<td>70</td>
<td>0.25</td>
<td>126</td>
</tr>
<tr>
<td>8</td>
<td>2/36</td>
<td>2/60</td>
<td>122</td>
<td>56</td>
<td>0.39</td>
<td>191</td>
</tr>
<tr>
<td>9</td>
<td>2/38</td>
<td>28.0</td>
<td>120</td>
<td>56</td>
<td>0.37</td>
<td>192</td>
</tr>
<tr>
<td>10</td>
<td>2/38</td>
<td>2/40</td>
<td>118</td>
<td>50</td>
<td>0.41</td>
<td>197</td>
</tr>
<tr>
<td>11</td>
<td>2/38</td>
<td>20.0</td>
<td>114</td>
<td>50</td>
<td>0.41</td>
<td>197</td>
</tr>
<tr>
<td>12</td>
<td>2/38</td>
<td>2/30</td>
<td>116</td>
<td>42</td>
<td>0.43</td>
<td>213</td>
</tr>
</tbody>
</table>

Fig. 6 – Line diagram of Alambeta
Table 2—Errors between the thermal resistance values obtained from Alambeta and the theoretical model

<table>
<thead>
<tr>
<th>Sample No.</th>
<th>Alambeta model</th>
<th>Theoretical model</th>
<th>Deviation*</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0088</td>
<td>0.0093</td>
<td>5.68%</td>
</tr>
<tr>
<td>2</td>
<td>0.0083</td>
<td>0.0075</td>
<td>9.64%</td>
</tr>
<tr>
<td>3</td>
<td>0.0072</td>
<td>0.0062</td>
<td>13.89%</td>
</tr>
<tr>
<td>4</td>
<td>0.0069</td>
<td>0.007</td>
<td>1.45%</td>
</tr>
<tr>
<td>5</td>
<td>0.0094</td>
<td>0.009</td>
<td>4.26%</td>
</tr>
<tr>
<td>6</td>
<td>0.0091</td>
<td>0.0087</td>
<td>4.40%</td>
</tr>
<tr>
<td>7</td>
<td>0.0088</td>
<td>0.0075</td>
<td>14.77%</td>
</tr>
<tr>
<td>8</td>
<td>0.0095</td>
<td>0.0106</td>
<td>11.58%</td>
</tr>
<tr>
<td>9</td>
<td>0.0098</td>
<td>0.0102</td>
<td>4.08%</td>
</tr>
<tr>
<td>10</td>
<td>0.0098</td>
<td>0.011</td>
<td>12.24%</td>
</tr>
<tr>
<td>11</td>
<td>0.0102</td>
<td>0.011</td>
<td>7.84%</td>
</tr>
<tr>
<td>12</td>
<td>0.0114</td>
<td>0.012</td>
<td>5.26%</td>
</tr>
</tbody>
</table>

*Average, 7.92; min, 1.45; and max, 14.77

Fig. 7 – Correlation between the values obtained from Alambeta and the theoretical model

than the theoretical approximation, because of the increased surface area. Therefore, the thermal resistance in these cases is less.

4 Conclusions

The single cell plain woven model gives a good prediction of the thermal resistance of plain woven fabrics. The correlation obtained with experimental values is found to be high. Convective heat transfer has not been taken into account in this case as the fabric is held between two plates at a pressure of 200 Pa.

Fig. 8 – Comparison of thermal resistances obtained from the theoretical model and Alambeta

References