Diagonal cracks of structural concrete beams under short-term load with unloading

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Characteristics of the process of diagonal cracking have been studied and their evaluation based on the experimental investigation of reinforced and prestressed concrete beams subjected to short term load with unloading has been carried out. The relations between shear deformations and the characteristics of cracking are explained in the paper. The relationship between the reversible part of diagonal crack opening density and the diagonal crack opening density immediately before unloading is also presented.

Considerable attention is paid to the limitation of crack widths in view of their unfavourable effects on the serviceability and durability of concrete structures. This is observed from the fact that R&D efforts have been directed towards reliable estimation of crack widths and subsequently to the design of construction rules limiting their widths. Mostly the development of vertical (bending) cracks has been studied. The issue of diagonal cracks has not been addressed adequately because it has been assumed during the design stage of a structural element, that the width of diagonal crack would be restricted by the reinforcement. Still, in some cases, such as deep or slender beams, the diagonal cracks are not sufficiently arrested by means of reinforcement. Therefore, in such cases, several studies have been concerned with the more detailed investigation of diagonal cracking. In cases, when a beam is subjected to moving load (e.g. bridges, rails, beams in movable structures like cranes and gantries), intersections of inclined cracks take place even in the middle part of the beam. An experimental research on the influence of moving load on the deformational properties of concrete members has been carried out at the Institute of Construction and Architecture in Bratislava. As a logical extension of the above research an experimental investigation of the reinforced and partially prestressed beams under gradually increasing load with unloading branches has been carried out. The tests were mainly concerned with the study of the relation between the elastic deformations and the deformations immediately before unloading. During this study, development of vertical and diagonal cracking was observed and studied. This paper deals with the studies related to the diagonal cracking of reinforced and prestressed concrete members.

Experimental Procedure

The diagonal cracks were evaluated from the test results of three structural concrete (SC) beams with I and T cross-sections. The span/depth ratio (l/h) of the I-beams was 7.5, and that of the T-beam was 11.4. One of the concrete I-beams (I-PCB) was partially prestressed. The degree of prestressing was 0.614 (ref. 8). Details of the cross-sections of the tested beams, reinforcement and side-views are shown in Fig. 1.

Average material characteristics of the deformed bars of the non-prestressing reinforcement were: yield strength $f_y = 454$ MPa, tensile strength $f_t = 649$ MPa, and modulus of elasticity $E_s = 213$ GPa. The characteristics of prestressing tendons with the nominal diameter of $12.5$ mm were: $f_y = 1700$ MPa, $E_s = 202$ GPa. The average mechanical characteristics of the concrete are given in Table 1.

Simply supported beams were tested under a concentrated force $F$ in the middle of the span. The force was induced by means of the two tie rods which connected a steel rolled I-beam with the hydraulic jack placed under the tested beam. The test set-up is shown in Fig. 2. The displacement rate control loading was applied on both the loading as well as unloading branches. For the control value, step-like increase of the displacement of the order of $4$ mm of the loading of hydraulic jack was taken. The measurements (deformations of concrete, cracks development) on the all loading and unloading branches of a beam were performed at the loading steps with the same deflec-
Fig. 1—Cross-section, reinforcement of tested elements and position of measuring bases

Table I—Mean values of strengths of concrete and moduli of elasticity

<table>
<thead>
<tr>
<th>Designation of beams</th>
<th>Compressive strength on cubes [MPa]</th>
<th>Direct tensile strength on prisms [MPa]</th>
<th>Modulus of elasticity [GPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>I-RCB</td>
<td>55.32</td>
<td>2.42</td>
<td>40.83</td>
</tr>
<tr>
<td>I-PCB</td>
<td>52.34</td>
<td>2.35</td>
<td>37.81</td>
</tr>
<tr>
<td>T-RCB</td>
<td>41.43</td>
<td>2.09</td>
<td>39.74</td>
</tr>
</tbody>
</table>

Fig. 2—Test set-up

Fig. 3—Loading force versus deflection relationship of beam I-PCB

reinforced concrete specimens (I-PCB and I-RCB), respectively.

At every control displacement level, the strains of continuously linked-up base-lines at the compressed and tensioned edges as well as at the crossing diagonals were registered. The metal plugs embedded into the surface layer of the beam served to measurements of the distance changes between two adjacent ones by means of the mechanical strain gauges. The base-lines at the top and bottom edges were of the nominal length $s = 360$ mm and their vertical distance was $v = 460$ mm. The base-lines formed an imaginary truss consisting of "struts" and "ties" enabling the calculation not only of the strains but also of the deflections. The network of base-lines (Fig. 1) was decomposed into two truss systems. The metal plugs were located in the vertices of trusses. The measurements of elongations between two adjacent metal plugs were then evaluated by using a method based on Williot-
Mohr translocation polygons (in numerical form). The input data consisted of co-ordinates of vertices of a truss, elongations of struts, and of an incident matrix formed by the number of vertices adjacent to a given vertex. From the input data, the vertical and horizontal displacement of the vertices (deflections) were calculated. The widths of cracks were registered at the two (or four) loading steps before unloading and after unloading to the zero level at each loading cycle. The widths of vertical and diagonal cracks were measured at the position of tensile reinforcement and in the middle of the web, respectively. The following characteristics of cracking were evaluated, namely: the crack density, the crack opening density, and the crack widths. The detailed loading procedure has been reported earlier.  

Results and Discussion  
In accordance with the theory of elasticity, the differential equation of the deflection line due to shear forces is

$$\frac{da_y}{dx} = \gamma_m$$

(1)

where \(\gamma_m\) is the shear deformation resulting from the relative displacement of two adjacent cross-sections at the distance \(dx\). According to the known relations the shear deformations may be calculated as follows:

$$\gamma_m = \frac{\kappa V}{GA}$$

(2)

where \(V\) is the shear force, \(G\) is the shear modulus of elasticity, \(A\) is the cross-sectional area and \(\kappa\) is the non-dimensional coefficient depending on the shape of the cross-section. The quotient of the shear force and shear deformation \(V/\gamma_m\), i.e., the expression \(GA/\kappa\) represents the shear stiffness \(B_q\). However, the calculation of the value of shear stiffness after cracking is more complicated.

Following the method of measuring the deformations as described, the deflections of the beams at specified locations are computed by separating the effects of bending moments and those of shear forces. In a similar manner, vertical displacements (deflections) \(a_q\) due to shear force \(V\) for each node of the bottom chord of the imaginary truss, and subsequently the shear deformations \(\gamma_n\) as a difference between two adjacent deflections \(a_{q,i} - a_{q,i-1}\) divided by the length of the base line \(l\), are obtained. The experimental values of shear stiffness \(B_q\) of the beam I-RCB versus the shear force \(V\) are plotted in Fig. 4. The course of this relationship may be quite satisfactorily approximated by the trilinear regression (dashed lines). The first part represents the stage up to cracking, the second part corresponds to the formation of diagonal cracks, and the third part approximates the stabilised stage where only the crack widths increase. As a matter of fact, the shear stiffness has to be constant up to cracking but the scatter of results in this part is large as the measured values are relatively small. It seems to be suitable to replace also the third part of the relationship representing the stabilised cracking with a constant. The value of this constant may represent the minimum shear stiffness related to the service load. Similar approach of calculation of shear stiffness has been used as reported earlier. The shear deformations after cracking may be also determined as specified elsewhere. More detailed description of the CEB method is reported earlier.
the shear deformation and shear force for beam I-RCB is shown in Fig. 5.

The next step is to find the relation between the shear deformations and the crack widths. In order to obtain such a relation, the diagonal crack opening density $\alpha_{cr,d}$ was evaluated, which is the sum of widths of diagonal cracks occurring between two diagonals of the imaginary truss divided by the length of the diagonal. Even if the cracks were, as a rule, not perpendicular to the direction of the diagonals, the difference of inclinations were small and these could be neglected. To mitigate the effect of the choice of the base lines, the moving average method of second degree was applied in the evaluation of both the shear deformations as well as the diagonal crack opening densities. The diagonal crack opening densities as a function of shear deformations of the beam I-RCB are plotted in Fig. 6. For approximation of the experimental values the regression straight line of the shape $y = ax + b$ was applied. Similar relationships were obtained for the beams I-PCB and T-RCB. The parameters of the regression straight lines are given in Table 2 together with the correlation coefficients for the three beams.

As in the case of vertical cracks, the diagonal crack opening density is related to the average crack width. But the crack width limit is usually applied to the maximum crack width. Therefore, the relationship between the diagonal crack opening density and the maximum diagonal crack width was established from the tests. Unlike the linear dependence between the vertical crack opening density and maximum vertical crack width, the relationship between the diagonal crack opening density and the maximum diagonal crack width cannot be approximated by a straight line. The third degree polynomial of the shape $y = ax^3 + bx^2 + cx$ was applied. As earlier, the span/depth ratio of the tested beams or their reinforcement were different and accordingly also the ratio of shear force induced deflection to the total deflection differed considerably (Table 3). Despite these differences, it was possible to apply one regression curve for all the three beams. Its parameters of regression were as follows:

$$w_{cr,d,\text{max}} = 2.83E - 08\alpha_{cr,d}^3 - 1.993E - 04\alpha_{cr,d}^2 + 0.5115\alpha_{cr,d}$$

... (3)

where $\alpha_{cr,d}$ is diagonal crack opening density and $w_{cr,d,\text{max}}$ is maximum width of diagonal cracks.

<table>
<thead>
<tr>
<th>Designation</th>
<th>Parameters of regression straight lines $y = ax + b$</th>
<th>Correlation coefficient $r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I-RCB</td>
<td>$a = 655.85$, $b = -121.17$</td>
<td>$r = 0.9251$</td>
</tr>
<tr>
<td>I-PCB</td>
<td>$a = 532.53$, $b = -99.52$</td>
<td>$r = 0.8097$</td>
</tr>
<tr>
<td>T-RCB</td>
<td>$a = 400.94$, $b = -163.75$</td>
<td>$r = 0.5841$</td>
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</table>

Table 3—Ratio of the shear force induced deflections to the total deflection

<table>
<thead>
<tr>
<th>Designation</th>
<th>Ratio $\alpha_{d}/\alpha_{cr,d}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I-RCB</td>
<td>Service load: $0.30$, Maximum load: $0.342$</td>
</tr>
<tr>
<td>I-PCB</td>
<td>Service load: $0.19$, Maximum load: $0.280$</td>
</tr>
<tr>
<td>T-RCB</td>
<td>Service load: $0.08$, Maximum load: $0.118$</td>
</tr>
</tbody>
</table>

Fig. 6—Diagonal crack opening density versus shear deformations

Fig. 7—Maximum diagonal crack width versus diagonal crack opening density
correlation coefficient was \( r = 0.9655 \). The experimental values of maximum diagonal crack width against the diagonal crack opening density and regression polynomial are shown in Fig. 7.

As the beams were tested for unloading it was possible to establish the relationship between the diagonal crack widths immediately before unloading and those after unloading to zero level. The loading steps with measured crack widths are illustrated in Fig. 8, where the relationship between the loading force and the sum of diagonal crack width of the beam I-RCB is plotted. The diagonal crack widths measured at the loading steps immediately before unloading and after unloading to zero level and consequently the diagonal crack opening density known for each base line provided sufficient amount of data to investigate the relationship between the total and reversible diagonal crack opening densities. This relationship shows relatively small scatter and could be approximated by a straight line with sufficient accuracy. The parameters of regression straight lines are given in Table 4 both for reversible and irreversible part of diagonal crack opening density. As the values of parameters \( a_{\text{rev}} \) (\( a_{\text{irrev}} \)) differed only slightly, it was possible to approximate the results of all three beams with a straight line as follows:

\[
\alpha_{\text{cr,d,rev}} = 0.76\alpha_{\text{cr,d,before-unloading}} \quad \ldots (4)
\]

The correlation coefficient \( r \) was 0.9887. In the case of the irreversible part of diagonal crack opening density the scatter of results is bigger \( (r = 0.8773) \) because the measured values are small. In Fig. 9, the reversible part of diagonal crack opening density is plotted against the diagonal crack opening density immediately before unloading for the three beams. In Fig. 10 the relationship of the irreversible diagonal crack opening density versus the diagonal crack opening density immediately before unloading for the three beams is shown. The linear relationship between the reversible/irreversible diagonal crack opening density and the diagonal crack opening density before unloading are in a good agreement with the results.

**Conclusions**

On the basis of experimental results, it may be concluded that there is a relationship between the shear

<table>
<thead>
<tr>
<th>Designation of beams</th>
<th>Reversible part</th>
<th>Irreversible part</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Parameter of regression straight lines</td>
<td>Correlation coefficient</td>
</tr>
<tr>
<td>I-RCB</td>
<td>( a_{\text{rev}} ) = 0.7741</td>
<td>( r_{\text{rev}} ) = 0.9908</td>
</tr>
<tr>
<td>I-PCB</td>
<td>( a_{\text{rev}} ) = 0.7459</td>
<td>( r_{\text{rev}} ) = 0.9833</td>
</tr>
<tr>
<td>T-RCB</td>
<td>( a_{\text{rev}} ) = 0.7280</td>
<td>( r_{\text{rev}} ) = 0.9866</td>
</tr>
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</table>

![Fig. 8—Loading force versus diagonal crack widths](image1)

![Fig. 9—Reversible part of the diagonal crack opening density versus diagonal crack opening density before unloading](image2)
deformation and the diagonal crack opening density which may be expressed by a straight line. From the diagonal crack opening density the maximum width of diagonal cracks may be obtained by means of the Eq. (3). However, to generalise the parameters of relationships between the above-mentioned quantities would require more experiments.

The reversible part of the diagonal crack opening density can be determined in accordance with the Eq. (4) as a multiple of the diagonal crack opening density immediately before unloading. The coefficient $\alpha = 0.76$ of the Eq. (4) corresponds with the one derived for the relationship between the elastic and total deflections.

Acknowledgement
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References

Nomenclature
- $A$ = area of the cross section
- $a$ = parameter of regression functions
- $a_d$ = deflection due to shear forces
- $a_{tot}$ = total deflection in the middle of the span
- $b$ = parameter of regression functions
- $c$ = parameter of regression functions
- $dx$ = relative displacement of two adjacent cross sections
- $E_c$ = modulus of elasticity of concrete
- $E_p$ = modulus of elasticity of prestressing reinforcement
- $E_{nc}$ = modulus of elasticity of non-prestressing reinforcement
- $F$ = loading force
- $F_{max}$ = capacity of loading equipment
- $f_{cu}$ = cube strength of concrete
- $f_{ct}$ = tensile strength of concrete
- $f_{pm}$ = prism strength of concrete
- $f_{ps}$ = tensile strength of prestressing reinforcement
- $f_y$ = yield stress of non-prestressing reinforcement
- $f_{ns}$ = tensile strength of non-prestressing reinforcement
- $G$ = modulus of elasticity in shear
- $h$ = depth of beams
- $l$ = span of beams
- $l_0$ = length of the base line
- $r$ = correlation coefficient
- $V$ = shear force
- $w_{cr,i}$ = width of the $i$-th diagonal crack
- $w_{cr,max}$ = maximum diagonal cracks width in a base line
- $\alpha_{cr,d}$ = diagonal crack opening density
- $\gamma_s$ = shear deformation
- $\kappa$ = non-dimensional coefficient depending on the shape of the cross-section
- $\lambda$ = loading level